*Musical Mathematics: On the Art and Science of Acoustic Instruments*

Text and Illustrations by Cris Forster

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Seven Extensive Reviews

**Review by Carlo Serafini**

First, an anecdote: I was once in Amsterdam at the Van Gogh museum. In front of "Wheatfield with Crows" the force of the painting was so strong that I had to turn away from it. I could feel the power of the painter and it was almost unbearable.

*Musical Mathematics* is a great work of art too but in order to appreciate Mr. Forster's "power" the reader has to read at least a few hundred pages of this outstanding tome. I can definitely feel it, recognize his encyclopedic knowledge and his dedication to the reader. Not only that, Mr. Forster too has been "seduced into carpentry," like Harry Partch, and that means he is not only a very erudite man because his sources for writing *Musical Mathematics* were not only a myriad books but his own direct experimentations building instruments, measuring string length ratios, tuning pianos, etc.

I am sorry to disappoint him but I am not going to build a *canon* after reading his book. Carpentry is not my thing.

He says: "Acoustic music is the most difficult music." Probably that is why I am an electronic musician. I can easily retune my synthesizers (hardware and software) to any imaginable tuning system and those who follow my blog know my favorite one is Carlos Gamma (20th root of 3/2, a non-octave one), I can instantly convert ratios to cents and *vice versa* too.

Heidi Forster, his wife, says it took him ten years to write it and more years to prepare it for publication but this is obviously the work of a lifetime.

Something the reader can immediately sense is Mr. Forster's care for him/her. This is certainly not an elitist book. Mr. Forster really has done all he could to make possible the transfer of some of his vast knowledge to the reader. The subject of this book is clearly not easy and the territory to cover is amazingly vast but Mr. Forster takes the reader through this journey making sure he/she does not get lost or discouraged. That does not mean reading/studying it is a simple task. There are chapters I am not going to read because (1) they are too difficult for me, (2) I am not interested (I know it sounds like Aesop's fable "The fox and the grapes" but that is the truth).

Mr. Forster knows that not all readers will go through his book from A to Z and for this reason suggests possible paths to follow. My main path has been Chapters 3, 9, 10 and 11 (see Table of Contents) but I started from the Epilog written by his wife where she tells the story of her husband's life. No doubts she too spent countless hours working on this book. My reverence to her for having supported this project for many years. I, then, jumped to Chapter 3, parts of Chapter 5 (because I am a keyboard player); then to Chapter 9 to finally land to Chapters 10 and 11 that were my real targets. These two chapters alone could be a massive book, almost 500 pages long,

on the history of the tuning of musical instruments. I, then, ended reading Chapter 12 admiring Forster's amazing musical instruments.

Saying that this book is the ultimate one on the subject of musical mathematics is an understatement. The level of details, of deep understanding, supported by Mr. Forster's hands-on experimentations is almost unimaginable.

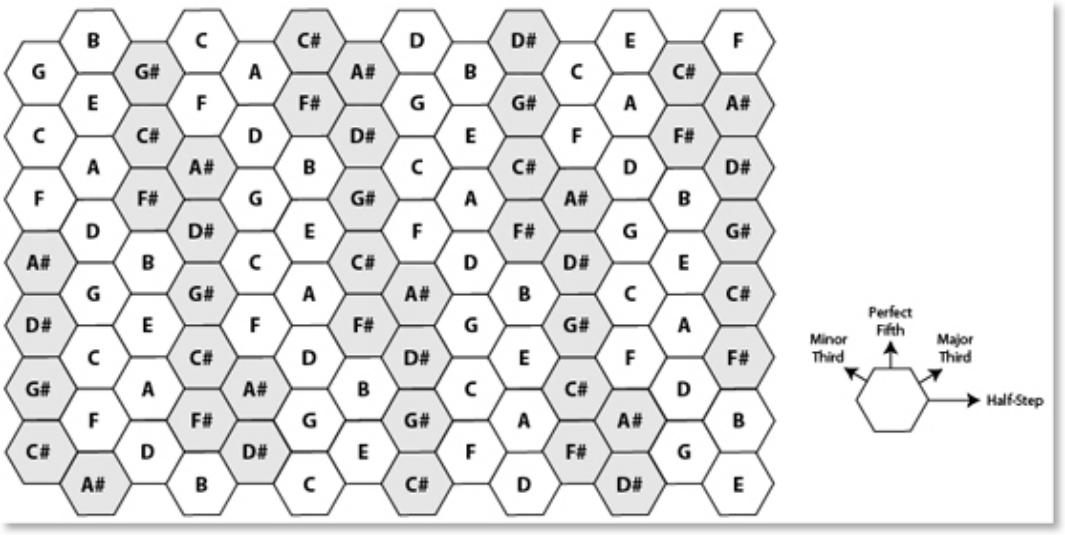
Mr. Forster is a composer, a musical instrument builder and an innovator. He is like one of those ancient scholars able to seamlessly move from one discipline to another surprising you at every step for their immeasurable knowledge.

My praise to Chronicle Books for undertaking such a venture. The book has been printed on very nice paper and with a very good binding.

This is a book that needs to be read over and over again to really gain something from Mr. Forster's work.

Someone could ask why an electronic musician like me should read this book considering that it deals exclusively with acoustic music. Paraphrasing Heidi Forster I could answer that "if we are to influence the course of musical history, the full history must first be learned!"

From my point of view I would like to add something about "the structural limitations of keyboard instruments" and "the physical limitations of the human hand" (Section 10.30, p. 349). Modern technology makes possible the creation of so-called isomorphic/generalized keyboards that can overcome both limitations mentioned by Mr. Forster, but these are electronic keyboards, so that is another story!




-C. Serafini, 2010
Electronic Music Composer

https://www.seraph.it/blog_files/0e4253e6d22fc393e0a33e044fdfe12e-74.html

## Review by Hajo Dezelski

When writing an article about the transformation of "light" to sounds I looked around to find other sources of sound production than the usual music channels. The first one was a blog of Steven Speciale "Mostly Noise" where I found a lot of modern music seldom heard. But I was looking for some microtonal music and found a lot of thoughts and music on the pages of Chris Vaisvil's "Music & Techniques." He is composing and performing microtonal music on a variety of instruments. And here I found also the book of Cris Forster which was released in 2010. After a look at his homepage I thought that it could not be a fault to order this book.

*Musical Mathematics* does not tell stories about hidden mathematical symbol patterns in the music of J.S. Bach. It translates our musical ear training into a down to earth language: Mathematics. In this book you find no guessing or flowering words for the incomprehensible. You will find formulas and numbers.

It starts with the basics of sound described in physical/mathematical formulas: The principles of force, mass and acceleration. But this is not a walk-through like in normal mathematical books on one page. He takes his time to describe in words and drawings what is going on. And this is one advantage of the book. Although it deals with mathematics, it uses also its elegant language, but never lets you alone guessing what that formula means. Forster describes step by step how to achieve a solution. He never takes the attitude of a professor who throws the equation on your head and says: "Here is the solution, see for yourself how it works."

He introduces the MICA mass definition: **M**ass (unit based on the) **I**nch **C**onstant (for) **A**cceleration; but a formula for conversion to kg is given.

Before I continue: This is my only complaint about the book. I thought that the USA changed their measurement system to metrics. But Forster stays in the old system. And for non-US readers that is really a pain in the back: You have to use your calculator anyway but you constantly have to convert *e.g.* $13^{15}/_{16}$ inches to mm. And I can assure you: On these over 900 pages there are inches, inches, inches . . .

Next you will find the correlation for plain strings and wound strings between frequency, length, tension and mass. It is the basis for the actual sound you get from a string. Of course the string's break strength for different materials is also discussed. Another point is the geometric structure of wound strings with various materials.

After these preliminary remarks, Forster continues with the behavior of strings. He shows transverse traveling and standing waves and simple harmonic motion in strings with pulses from different directions. The explanations are clear and facilitated by drawings.

You will find tables with movement of strings fixed on both ends; period and frequency equations; length, frequency and interval ratios of the harmonic series; all you ever wanted to know.

The next block described in the book is string harmonics. Harmonics are normally described in musical terms: A "fifth" is the fifth tone in a scale. This does not mean a mathematical length ratio $\frac{1}{5}$. Forster will throughout the book talk about mathematical ratios in order to describe other scales. So he describes in detail the relation between harmonics, note names, frequencies, cents, modern length ratios, frequency ratios, interval ratios, interval names and cents.

This is much theory, so in Chapter 13, Forster proposes to build a *Canon* (Monochord). This is a simple instrument which you can build by yourself (plans, pictures and measurements are included). With this *canon* it is easy to learn other scales. The instrument has a nut and a stationary bridge, 6 strings of identical length and form, and you can subdivide the strings with movable bridges. So it has a theoretically infinite number of notes and can produce any number of new frequencies without changing the overall design of the instrument.

You will not only learn about mathematics. Forster digs deep into history and researches musical, mathematical, and linguistic origins of length ratios from the old Greeks to modern systems. You will find all the old systems used in former times translated to measurements you are used to.

Until now the world was in harmony: All harmonics were exact integer multiples of the fundamental frequency. But that does not fit into the actual sound you get. In reality stiffness of the material increases frequency. You get inharmonically induced dissonance. And you have to consider this. In the workshop the luthier knows that you have to give or take a little to produce the right sound. But Forster shows how to calculate the differences, the relations between soundboards, strings, bending waves, and also geometric versus acoustic lengths.

To ease the calculations he introduces step by step conversion to the cent system (1200 cents = 1 octave).

The next big chapter after the mathematics of strings introduces bars, rods and tubes. It is a parallel description of the world of resonant bodies. You will *e.g.* learn how to tune marimba bars, acoustic resonators and tube resonators. You will find the harmonic motion of longitudinal traveling waves in air and standing waves in tubes resonators; and tuning techniques for cavity resonators using dowels to stiffen the walls. Forster explains the placement of tone holes for simple flutes, gives equations for analyzing the tuning of existing flutes; then gives logarithmic equations for guitar frets, 12-tone equal temperament to just intonation.

You will notice that I start to summarize. The material is extensive and each chapter would be worth an in-depth description. When you study this book up to p. 281, you should have the following knowledge:

- the mathematical structure of the harmonic series
- the distinction between different ratios (length, frequency, interval)
- mathematical methods used in the division of *canon* strings
- distinction between arithmetic and geometric progressions
- how to add and subtract musical intervals
- how to convert length and frequency ratios into cents

Chapters 10 and 11 are in my eyes the most important parts of this book and I must confess that most of it is *terra incognita* for me. It is an encyclopedic overview of all known tuning systems and scales ever invented by musicians all over the world. It covers scales from times gone by to actual modern tunings. You will not only get a description and an historical evolution but also a mathematical declaration based on the explanations of the former chapters. So you are able to reproduce these scales.

Chapter 10 is titled: "Western Tuning Theory and Practice." In the first part, Greek classifications of ratios, tetrachords, scales and modes are mathematically and practically explained by tuning *canon* strings. These differentiations continue with the systems of Philolaus, Euclid, Aristoxenus, and Ptolemy. Of course, these tunings are also broken down to length ratios on a *canon*. Tables with Greek Enharmonic, Chromatic and Diatonic scales are provided. Meantone temperaments, well-temperaments and equal temperaments are treated in the same way. You will find for all these scales mathematically exact formulas and tables. This is, of course, no easy reading material although the math required could be done with a simple calculator. In the following sections, different 12-tone Western scales are calculated and translated into cents. It is really astonishing just looking at the tables to see how different these scales are.

I had my nightmare when I tried to tune my clavichord with a tuning aid. The tuning was not the problem, but to hear the difference. I must admit that my memory was too short to remember the differences. So the proposal of Forster to build a *canon*, where you can quickly change the tuning and hear the difference, might be a better approach than mine. ;-)

I could continue to drop words about the next 100 pages where Forster goes through more scales than I ever heard of. For me it is more a chapter for reference. I can try to understand the differences of tunings, but I have to hear them in comparison to the scale we use to "understand" what is going on. And please do not think that this is a book with a popular background. Only for this chapter there are 364 notes.

And the encyclopedia goes on with "World Tunings" with "only" 300 pages. You may have noticed it already. This is not really a review. I scanned this book, got lost by reading interesting chapters, turned pages, took my calculator, read on, found another interesting point . . .

But if you are interested in non-Western music, this book is definitely for you. Of course Forster is not talking about music. His subject is still "only" tuning and I would bet that you will find nearly every tuning which is used in this world.

- Chinese Music
- Indonesian Music (Java, Bali)
- Indian Music (Ancient, South India, North India)
- Arabian, Persian and Turkish Music

So if you hear some ragas and on the CD-booklet you find the name of the instrument you will find the tuning of this instrument in this book. I cannot even write the names of the instruments: My character set does not allow it. I do not even want to know how long it took Forster to gather that wealth of information and transform it into today's mathematics. I am sorry: I am not the one to make a critical review of these two last chapters. But you can be assured, I have never seen such a compilation of musical tunings and scales and if I make a conclusion from the first chapters: The scales are right to the point. Otherwise he would not be able to turn these scales into playable tunings.

You can take your *canon* and tune it accordingly and you will hear the magic of this tuning. I sometimes played on a Turkish *'ud.* Now I know how it worked. My former experiences with a *sitar* are now more transparent.

So this book is a culmination of what I have seen about mathematics and music. You will understand what system is behind tone, harmonics, scales . . .

It is a summary of tuning systems through the centuries in the Western world.

It is a compilation of musical scales of the world never seen before.

Who must own this book?

* Musical libraries
* Musicians, who have a brain for mathematics
* Composers, who dive into new scales, sounds, systems
* Freaks, who like to listen to instruments from other worlds
* Me, but I still have a lot to cover in this book

-H. Dezelski, 2011

PS: Of course there is an extensive bibliography, index, sixteen color plates . . .
PPS: Another critical point: You can't read it lying on a sofa. It is far too big and heavy. ;-)

## Review by Margo Schulter

Two measures of the monumental achievement represented by Cris Forster's *Musical Mathematics* are the vast musical spaces encompassed by the palatial precincts between the covers of this book, and the yet vaster musical spaces to which the book is a beckoning open portal or gateway. It is a palace with paths leading to many other palaces, whether previously unknown to the reader, or revisited with the benefits of a new perspective gained from Forster's breathtaking survey.

As I will not be the first to observe that this work is at once a compendious reference and a friendly companion, with readable type and a binding and generous page margins which permit opening the pages wide and making the best use of the information which is there. Getting this material to fit in "only" something like 950 pages was itself a delicate task, and happily the author and publisher succeeded without compromising an attractive and engaging format which is a feast for the eyes, mind, and ears.

Yes, a feast for the ears, because this is a book which is meant to be put into action, "action" here meaning the production and experiencing of actual musical sounds. While the focus, as the full title announces, is *Musical Mathematics: On the Art and Science of Acoustic Instruments*, even a digital synthesizer can permit a fascinating first-hand exploration of the aural territory that this book maps with expertise and scholarly humility.

For example, I am having great fun exploring some of the genera of the great Islamic theorist al-Farabi as summarized at pp. 650–654, including a number derived from the classic Greek theorist

Aristoxenus, whose mathematical imprecision is duly explicated elsewhere in these pages. There is the fascination of knowing that the instrument on which I am seeking these genera, a 24-note digital archicembalo with two 12-note chains of fifths tempered slightly wide (and the fourths slightly narrow), would hardly have been envisioned either in ancient Greece or in the 10th-century (by a European reckoning!) world of the Islamic Renaissance. Yet the genius of the imprecise "geometric" concept of musical space by which Aristoxenus devised his chromatic tetrachords may be their very impressionistic nature, inviting a variety of interpretations, some of which Forster lucidly explores in the context of al-Farabi.

Perhaps there is no deeper praise for this book than to say that I hope its readers will early and often put it into action, whether modestly on a synthesizer or more boldly on an acoustic instrument of one of the various kinds addressed by Forster, and possibly even built according to a plan he here provides.

By this point, I hope to have conveyed my overwhelmingly positive response to this comprehensive and yet amazingly compact volume as at once a guide to the mathematics of musical acoustics and tuning, and an engaging survey and reference on a number of world musical traditions ranging from ancient Greece, Europe in the meantone era of the 15th–18th centuries, and China over this same long span of time, to the Balinese and Javanese art of gamelan, the intricate raga tradition of India, and the Near Eastern musical genius of the 8th–14th century Islamic Renaissance as expressed today in the Arab and Turkish maqam traditions, the closely related dastgah tradition of Iran, and a Kurdish tradition evidently drawing on maqam and dastgah music alike.

Here I will be addressing especially some topics relating to the medieval and Renaissance/Manneristic eras in Western Europe, say roughly from the 6th to the early 17th century, and to Near Eastern music. My viewpoint will surely be influenced, not to say biased, by my own primary background and training (mostly by self-education) in 13th–14th century medieval European polyphony, supplemented over the last nine years or so by an interest in the maqam and dastgah traditions as reflected by the tunings of the Islamic Renaissance and also modern Near Eastern practice and theory.

Before delving into some topics raised by this book or spinning off from it, I would like to say that Cris Forster has scored a stunning success in making some basic material from theorists of the Islamic Renaissance such al-Farabi, Ibn Sina, and Safi al-Din al-Urmawi available in English for the first time, and in a handy compilation skillfully narrated and generously illustrated with charts, diagrams, and musical examples. This, alone, would make *Musical Mathematics* an invaluable achievement.

Seeking to share in the spirit of scholarly humility which Forster expresses, I must tell my delight at actually learning, for example, the forms in which Ibn Sina himself gave certain tetrachords whose rotations or permutations are often ascribed to him. Having encountered different cited forms over the years both on the printed page and in sources such as the scale archive offered along with Manuel Op de Coul's outstanding and freely available program Scala, I was deeply moved to encounter his own original versions of "the very noble genus" with ratios of 104:91:84:78 (8:7-13:12-14:13) and 16:14:13:12 (8:7-14:13-13:12). Knowing these two versions set down with admiration by Ibn Sina about a millennium ago deepens my sense of wonder and admiration that all permutations or arrangements of these three melodic steps (14:13, 13:12, 8:7) offer beautiful realizations of or variations upon Near Eastern modalities both old and new.

An advantage of such a handy reference is that it provides a highly accessible overview in itself while also directing the interested reader to sometimes more detailed sources such as the Baron Rodolphe d'Erlanger's French translations of Islamic Renaissance texts in the first four volumes of his classic six-volume collection *La Musique Arabe*. I say "sometimes more detailed" because Forster manages to include often rich and lavish detail within the constraints of space set by his framework. Thus while his book supplements rather than substitutes for the longer sources, the reverse is also true.

First focusing in the supplementary topics which follow on some points relating to early European music of the 13th to early 17th centuries, I will then turn to the very extensive and admirable treatment of Near Eastern music.

[The full review and commentary will include these supplementary topics.]

Most appreciatively,

-M. Schulter, 2010

## **Review by Jeff Scott**

This is the book on tuning we have been waiting for. It was just released in July 2010 after more than a decade in preparation.

Cris Forster is an instrument designer, builder and composer who lives out in California. He's originally from Brazil and has lived in Germany as well. Cris even spent time restoring the old Harry Partch instruments for the Harry Partch Foundation, and once packed them up and shipped them to Germany and back for a concert.

Cris is a very serious and conscientious guy who always does the best possible work. He's a perfectionist and a rare craftsman in this regard. His instruments are each gorgeous works of art.

Cris has always been a microtonalist but for 12 of the last 20 years he has been researching tunings very intensively for the purpose of a book he was working on. It took ten years to research and write, and then it took another two years to format it, create all the hundreds of necessary diagrams and graphs, index it, footnote it, and get it published. That publication was just a few weeks ago, by Chronicle Books in San Francisco who has made a well made book physically, very well bound with sewn folios and high quality paper.

*Musical Mathematics* clearly reflects 12 years of work and is the most comprehensive book on tuning I have seen. For many years people have asked me if I know of a single excellent reference that will orient them in all this tuning stuff. There hasn't before really been a single book that contained a real breadth of coverage, and much of the most interesting material, particularly regarding ethnomusicology, was buried in obscure out of print publications or extremely expensive references. Finally I have a single book I can unequivocally recommend. Whew. Thank goodness. At last.

It's not just about tunings; the book begins with many chapters extensively discussing acoustic musical instrument physics, providing the solid foundation needed not just to build instruments, but to understand where scales came from and why we hear things the way we do.

It then moves through 200 pages on Western tunings, with a comprehensive and extensively cited coverage of Ancient Greek music before heading into the history of Western European tunings that most of us are somewhat familiar with.

It finishes up with 300 pages of coverage of ethnic tunings covering four different major traditions: Chinese, Indian, Middle-Eastern (distinguishing between Arabian, Turkish and Persian), and Indonesian tuning systems, each which have unique approaches. There is coverage of Cris' own instruments, a brief biography at the end, and also directions for building your own small harmonic canon, which will be helpful in understanding tunings in a direct and sensory way. The canon is referred to occasionally in the text with suggested experiments relevant to the tuning discussion.

The book is giant. Originally nearly 2000 pages long, the format of the book was reworked so it would fit into 1000 pages. The result is pretty dense text with long scan lines in a large and heavy book. If there is one thing that could have been done differently it would be to break this wonderful monster into a three volume set as reading it can be backbreaking work due to the sheer weight. But that would have increased the cost surely. Reading it pretty much needs to be done lying on the floor as holding it up will hurt your wrists after a while. Just letting you know that you get your money's worth with this book, it has so much useful information.

One tip is to bookmark the footnotes at the end of each chapter and read them in tandem. In addition to the usual citation information which comprises most of the footnotes, the notes also have quite a bit of secret and fascinating information in them that probably would have distracted from the flow of the text if placed in line. Reading these is like going down the rabbit hole though. I quickly was led off into other references and found I was spending days and days following up on interesting things I didn't know about just to advance a single page in the book.

Cris's style is dry. There's not a lot of blatant humor here or hilarious anecdotes. If you crave that sort of stuff there are other books that can be used as a supplement. But it's also not useless or obscure and impenetrable academic information like we see in so many of those journal articles and university theses about scales that come out nowadays which try to make tuning into some sort of incredibly abstract thought experiment. In contrast to that, the math in here is necessary and describes real things. There are formulas or numbers on almost every page, but much of this is because he doesn't skip steps but tries to explain everything in detail. The math is junior high or high school level depending on where you went to school. Possibly elementary level if you went to school outside the US. You need to know fractions, multiplication, division, and a little about logarithms. No calculus. It's not pure math formulas for page after page either, I don't want to give the wrong idea. There is a great deal of historical information as well that is clearly relevant to the history and use of tuning. In fact, the math serves these parts rather than the other way around.

As to whether you should get this book or not there is no question: you should get it. You're here on this site, you have an interest in tuning, you probably have written or performed microtonal music, well oh yes oh my certainly you definitely should be getting this one without a doubt. It's a brilliant work of scholarship by someone who really knows what he is talking about. We have really needed someone to take this approach and spend the time needed to assemble all this information

in just such a giant book and present it in a straightforward, conscientious and accurate manner. Eventually you are going to buy it and if you wait too long it could go out of print since this is definitely specialist fare and not for the general reading public. I recall years ago I bought the other giant red book of tuning, Owen Jorgensen's amazing manual of bearing plans and common practice tuning history called "Tuning". It was about the same price, just under $70. Couldn't afford it at the time as I was a starving student, but I bought it anyway. Nowadays "Tuning" runs $250–$700 for copies in poor to middling condition assuming you can find it at all. So, you should get it right now. And you should not be surprised if it takes you 10 years to read if you do it like I am doing, one page at a time, trying to absorb everything.

I've been going through the Greek and non-Western sections and the coverage is excellent and just the sort of specific and accurate detail I have been craving for so long. I am so glad this book has at last been published and I think you will be as well.

-J. Scott, 2010

Chief Musical Instrument Designer
Red Barn Goat Farm — Mac tuning software

http://www.nonoctave.com/tuning/book-reviews.html
http://www.nonoctave.com/tuning/biblio.html

**Review by John Schneider, Ph.D.**

Sir Thomas Beecham once famously quipped that, "…the British public doesn't really understand music, they just like the noise that it makes." Sadly, the same can be said for most musicians when it comes to their instruments. We all spend untold hours dealing with HOW to play them, but rarely consider WHY these marvelous inventions do what they do. In fact, the simplest concepts of rhythm, pitch, & volume, the basic building blocks of music, are all determined by numbers. But the very nature of instruments themselves — the scales they play, the colors they produce, how well they project their sound, etc., all of these elements are also deeply determined by math. For thousands of years, this has been the realm of the instrument builder, who, more often than not, relied on intuition and received knowledge to practice their craft. Now Cris Forster has cracked the code, and reveals with uncanny scientific accuracy the principles behind instrument design, the vibrations of strings, bars and tubes and the very scales they produce.

This is not the work of some ivory tower theorist, a mathematician with music as an after-hours hobby. This author has sawdust in his hair and the strong hands of an artisan, having earned this knowledge the hard way, spending decades creating museum quality instruments of striking originality. He begins with basic principles: how do strings, rods & bars vibrate? We get the mathematics for all of these, and their application on real musical instruments. These formulae are absolutely essential for luthiers & craftsman, who will be amazed that the author has also created a new method for measuring Mass called the Mica mass unit. This revolutionary approach will save builders hours of time when measuring materials by bypassing conversions from the traditional English mass unit to a new inch-based unit. This new discovery is so important that the book

begins with it's explanation, which may scare off quite a few non-numerate readers, but for many, it alone will be ample reason to own this groundbreaking volume.

The realm of **Music Theory** (remember it's antique name: 'Musical Science'?) has always promised to explain the magic of music by parsing the mystical alchemies of melody & harmony. But it has really only been in the past generation that we have developed the tools to adequately explain the complex interaction between mechanical vibration and human perception. The mathematics of human hearing are explained (how we *really* hear), as are the methods and units used to accurately measure pitch. It is now common knowledge that the twelve equal-tempered tones of Western music are but a few of the hundreds available to the musician. How we have arrived at this historic compromise is a fascinating story that the author reveals in exquisite detail, tracing the trajectory from the ancient Greeks (Archytas, Ptolemy, Pythagoras, Aristoxenus, *et al.*) through the Renaissance & Baroque eras, to the present day. Happily, Forster is also multilingual, translating many critical primary and secondary texts previously unavailable to the English reading public. Of course the mathematical complexities behind *meantone* tunings and the numerous evolutionary *well-temperaments* are explored in great depth before we arrive at modern equal temperament, a tuning so challenging that it's adoption over a century ago created a new profession: the piano tuner. Previously, tunings were simple enough to be done by the instrument's owners, since most included many pure intervals of *Just Intonation*, another subject discussed at great length and of profound importance to the development of both the language of music and its instruments.

The beauty of Forster's presentation is not only the length and depth to which he goes to explain the history and detail of these fascinating European tunings, which have all but determined the very repertoire that we hold so dear, but that is only the beginning: the section on **World Tunings** is alone worth the price of admission. The author admits:

*"Long before I began writing this book, I envisioned discussing Chinese, Indonesian, Indian, and Arabian tuning theory in the same breath as Greek and Renaissance tuning theory. Why not? Music is a universal language not only because human beings have ears and a desire to make music, but also because people all over the world cultivate and investigate the subject of musical mathematics."*

What follows is over 300 pages of superb detail devoted to the notes of the *ch'in*, fretting of Arabian *'ud* (Al-Farabi, Ibn-Sina, Al-Kindi, *et al.*), various *pélog* & *sléndro* scales of Indonesia, the Turkish *tunbur*, Indian *vina* and *sitar*, and more.

If you have ever pondered the intricate diversities of the Arabian *makam* or Persian *dastgah* you will find them not only beautifully diagrammed and explained, but also compared and contrasted, a clearly 21st century approach that embraces our increasingly global musical resources. How do the ancient Greek modes compare with the *ragas* of India? Look no further. And these are not simply diagrams of the results of research — in most cases the scores of tuning tables include the musician's original descriptions of their tunings, not just the results. In other words, we benefit from the exact description of ancient modes in modern terminology (ratios & cents), but also get specific instructions as to how they were achieved in contemporary practice, regardless of century. The result? A truly three-dimensional view of our shared musical heritage.

*Musical Mathematics* culminates in the tunings and instruments that Forster knows best: his own. Though his credo is simple: *"I build because the tunings and timbres I want to hear do not exist on store shelves,"* the results are anything but. From the extraordinary double-sided spinning *Chrysalis*

to the elegant choreography of the *Glassdance*, Forster's instruments glisten with originality. Of course the canons and marimbas immediately recall the instruments of Harry Partch (1901–1974), and the connection is not casual, since the author spent several years as the curator for the Harry Partch Foundation, eventually rebuilding the entire 'orchestra' of original instruments. Forster's versions, however, transcend the originals in both skill & scope, expanding the tonal resources to include pitches based on the 13th harmonic of the overtone series (Partch's stopped at the 11th), and reaching a stunning level of craftsmanship that one can only described as first-class.

What's missing? The most obvious thing of all: *sound*. A compact disc or two would have gone a long way towards making these numbers live. If ears could salivate, yours will be drooling at the chance to actually taste these vibrational delicacies, having read about them in such detail. Luckily, Forster's own music can be heard in over a dozen MP3 excerpts found on the website www.chrysalis-foundation.org. Or, if seeing is believing, look no further than the Chrysalis Foundation's recent DVD ***A Voyage in Music*** to see the instruments in action and meet the man that made them, as well as the people that play them.

There are also discussions of numerous compositions from various traditions, some of them painstakingly transcribed from recordings. Sadly, several of these musical jewels are long since out-of-print, but the author helpfully includes all references to those still available. Clearly, a compilation disc drawing from Forster's listening library would be an excellent supplement to the book. As for the pages & pages of scales, modes, *dastgah*, *raga*, etc., it would surely have taken another few years to actually record all of the material, let alone on the appropriate instruments. Perhaps the answer as to why this was not done can be found in Chapter 13: "Building a Little Canon" — it seems as if the author is challenging the readers to make a little sawdust themselves if they want to taste the fruits of all of this hard-earned knowledge!

*Musical Mathematics* is essential reading for anyone interested in music, innumerate or not. For the builder, it's details are simply indispensable. For the musician, the historical and practical perspective on the materials of music should be basic reading. This is surely the guidebook for 21st century music making, and as such, belongs in every library. Where Partch's *Genesis of a Music* is in equal parts a manifesto, autobiography, intonation history & cookbook (yes, you really can recreate the instruments with those recipes), it's author once admitted that the pioneering work was written for the rebellious young man that he was back in the 1920's. Forster's *Musical Mathematics* is clearly the next step: it goes deeper and farther than *Genesis*, keeps the editorializing to a minimum (though when it comes, it is no less piquant & provocative), and offers sophisticated & exactly practical answers to anyone daring to pick up a piece of wood or wire and ask those materials to make music. Whereas Partch was a self-proclaimed "Philosophic music man seduced into carpentry," Forster is surely a craftsman of the highest order, seduced into mathematics to answer his own burning questions as to how physical and musical materials behave, in order to bring them to the highest level of perfection. We will be thankful for generations to come that he took a decade of his own productive life to write down the conclusions.

-J. Schneider, 2010

Professor of Music, Pierce College
Director, MicroFest

## Review by Kraig Grady

*Musical Mathematics* by Cris Forster is a rigorous and highly organized book that deals with the construction and tuning of acoustic instruments. In a clear and graspable way, the book first tackles the physics of instruments, a subject that is often the greatest stumbling block for readers interested in building instruments of their own design. After a detailed examination of the subject of mass, Forster guides us through his knowledge of strings, which includes their physical properties and different usages on musical instruments. Only in retrospect does one realize what a careful choice as a starting point this is because it easily leads us to a more complex study of bars, rods, and tubes. Resonators follow, with thematic connections that reach back to earlier chapters and forward to air columns and flutes. A chapter on geometric progressions, logarithms, and cents concludes the first part of the book, and at the same time acts as a bridge to the study of tunings. The second part presents the reader with a strong foundation of the history of tuning in Western civilization and throughout the world, and the methods employed to realize these tunings. The book ends with an examination of Forster's own instruments, which are extremely beautiful in both design and sound. He remains one of the greatest practitioners among instrument builders.

Although I have spent many years in the field, I discovered in *Musical Mathematics* a fresh and above all generous presentation of knowledge both with regard to acoustics and the history of scales. For example, the chapter on Chinese music discusses an approach to string tuning that I have never encountered in any other sources. Because of his own translations from other languages, Forster's research is not limited to English texts; for this reason, his book is filled with many new sources that provide fresh perspectives of the historical record. The subjects of Indonesian, Indian, Arabian, Persian, and Turkish tunings are likewise treated with much care and depth. Perhaps the book might be compared to Harry Partch's *Genesis of a Music*, but there are marked differences. The latter was written to explain Partch's music and instruments, and only secondarily to help others build their own unique instruments. *Musical Mathematics*, on the other hand, focuses more on the needs of creative individuals; it encourages musicians to discover and explore aspects that are most useful and fruitful to their own work. It is toward this goal that Forster shares the benefit of his knowledge and experience.

Yes, here is a book I surely wish I would have had 30 years ago when I first started out as a just-intonation composer and instrument builder. *Musical Mathematics* is truly as useful to the beginner as to the most accomplished expert in the field; both will find much value in this book. Also, it is obvious from his thoroughness and practical insights that Cris is an authority who has actually worked with the materials — an important aspect that sets this publication apart. This is a work of depth and breadth written in a spirit of sharing and helpfulness for those interested in the subject. *Musical Mathematics* is a watershed book that will, without doubt, change acoustic instrument building for the better, and change our views on the history of mankind's intonational practices.

-K. Grady, 2010

http://anaphoriasouth.blogspot.com/2010/04/cris-forsters-musical-mathematics.html

**Review by John H. Charmers, Ph.D.**

For over thirty years, I have been following the pioneering work of Cristiano Forster in musical instrument design and construction as he has painstakingly designed and constructed the ensemble of instruments necessary to embody his highly personal musical concepts. Cris is a gifted composer, a superb engineer of new musical instruments, a scholar of great depth, and the author of a unique book on the practical application of mathematics to the construction of new musical instruments for new 21st Century music.

If I had to choose just a few words to describe *Musical Mathematics*, they would clarity, explicitness and comprehensiveness. From the first chapters where Cris addresses the problems of the lack of appropriate units of mass and distance in the English System and the necessity for musical engineers to continually juggle several systems of measurement without error to the finally chapters, where his own beautifully designed and crafted instruments are depicted, all the cognitive and mathematical steps are described with such clarity that no beginner should have any difficulty whatever in following the argument and learning the requisite skills to become a competent instrument designer and builder.

Following this general introduction to musical engineering, concrete examples of the acoustics of various sounding bodies, including both wound and unwound strings, pipes open and closed, free and clamped bars and volume resonators are covered in extenso. This kind of information in sufficient detail for the non-physicist to make use of is very difficult to find elsewhere and potential instrument builders will find this section extremely valuable.

Similarly, the chapters on tuning theory are models of exposition. All the difficulties students may have had conceptually and operationally with ratios, logarithms, cents, etc. are identified and clarified in such detail that they are abolished forever.

Cris then reviews the history of musical scales starting with the Ancient Greeks. Greek music theory is amazingly relevant to 21st Century music as the Greeks were not only the first to apply mathematics to music, the tetrachords, scales, and modes they left for us are a fascinating *materia musica* for contemporary composition. Furthermore, Greek theory forms the basis for much of historical and extant Islamic and Byzantine music as well, so a knowledge of it is essential for understanding this portion of World Music.

Other types of World Music are then discussed in detail, including a very welcome section on Indonesian tunings as they differ greatly, not only from each other, but especially from other ratio or cycle of fifths based traditions. In this case, the use of cents and logarithms is essential but, happily, the reader will be well-familiarized with them from the earlier chapter. This section ends with an introduction of various kinds of equal and linear temperaments, of both historical and contemporary interest.

The remainder of the book is devoted to the documentation of Cris's instruments, including his own custom string-winding machine. Forster's artistic sensibilities, as much as his engineering skills, are shown in this section. In a word, his instruments are simply stunning visually, as well as acoustically. There is none of the rough-hewn, amateurish construction of other experimental musicians here. Let his be an example to others.

The book ends with extensive bibliographies and appendices of information for musical instrument designers and builders. Together with the chapters on the acoustics of musical instruments, they form a collection of essential data not easily acquired elsewhere.

It has also been a pleasure to read Cris's *Musical Mathematics*, and if I can be of any further assistance in evaluating Forster's book for publication, please do not hesitate to contact me. Though currently working on the Urey/MOD Mars biomarker detection instrument system as an astrobiologist/prebiotic chemist at the Geosciences Research Division of the Scripps Institution of Oceanography at the University of California, San Diego, I am the founding editor of the journal Xenharmonikôn whose Spring, 1979 issue featured Cris Forster's instruments, and the author of *Divisions of the Tetrachord*, a book on the application of Ancient Greek musical concepts to contemporary composition.

-J. Chalmers, 2007

http://www.tonalsoft.com/sonic-arts/chalmers/links.htm

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For more information about
*Musical Mathematics: On the Art and Science of Acoustic Instruments*
please visit:

https://www.chrysalis-foundation.org/Musical_Mathematics_Pages.htm

https://www.amazon.com/Musical-Mathematics-Science-Acoustic-Instruments/dp/0811874079