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Modeling and Measuring of the Oriental Musical Scale

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Summary
After a brief survey of the western system, we present a study of the oriental musical scale, based on the (diatonic) models built by European mathematicians. The main feature being the presence of the three-quarter-tone interval or medium third, we explain why a modeling by an equal-tempered scale of 24 quarter-tones of exactly 50 cents is not suitable. Our model starts with a combination of equal and pythagorean scales, displayed as a series of whole-tone and ¾-tone intervals. Practical music performance on lute and violin, when measured, show current pitches higher than equal-tempered theory would suggest. Some empirical adjustments of about 15 cents were needed to reach these practical pitches. Checking of some of the most common oriental keyboards proves that their ¾-tones are not in good accordance with real performance.

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1. Introduction

"Oriental" refers to music and tonalities used by the arabs, turks, persians, kurds and other peoples of the Middle-East and Central-Asia. They are based on the same scale. "Oriental keyboards" refer to keyboards designed to offer quarter tone intervals.

The oriental musical scale, unlike the European/Western [1, 2], has not received an extensive study from an acoustical or mathematical viewpoint. In the past, ancient music theorists tried to build some empirical methods, but they did not have the necessary mathematical tools. Others, in the 19th and 20th centuries, were more practicians than theorists and have neglected the scientific approach. All their scales lack rigor and none has succeeded in being adopted as a standard. Nowadays, only keyboard makers are concerned with this problem but are not inclined to carry out deep research to address the problem. Few works in English language deal with this subject; one interesting reference is a manuscript by C. Forster on the Web [3].

The purpose of this paper is a scientific study of the oriental musical scale, with particular attention to the comparison between the theoretical and empirical value of the so-called ¾ tone interval (often called minor tone in western writings), whose frequency ratio has never been theoretically determined with enough accuracy and which, empirically, is slightly different according to regions and styles.

The Greeks defined the western musical scale in an acoustical/mathematical way and since the 16th century several physical studies have been carried out that gave birth to different models (Zarlino, mean-tone, equal-tempered scale, Kirnberger, Rameau, Werckmeister, Holder ...). Middle-East peoples use something similar, but in practice they perform some notes not existing in the theoretical scale. Their lute, unlike the European version, was not fretted, so it allowed pitches such as E♭̸ located between Eb and E and divided this semitone into two smaller intervals roughly equal to one quarter of a tone. This reminds us of the blues tonalities used in American black music. These notes, found in an empirical way, do not have precisely defined frequencies.

We have been inspired by the Western model to build an oriental scale based on a combination of equal-tempered and Pythagorean temperaments. The resulting minor tones are well defined, their frequencies are accurately evaluated, and the values are compared with some existing scales and with practical performance.

2. The Western/European musical scale (survey)

The diatonic musical scale is a series of 7 notes or pitch classes, from low to high. The name of these 7 notes varies according to countries, the most used are Do, Re, Mi, Fa, Sol, La, Si in latinic, arabic and middle-eastern countries. For the anglo-saxon countries, they are designated as C, D, E, F, G, A, B.

Since the Ancient Greeks, an equivalence has been set which links the interval between two notes and the string lengths sounding these two notes: if you divide a string in two equal parts you will obtain the octave, in three equal parts the (perfect) fifth, in 9⁄8 a tone, etc.... Otherwise, if

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2 string lengths l_1 and l_2 give 2 sounds of frequencies f_1 and f_2, we get $l_1/l_2 = f_2/f_1$. Music theorists use the cent, a unit which defines the interval between these 2 notes as:

$$I = 1200 \log_2 f_2/f_1 = 1200 \log_{10}(f_2/f_1)/\log_{10} 2.$$

So, an equal-tempered half tone has 100 cents, and an octave 1200 cents. To raise the pitch by one cent we must multiply its frequency by $2^{1/1200} \approx 1.000578$.

2.1. Pythagorean scale

Starting from any note (C for instance), we can form the ascending fifth with another note whose frequency is 1.5 times higher (G). Another fifth gives us D ($1.5 \cdot 1.5 = 2.25$), etc… Continuing, we obtain a set of 7 notes whose frequencies brought inside the span of an octave are shown Table I.

If we continue this procedure beyond the 7th note by forming another circle of fifths, we will find a new set of notes higher than the first one by about half a tone. These notes are expressed by C#, G#, D#, A#, E#, B#, F#. Notes designated as E# and B# are very slightly higher than F and C whereas they must be their enharmonics. The difference

$$1200 \log_2(1.5^{12}/2^7) = 23.5\,\text{cents}$$

is about one ninth of a tone and is known as a (Pythagorean or ditonic) comma.

The Pythagorean scale is the basis of all musical systems. Except E–F and B–C, the interval between 2 successive notes is 9/8 = 1.125 or 203.9 cents, and this defines the (Pythagorean) “whole-tone”. E–F and B–C both have a ratio of 256/243 ≈ 1.0535 (90.2 cents), which is slightly smaller than half a tone and is called a “limma”. This “anomaly” is the cause and origin of the existence of several different tuning systems; the most common is the Equal Temperament used for keyboard and fretted instruments.

2.2. Equal temperament

Very suitable for keyboard and fretted instruments, the equal tuning has diatonic and chromatic semi-tones that are all equal. One octave is then composed of 12 equal semitones, whose height is expressed as the twelveth root of 2.

$$\sqrt[12]{2} = 2^{1/12} \approx 1.0594631 \quad \text{or}$$
$$I = 1200 \log_2(2^{1/12}) = 1200 \cdot \frac{1}{12} = 100\,\text{cents}.$$

2.3. Holderean scale (also called Holder-Kaufmann scale [4, 5])

The 12-tone equal tuning is not so appreciated by string performers who want more freedom to choose the pitch of their sounds and prefer a chromatic semi-tone higher than a diatonic one (E#>F). The Holderean scale can accomodate this preference and has been favourite by some oriental theorists. Designed in Europe in the 18th century, it is often mentioned by the German and the French, but fully ignored in the English writings.

The Holderean comma (or turksent) is a microtone defined as exactly the ninth part of a tone or the fourth part of a limma: the octave will then be divided into 53 equal commas whose ratio is

$$\sqrt[53]{2} = 2^{1/53} = 1.013164 \quad \text{or} \quad 22.6\,\text{cents}.$$

This yields 203.8 cents for a whole-tone, 90.57 for a diatonic semitone and 113.2 for a chromatic semitone.

Holder and Pythagoras temperaments have the same diatonic scale, the difference appearing only in chromatic notes.

More important is the comparison between Pythagoras (strings) and Equal (keyboards) systems. They are based on two deeply different concepts but yield a quasi-identical diatonic scale; the gap between two similar notes and their average is at most 0.17% (except for E natural and B natural, not of concern in our modeling § IV). A significant difference however appears when it comes to the chromatic scale.

3. The Oriental musical scale

The main feature of oriental (Arabic, Turkish, Persian, Kurdish …) music is the existence of the ¾-tone interval (sometimes called a minor tone), which results in a great variety of tonalities or modes (maqam in arabic as well as in turkish, dastgah in persian) and which can be found in almost all oriental works. Referring to the half or semi-flat by the sign ƀ (denoted 𝄳 or 𝄲 in oriental sheets, the semi-sharp exists also but it’s scarcely used: ≠[1]), the rast tuning [6] will be the basis of our study:

C, D, Eƀ, F, G, La, Bƀ, C.

There has never been an accurate definition of this interval. It was discussed by ancient arab theorists (Al-Kindi, Al-Farabi, Avicenna, Safyuddin or Safi Addin) from the 9th to the 13th century. Several varieties of this flat accidental (decreasing the pitch by 20, 30 or 40 percent of a tone and denoted 𝄳, 𝄳 or 𝄳 [6]) existed untill the Conference of Cairo in 1932 [7] decided to keep only one to be considered unique and denoted 𝄳, though its precise frequency ratio had never been defined and no consensus as to its value had emerged.

In practice, the ¾-tone interval varies slightly according to regions and styles. In this study we base our definition on the 19th century interval, that originated in ancient Syria (including Lebanon, Palestine …) and Egypt, which then spread to the rest of the Arab world in the early 20th century.

1. The first wave of Arab theorists (9th to 13th century) left many books dealing with musical tuning; the most important of them are translated in the 6 volumes work of R. d’ Erlanger [8]. Because of the lack of mathematical tools at their disposal, their models consisted,

[1] This sign is used in Spectral and Micro-tonal musics.

Table I. Pythagorean scale and its intervals.

C	D	E	F	G	A	B	C
1	9/8 = 1.125	81/64 ≈ 1.2656	4/3 ≈ 1.3333	3/2 = 1.5	27/16 = 1.6875	243/128 ≈ 1.8984	2
0	203.9	407.8	498	702	905.9	1109.8	1200

C–D	D–E	E–F	F–G	G–A	A–B	B–C
9/8 = 1.125	9/8 = 1.125	256/24 ≈ 1.0535	9/8 = 1.125	9/8 = 1.125	9/8 = 1.125	256/24 ≈ 1.0535
203.9	203.9	90.2	203.9	203.9	203.9	90.2

Table II. Equal scale and its intervals.

C	D	E	F	G	A	B	C
$2^{0/12}$	$2^{2/12} \approx 1.1225$	$2^{4/12} \approx 1.2599$	$2^{5/12} \approx 1.3348$	$2^{7/12} \approx 1.4983$	$2^{9/12} \approx 1.6818$	$2^{11/12} \approx 1.8877$	$2^{12/12}$
0	200	400	500	700	900	1100	1200

C–D	D–E	E–F	F–G	G–A	A–B	B–C
$2^{2/12} \approx 1.1225$	$2^{2/12} \approx 1.1225$	$2^{1/12} \approx 1.0595$	$2^{2/12} \approx 1.1225$	$2^{2/12} \approx 1.1225$	$2^{2/12} \approx 1.1225$	$2^{1/12} \approx 1.0595$
200	200	100	200	200	200	100

Table III. Holderean scale and its intervals.

C	D	E	F	G	A	B	C
$2^{0/53}$	$2^{9/53} \approx 1.1249$	$2^{18/53} \approx 1.2654$	$2^{22/53} \approx 1.3334$	$2^{31/53} \approx 1.4999$	$2^{40/53} \approx 1.6873$	$2^{49/53} \approx 1.8981$	$2^{53/53}$
0	203.8	407.5	498.1	701.9	905.7	1109.4	1200

C–D	D–E	E–F	F–G	G–A	A–B	B–C
$2^{9/53} \approx 1.1249$	$2^{9/53} \approx 1.1249$	$2^{4/53} \approx 1.0537$	$2^{9/53} \approx 1.1249$	$2^{9/53} \approx 1.1249$	$2^{9/53} \approx 1.1249$	$2^{4/53} \approx 1.0537$
203.8	203.8	90.6	203.8	203.8	203.8	90.6

mostly, of empirical charts based on the division of a string length.

2. In the 19th century the Lebanese Michel Mushaqah tried to build (1820) some new models, but he was rather practician, and theory was beyond his scope [9, 10, 11]. The turc Raouf Yekta Bay exploited the Holderean scale with its 53 commas or turksents to evaluate some non-equal intervals

$$\mathrm{C} \underset{9}{\text{——}} \mathrm{D} \underset{8}{\text{——}} \text{Eƀ} \underset{5}{\text{——}} \mathrm{F} \underset{9}{\text{——}} \mathrm{G} \underset{9}{\text{——}} \mathrm{A} \underset{8}{\text{——}} \text{Bƀ} \underset{5}{\text{——}} \mathrm{C}$$

but as we will see, dividing the 3/2 tone D-F and A-C in 8 and 5 does not comply with practice.

4. Modeling of an oriental scale

The ¼-tone is rather scarce; what really exists is the ¾-tone (minor tone). We can find the most common ¾-tone intervals in the widely used rast tuning [1, 3, 6] (something intermediate between minor and major modes, very similar to the blues tonality [12]). With T denoting a whole-tone, rast tuning can be represented as follows:

$$\mathrm{C} \underset{\mathrm{T}}{\text{——}} \mathrm{D} \underset{3/4}{\text{——}} \text{Eƀ} \underset{3/4}{\text{——}} \mathrm{F} \underset{\mathrm{T}}{\text{——}} \mathrm{G} \underset{\mathrm{T}}{\text{——}} \mathrm{A} \underset{3/4}{\text{——}} \text{Bƀ} \underset{3/4}{\text{——}} \mathrm{C}.$$

The idea of dividing an octave into equal parts (24 quarters of tone) may seem adequate and has been exploited in designing low price musical keyboards. Several arguments, e.g. the need of an accurate method in order to design instruments and computer cards and the convenience of the equal-tempered scale for transposition and modulation, led us, initially, to experiment with such a scale.

But, unlike blues tonality, an underlying equal temperament cannot work; oriental musicians always refer to their favourite (unfretted string instrument) lute and prefer chromatic semi-tones higher than diatonic ones. However, it may be used as a basis which needs further little adjustments, and for the same reasons as in European music, we may not ignore the existence of the Pythagorean scale. We have finally chosen a scale that combines features of both equal and Pythagorean ones by merely taking the average of both corresponding frequencies of Table I and Table II.

The gap between this new tuning and each of the previous ones is ≤0.17% (except for E and B), avoiding any further debate about the choice among the two main scales. This debate will not affect our work because the main concern in oriental scale is to determine the length of the minor tone interval. So, because of the variation in practice, we do not need to achieve such an accuracy.

Considering E semi-flat and B semi-flat, we proceed by taking the geometric mean of the frequency ratio of (D–F) and (A–C).

We then obtain the following intervals which form the rast tuning:

1 tone: 1.1237 (201.9 cents), C–D, F–G, G–A,
¾ tone: 1.0896 (148.6 cents), D–Eƀ, Eƀ–F,
A–Bƀ, Bƀ–C,

Table IV. Theoretical Rast tuning.

C	D	E ♭	F	G	A	B ♭	C
1	1.1237	1.2244	1.3341	1.4992	1.6846	1.8356	2
0	201.9	350.5	499	701	903	1051.6	1200

Table V. Roland MIDI sequencer Arabian temperament.

C	D	E ♭	F	G	A	B ♭	C
1	1.1251	1.2276	1.3333	1.5000	1.6873	1.8414	2
0	204	355	498	702	906	1057	1200

Table VI. Practical/experimental Rast tuning.

C	D	E ♭	F	G	A	B ♭	C
1	1.124	**1.234**	1.334	1.499	1.684	**1.852**	2
0	202	**364**	499	701	903	**1067**	1200

½ tone: 1.0600 (101.0 cents),
¼ tone: 1.0296 (50.5 cents).

Our medium third C–E♭ (often quoted by theorists) has a value of 1.224 or 350.5 cents (1.26 for the major and 1.19 for the minor one).

5. Oriental keyboards [1]

Almost all keyboard makers have their “oriental” models intended for Middle-East and North Africa markets. Their main features is that they provide ¾ tone intervals and related tonalities, but also some typical instruments and rhythms. Low-price models have a pre-set (equal) oriental tuning, i.e. with a quarter tone of exactly 50 cents, which is not suitable for performers. Some advanced models allow the musician to set the tuning himself and find the ¾ tone that suits his music, and dare not give a fixed value that, maybe, not everybody agrees with [1].

Very few attempts have been made to offer non equal pre-set tuning to account for real pitches. Let us see the example of Roland MIDI sequencer. Its user’s manual [13] gives a so-called Arabian temperament compared to an equal tuning. We have listed this scale, expressed in terms of (relative) cents and frequency ratios in Table V:

I have chosen this example because it raises the real problem [1]. The medium thirds (C–E♭ and G–B♭) of 355 cents are higher than the equal ones of 350 cents, but not enough. The 3⁄2 tone intervals (D–F and A–C) are composed of non equal ¾ tones of 151 and 143 cents but their size must be rectified.

6. Experiment

Although the interval of ¾-tone varies very slightly according to regions and styles, we can consider that a standard does exist: it was imposed by the strongly invading Egyptian style which has been adopted by many other countries.

To measure the acoustical frequencies many devices are available, ranging from old sonagrams to advanced computer-controlled spectrographs. But the simplest method is the best one, which has been used since Euclid and his “Division of a monochord” [14]:

The Arabic lute has 5 or 6 strings whose length is exactly 60 cm: (D2,) E2, A2 (220 Hz), D3, G3, C3 (according to European convention and MIDI standard), and the neck of our model has been graduated in millimeters like a scholar’s ruler. Let us tune one of these strings to a tonic or reference frequency, for instance 100 Hz (the exact value doesn’t matter), and let us call it C; G will be then 150 Hz with a length of 40 cm. All we have to do is to spot the position of our wanted notes. After playing many times some widely spread oriental themes, I found that the average measured lengths for E♭ and B♭ are respectively 48.6 cm and 32.4 cm . So, applying the relation $f_2/f_1 = l_1/l_2$, the frequency ratio of the two medium thirds are 60/48.6 and 40/32.4 and are both roughly equal to 1.234:

$$f_2/f_1 = l_1/l_2 \approx 1.234.$$

After empirical adjustment, our work led us to divide each minor diatonic third D-F and A-C into 2 non equal ¾ tone intervals (Table VI). The first ones (D-E♭ and A-B♭) have a size of **163** cents, and are higher than the second ones whose size is **134** cents. The rest of the notes, i.e. the diatonic scale, are quite in accordance with Table IV.

The value of the medium thirds (C–E♭ or G–B♭) is **365** cents implying a gap of about +15 cents, compared with equal temperament, which can be easily detected by beginner musicians.

As for the accuracy, the measured values have 3 significant figures: 486 mm (E♭) and 324 mm (B♭). Both of them are means of several other values, which vary very slightly according to players and melody styles, and are already rounded to the nearest whole millimeter. The apparent accuracy shown in Table VI results only from arithmetic operations.

7. Conclusion

This work presents some theoretical and practical research on oriental musical scale. We first designed a quasi-standard diatonic scale from a combination of equal-tempered and Pythagorean ones. We then derived the values of the whole-tone, half-tone and quarter-tone intervals from this theoretical standard scale (Table IV). After comparison with measured values given by musical performance we came to the conclusion that the real practical scale (Table VI) contains three intervals of 1 tone (202 cents) plus two larger intervals of 3/2 tone (297 cents): D–F and A–C.

D–F and A–C are both composed of a large ¾-tone (163 cents or 55%) and a small ¾-tone (134 cents or 45%). Also, the measured medium thirds (C–E♭̸ or G–B♭̸) of about 365 cents (Table VI) are 15 cents higher than equal medium third of 350 cents.

The theory based on the 53 turksents of the Holderean scale (section 3 and [4]), conceived a century ago, gives medium thirds of 385 cents. This value is too high for current practice and, in addition, a 53-division scale is no longer convenient for any temperament.

The conclusion is that our scale (Table VI) fits the practice very well, and is better than any published so far.

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