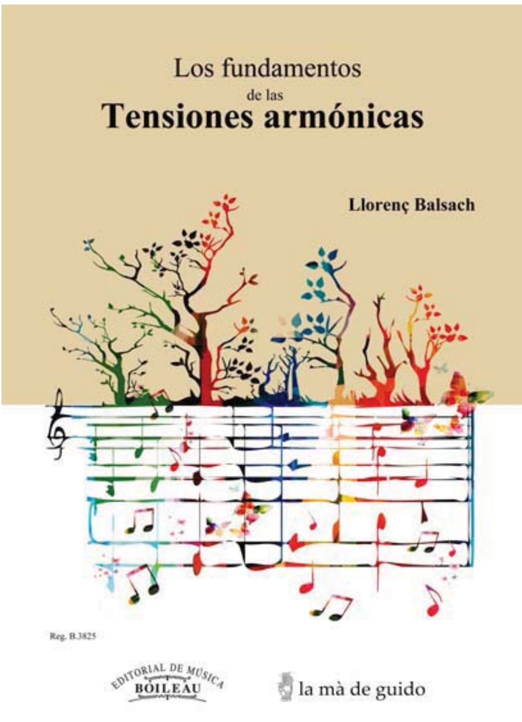
The foundations of harmonic tensions
(The fundamentals of harmonic tensions)

Llorenç Balsach

Provisional translation to english
of the book
Los fundamentos de las tensiones armónicas



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Per al Llorenç, el Roger, el Jan i la Sedes

Index

Introduction

Why when we hear a dominant seventh chord, or simply a major third, or else a tritone, are there notes that resolve the chord's tension or the interval's one? Why do we also notice relaxion* when we hear the so-called Phrygian cadence? Why does a specific succession of notes establish a (or several) tonic, and hearing this tonic (or tonic chord) produces a relaxation irrespective of the previous chord? Which tension do the chords of the Neapolitan sixth and those of the augmented sixth establish? Why is a specific succession of chords fluent and released and another produces tension?

In the following pages we will try to provide answers to these and other questions, always trying to look for the reasons accounting for musical tensions and relaxions . Furthermore, we will inquire into the nature of sound and our harmonic perception. We are aware of the fact that many of these questions could be answered in official theory by taking as a basis a scale and its associated concepts: degrees, tonal functions, cadences, etc.; nevertheless, there is usually no acoustic-harmonic explanation provided for the tensions and relaxions that are produced.

In my book *La convergència harmònica* (Balsach, 1994) and in the article "Applications of virtual pitch theory in music analysis" (Balsach, 1997), I already introduced many of the concepts and explanations that appear in the following pages. However, this time I have tried to present the main ideas in a more ordered and structured way, besides including new contributions and suggestions, mainly in the field of tonality.

This is the reason why the first chapter has been devoted to presenting in an abridged and ordered form the main and new ideas and conclusions of the book . In fact, if anyone wanted to get an idea of the book's contents, they could achieve it by just reading the first chapter.

I proceed on the basis —and this is not new— that the continuous perception of the harmonics phenomenon of a sound —from birth— is what shapes in the brain the sensations of harmonic tensions and relaxions when we listen to music. And, according to our theory, we will see that only the first seven harmonics play a role in it —at the most—, being the third, the interval of fifth (12^{th}), the main responsible for harmonic and tonal tensions. Nevertheless, curiously enough, this is in contrast

*I use the term 'relaxion' as antithesis to the term 'tension': tension-release, tension-discharge, tension-relaxation, tension-resolution, tension-relaxion. In spanish we use the term 'distension'.

with the fact that, apart from the octave, the fifth of a fundamental is the least important note in a chord; and this is so precisely because it does not produce tension in it, as it happens with the octave (the second harmonic), which does not produce tension in it either. Conversely, the intervals that are closer to the fifth (a difference of a semitone) produce tension, such as the tritone intervals (augmented fourth/diminished fifth) and the minor sixth/major third ones.

As we will see, this is a fundamental point in the theory, for the situation of these intervals in the chords will provide us with much information on the resolutive "preferences" of the chord to free this tension of the "quasi -fifth " interval that the auditory system perceives as "something is wrong here" in the chord (as a kind of "dissonance").

We will see that the tension of these "quasi-fifth" intervals and the tendency of the notes to their lower fifth are also responsible for the tonality (we will see that a major third and a tritone combined in a specific way create the most powerful tonal vector that may appear in a score). The formations of the major and minor scales with leading-tone will be the result of such tensions, but not the cause. That is to say, we do not base ourselves on the degrees of a scale to explain the harmonic or tonal discourse.

Regarding tonal functions, we could say that our theory is a neo-Riemannian theory that also agrees with some aspects of Ernö Lendvai's tonal axes theory.

Over the years, I have reached the conclusion that we may separately study three kinds of harmonic tensions: the purely local tensions or relaxions between two chords or arpeggios, irrespective of the tonal memory, which I call *homotonic* relaxions, the tonal tensions, and the chord tensions taking into account their dissonance or consonance (which I call *sonance* tensions). To these three harmonic tensions we should add the melodic tensions that, despite being intertwined with the harmonic tensions, have their own laws, among which the second's descending movements producing relaxion, stand out.

The second chapter analyses thoroughly the phenomenon of the harmonics and the virtual pitch (missing fundamental) theory. It shows that, in a perceptive way, harmonics may be reduced to those that are in prime position (that is to say, they are not harmonics of a prime harmonic); more precisely, it studies the effect of the second, the third, the fifth and the seventh harmonic when establishing music laws (for they are the sole harmonics that have a "functional" influence). It develops the "harmonic" meaning of the major and minor scales, and their triads.

In the third chapter we carry out a study and functional classification of the chords according to their internal tensions, that is to say, according to the fundamentals that represent the "quasi-fifth" intervals. These fundamentals tend to resolve mainly (and locally) to another fundamental a lower fifth (or upper fourth), or to another fundamental a lower minor second (or upper major seventh).

The fourth chapter analyses the secondary relaxions between chords and other successions of fundamentals, and presents a summary of homotonic relaxions.

The fifth chapter investigates the tonality and harmonic processes that originate a hierarchy among the notes (the tonal field). It analyses the cadences and modulations, as well as the tonal functions of the chords in line with the fundamentals representing them, which have been deduced in the previous chapters and that will provide us with much more information about the tonal vectors that are formed in a score.

In the sixth chapter we present chord progression examples with homotonic relaxions in weak tonal fields -that is to say, when there is no clear tonic, or when the latter changes constantly-, which happens when these local relaxions become more important, for, otherwise, the tonal tensions are predominant over the homotonic ones.

In the seventh chapter we may see some examples of homotonic and tonal analyses of fragments of musical works by several composers.

I have added four annexes that I already included in the work from 1994. The first one, which I have updated with new chord symbols for the most complex chords, carries out a functional analysis of every chord class that may be formed with the twelve-tone equal tempered tuning (that coincide with the "scale classes"). We prove that, altogether, there are 351 classes , and we select a representative for each one of them. In the second annex, we study and functionally classify all the modes that can be shaped with the eight main seven-note scales. In the third annex we organise the cyclic chords and cyclic modes , and in the fourth, the symmetric chords and symmetric modes .

Regarding the nomenclature that appears in the book, I often make use of the symbols that are used in English-speaking countries to specify the interval class:

m2: minor second / major seventh (also M7)

M2: major second / minor seventh (also m7)

m3: minor third / major sixth (also M6)

M3: major third / minor sixth (also m6)

P5: perfect fifth / perfect fourth (also P4)

Tritone: augmented fourth / diminished fifth

That is to say, if I use M2, I am both referring to the major second interval and to the minor seventh one (ex. C:D or D:C forms a M2 interval class).

When the chords appear in the middle of the text, the notes are written one after the other, with their eventual accidentals ; for instance, CEG♯B (the accidental always refers to the previous note, not to the next one, that is to say, ♯ applies to G, not to B).

1. Theoretical framework

1.1 The intervals that create more harmonic tension are those of M3 and tritone.

Chords (or arpeggios) tend to «resolve» locally to some specific notes and not to others, and the harmonic tension of the chord is determined by the intervallic structures formed by its notes. This tension has a local context , but also —keeping the previous musical tensions in our memory— it may also have a global sphere; in this case, to the strictly-speaking local tension of the chord, the «tonal tension» will be added, which will have more or less influence according to the grade in which the attraction towards the tonic has been established. However, these tensions are of a different kind, and thus we will analyse them individually.

The melody also has its harmonic and tonal tension, but this tension is shared with other variables of a more complex nature, and, sometimes, it is difficult to analyse it just in one theory. For instance, when the melodic line reaches a tonic or tonicized note, it produces a relaxion that may contrast with a harmonic succession of tension in the same place. The melodic logic is very important, but in this work we will not study other aspects apart from the purely harmonic ones. It is also important to make clear that when I refer to harmonic tension I am not referring to the greater or lesser consonance or dissonance of the chords (when I refer to it I will use the term "tension of sonance "). For instance, the dominant seventh chord GBDF is more consonant than the major seventh chord GBDF#, but the first chord produces more harmonic tension in the sense that it has a stronger need to be resolved than the second chord, which has more sonance tension (it is more dissonant), but is nevertheless more stable, tonally speaking. Many compositions end up with this dissonant chord. Therefore, despite the fact that they are interrelated, we will distinguish between these three kinds of tensions: tonal, local harmonic and "sonance" tensions.

In a chord, arpeggio or melody (regardless of the tonal memory), the intervals that produce more harmonic tensions are those of M3 and tritone (and their inversions). Furthermore, we will see that these two intervals are fundamental in shaping and explaining the local sensations of tension or relaxion between chords, and also, over a longer period of time, tonal functions. We find the explanation in the great strength the third harmonic has for the auditory system accompanied by its multiple harmonics (6, 9, 12, 15...), which form what is known as the fifth of the fundamental (for further details on the influence of harmonics, see chapter 2). The ear hears the intervals that are very close to the perfect fifth as "imperfect fifths", "impure fifths" or "out of tune fifths", the sensation being that of resolution when these “false fifths” match.[1]

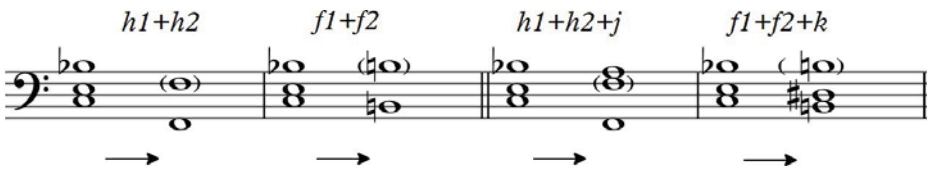
The interval classes that are closer to the perfect fifth (P5/P4) are the M3 intervals (major third or minor sixth) and the tritone (augmented third or diminished fifth). In the equal tempered tuning they are even closer.

The resolutions of these “quasi-fifths” intervals to perfect fifths (increasing or diminishing a semitone) are those that appear in figures 1 (M3) and 2 (tritone).

Fig. 1

Fig. 2

Fig. 3



As we can see in figure 1 in chord form, if we take the notes C-E as a sample of M3, E tends to resolve the «dissonance» to F, or C to resolve to B. With these two resolutions, the «quasi-fifth» is «tuned» as a perfect fifth. Later on we will see the harmonic consequences of it.

The case of the tritone is more complex, for, as we see in figure 2, it may resolve the «false fifths» in four different ways.[2]

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[1] The same could also be said for the second harmonic, the octave (see 2.3). But in this case we would be referring to the fact of “being out of tune” of the same note (the octave) that resolves to itself. The m2 interval is an interval that is close to “auditory masking “ (see page 33), which does not happen with M3 and tritone intervals.

[2] Be that as it may, we must say that when the tritone is accompanied by more notes in a chord, the auditory system quickly positions the correct enharmony of the tritone notes. For instance, if we hear B-D-F, the auditory system hears the virtual fundamental G with its M3 B and its m7 F, but if we hear B-A♭-F, the ear captures the virtual fundamental D♭ with its M3 F and its m7 C♭ (not B).

If, as an example of tritone we take the notes E-B♭, the notes that resolve the «false fifth» to a perfect fifth (P5/P4) are F, B, A and E♭.

To sum up, of the two eventual "quasi-fifths" or "false fifths",[3] one, the M3 interval (EC/CE), produces a harmonic tension that is resolved by the notes F or B, and the interval of the tritone (EB♭/A♯E) creates a harmonic tension that is also resolved by the notes F or B (in addition to A and E♭), in all cases to adjust the fifth. We should note that the other two resolutions of the tritone (A and E♭/D♯) are the major thirds of the main resolutions F and B; therefore, the simultaneous resolutions of F and A are compatible, for they are part of the F major chord (or, in other words, A is the 5th harmonic of F —as a "resonance"—). Something similar happens with B and E♭ (D♯), for, in the tempered tuning, E♭ may be heard as D♯, although in this case the joint resolution does not seem so natural.[4]

If we consider the two intervals together, we obtain the resolutions of figure 3. These resolutions are boosted in the tempered tuning.

I call the local resolutions of these intervals **homotonic resolutions**. As we have already mentioned above, and in order to make it more comprehensible, everything that is said in this chapter about some representative notes can obviously be applied to all their transpositions.

From all that has been previously said, we can now state that:

1.2 The notes that form the M3 intervals (CE/EC) and/or tritone (EB♭/A♯E) locally resolve to the notes F or B, or to the chords that have these fundamentals.

That is to say:

- The tone or the fundamental F resolves the tension of M3 (CE/EC) and the tension of the tritone (EB♭/B♭E). We will call it resolution of 1st order or **htonal resolution** (simplification of homotonic-tonal).

- The tone or the fundamental B resolves the tension of M3 (CE/EC) and the tension of the tritone (EB♭/A♯E). We will call it resolution of 2nd order or **Phrygian resolution**.

We should observe that these two resolutive notes F and B are located on opposite poles in the circle of fifths and forms a tritone interval.

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3 Formerly, the interval of diminished fifth was precisely known by this name: false fifth.

4 Scholastically, perhaps it would be possible to put A♯ instead of B♭, but I prefer to write B♭ if C is in the bass, for although B♭ and A♯ are at the same distance from E in the circle of fifths, the 5th and 7th harmonic of C are E and B♭ (not E and A♯). And this is so despite the fact that we know that the harmonic B♭ is lower than the tempered B♭. Similarly, B♭ is closer to C than A♯ in the circle of fifths.

I insist again on the fact that I take (C-E) and (E-B♭) as representative notes, but, obviously, the wording refers to every interval of M3 and tritone, and in any inversion. Generally speaking, we could also say that the M3 interval resolves to any one of its two "external" adjoining notes at a distance of a semitone when the interval is in the form of a third, or to any one of its two "internal" adjoining notes at a distance of a semitone when it is in the form of a sixth, and something similar for the tritone; however, it is a more complex wording than just putting specific notes as an example.

We should note that, in the two intervals that have the same resolution, a note appears twice, E; therefore, this note is of fundamental importance in harmonic tensions (provided that it is accompanied by the other two notes of the two intervals: C and B♭). We know that this note is known, in music theory, as the *leading-note* note of the tonality (in this case, F).

These resolutions of the "quasi-fifths" or "false-fifths" do not explain *per se* alone the phenomenon of tonality (tendency of a group of notes to resolve to some specific notes —more than to a specific scale—). To comprehensively explain tonality we should add to it the effect that the phenomenon of the harmonics of a sound provokes in the harmonic relations established by the auditory system.

The harmonic 2 (the octave, with its resonant "harmonics" 4, 6, 8, 10, 12, 14, 16, etc.) makes it possible for the auditory system to identify a sound with its octave, because this sound and its octave have many harmonics in common (50%), and, therefore, due to the effect of the virtual fundamental (see 2.2), the auditory system considers the octave as the "same" note, but "with a different 'timbre', a more acute one". This is the reason why this second harmonic is essential in establishing the laws of harmony, and it really simplifies its principles (what the brain does whenever it can in order to save a great deal of information and energy), since, by matching the notes and their octaves, it very much reduces the "number" of possible notes.

The next most important harmonic is the 3rd one (the fifth, with its "harmonics" 6, 9, 12, 15, etc.). The fact that the fifth has a third of the harmonics of the fundamental is not enough any longer to identify fundamental with harmonic, but it is the note (different from the 8th) that best fits the harmonics of the fundamental. The auditory system does not consider it "the same note" but is certainly "the more familiar" with it. To put it in a more colloquial form: if C is the fundamental, as if G were the "fruit" of C and were generated by it . Therefore, if we play G and then C, the auditory system seems to recognise C as generating G, for it perfectly fits its harmonics and therefore produces a "resolution" effect. Furthermore, as we will see, the main harmonics of a tone actually form a kind of dominant seventh chord, which also makes its tendency towards the lower fifth easier (see details in chapter 2).

Therefore, of the two resolutive tones F and B of the intervals CE and EB♭, one of them, F, has an added relaxion because it is the lower fifth of C. C is the fundamental of CE (E is the 5th harmonic of C), C is the virtual fundamental of EB♭ (E and B♭ are the 5th and the 7th harmonic of C, see 2.2), and C keeps on being the fundamental of CEB♭. Therefore, the F tone, which generates C, has another kind of "hierarchy" with respect to the other resolutive note B, and has a relaxion of a different kind to that of the "quasi-fifth" tuning. It is a resolution of a "tonal" type, and this is the reason why I have called this local resolution htonal resolution. I do not call it "tonal relaxion", for it could be confused with a succession towards the tonic. A local htonal homotonic relaxion may happen —in fact it happens habitually— even though the fundamental to which it resolves may not be the tonic. The tendency to F of the structure made up by the notes CEB♭ is the basis of the tonality.

These two attractive forces that we have studied above (that of the resolution of "quasi-fifths" and that of the recognition of a harmonic generator) are the principles that explain and generate tonality, the tendencies of a group of notes to resolve at each temporal moment to (one or several) specific notes.

1.3 The fifth of the fundamental is the less important note as regards the chord's harmonic function.

As we have seen, the note that has more harmonics in common with the fundamental is its octave. They have so many in common that the auditory system, due to the phenomenon of the missing fundamental (or virtual fundamental), regards it as being the "same note"; therefore, for a chord to have or not have the octave does not alter its harmonic or tonal function.

The following note that has more harmonics in common is the fifth, and, similarly, it is so strongly united with the fundamental that the fact of being or not being in the chord does not seriously alter its function (unless it forms new intervals of M3 or of tritone with other notes of the chord).

The meaning of the fifth as a powerful and consonant interval is often confused with the power to provide functional support to a fundamental. The intervals of octave and fifth are the more consonant and closer to the fundamental ones, and are therefore those which are **less** functionally important when they appear inside the chord. In fact, the fifths of the fundamentals are so powerful that they are heard as harmonics even when they are not in the chord. As we have seen and will see later on, the tendency to "tune" the perfect fifth builds up the tensions and relaxions between chords, but once we have obtained the perfect fifth, the tension disappears.

1.4 The 7M3 structure produces a tonal vector.

What I call 7M3 structure refers to the intervallic structure formed by the notes C, E and B♭ in any transposition and inversion. The 7M3 structure is the sum of the tensions of M3 and tritone (tensions which “want” to resolve to the notes F and B). Seen from the fundamental C, we have the intervals M3 and m7 (that accounts for the abbreviation 7M3).

The notes that form the 7M3 structure (CEB♭ in F) are known in music theory as dominant (C), subdominant (B♭) and leading-tone (E) of the tonality (in this case of F), and, as we have already seen, they have only been deduced from the common resolution of the “false fifths”. We have not made use of any scale. The presence of these notes or of these intervallic relations in a melody or in a chord produces a tonal vector towards the tonic F (a tendency to resolve to this note); that is to say, towards the lower fifth of its fundamental C (in addition to the homotonic local Phrygian tendency towards B). We should note than when I refer to subdominant or dominant, I am not referring to the chords of subdominant or dominant, but only to THREE notes. Obviously, the dominant chords (in the F tone) contain CE, and those of the subdominant contain B♭, but the chords of this last tonal function may have the major or minor 3rd (of B♭).

The M3 structure (C-E) alone also creates a tonal vector towards F, and the same happens with the tritone (E-B♭). When the two intervals appear at the same time, the vector is strengthened. The tonal vector of M3 is more powerful (because it contains the fifth of the tonic F, the dominant). In the case of a “competition” between two or more tonal vectors, the M3 of the last chord that can be heard always has the preference (always taking into account the Phrygian resolution of the 7M3 structure).

Fig. 4

In a short musical fragment we may easily find two or more 7M3 structures; for instance, a musical fragment containing the notes B♭-C-D-E-F# has the structures CEB♭, DF#C and F#A#E. We have three vectors towards F, G and B (see figure 4). The “winning” vector will depend on the position of the notes, on whether the auditory system hears B♭ as B♭ or A#, and on the last chord or arpeggio. Usually, the last M3 heard is determinant. In these specific examples of figure 4 we have also added the melodic Phrygian cadence in favour of B (E-D-C-B); therefore, most probably, if we were to write the five notes randomly, the more resolutive note would be B. If we put all the notes together as a chord, the more resolutive note is also B.[5]

There is another structure that also creates a (less powerful) tonal vector: the intervallic relation made up of two m3 (EGB♭) due to the phenomenon of the virtual pitch (see 2.2). The reason for it is that this structure makes the ear hear C as a chord's missing fundamental , and, therefore, we are actually hearing the (C)EGB♭ structure; that is to say, even though the C is virtual, we have incorporated the 7M3 structure (with the fifth of the fundamental added), C being however virtual. So, EGB♭ also creates a tonal vector towards F.

The well-known dominant seventh chord already creates alone a powerful tonal vector, for it has these three notes available that play a leading role in the same chord, being the fifth of the fundamental optional or even alterable.

In figure 5 we may find some examples of (atypical) cadences due to the tonal strength of the 7M3 structure, in this case transposed to GBF. A flat between parentheses means that we may play this natural note or flat without affecting the resolutive sense of the progression. Therefore, each measure of the examples has between 4 (2 parentheses) and 32 (4 parentheses) possible cadential combinations, and the double if we consider resolution in major or minor.

Fig. 5

5 We have seen that the 7M3 structure also resolves "in a Phrygian way" to B. F♯A♯E also resolves in the Phrygian way to E♯, but the ear does not identify the Phrygian resolution to E♯ with the tonal resolution to F of CEB♭ due to the other note D, which provides support to F♯ and not to G♭ (the ear recognises F♯A♯E and not G♭B♭F♭). If the note D were not there, CEB♭ and F♯A♯E would be two symmetric chords at distance of tritone, and then F and B, resolutively speaking, would be "tied".

In many combinations of figure 5, new 7M3 structures appear that may make the cadential sense «vibrate». For instance, if we have D♭ and E♭, we then also have the 7M3 E♭GD♭ structure, and we obtain a tonal vector added towards A♭ (figure 5c bis). or, also, in the cases of considering notes A and E♭, we then have the 7M3 FAE♭ structure with tonal vector towards B♭, which is «refuted» when hearing afterwards the B♮ of the dominant chord. We would even find more 7M3 structures hidden in these examples.

But if in the group of cadential notes where a new 7M3 structure is created we find its associated tonic augmented, we will note that this makes the power of the tonal vector diminish towards this new tonic; for instance, B♮ in FAE♭ or A♮ in E♭GD♭.

Furthermore, also in the examples of figure 5, the altered notes of the very GBF structure (G♯, B♭, F♯) could be added in the first chord, and even some in the second chord, although this would increase the possibilities of finding other stronger 7M3 structures (closer tonalities), and thus diminish or fully eliminate the cadential effect.

The only scales that contain a single 7M3 structure are the major diatonic scales and the minor scale with leading-tone (harmonic minor) (see 5.2). Probably, this was the reason why when all the modes were progressively incorporating the tritone into close notes (most probably since Glaureanus' *dodecachordon*) their decline began, because the incorporation of the 7M3 structure into the modes (in scarcely distant environments) entailed their amalgamation in the two aforementioned scales.

1.5 The htonal and Phrygian resolutions of M3 and tritone account for most cadences, secondary dominants and cadential harmonic progressions.

Besides the examples of (non-usual) cadences that appear in figure 5, many of the typical cadences and harmonic progressions that appear in the treatises on harmony also provide an explanation based on the resolutions of the tensions we have seen above. In figure 6 we may observe the most common cadential processes. We have used the notation (symbology) of chords and tonal functions that will be employed throughout the book and that will be explained in the following sections: 3.2-4 and 5.3. For the tonal functions of these chords, see chapter 5 devoted to tonality.

The **authentic cadence** is the htonal resolution of M3 (CE→F), with an increased effect when the fundamental has m7, because it also resolves the tension of the tritone that is formed with E (EB♭→F) (figure 6a).

The plagal cadence in the major mode does not take place due to the local resolution of a tension that creates a chord but (when a tonality is well-established) to the resolutive sense of resting on the tonic (chord), in this case from the subdominant . However, the Locrian secondary homotonic relaxion (A♭-C) (see 4.1) does happen in the minor mode. Plagal cadence is less resolutive than the authentic cadence, and, in effect, as a succession of fundamentals, it is a local succession of tension that, contrasting with the tonal relaxation, gives this very characteristic colour to it. In fact, there is the so-called plagal half cadence (I-IV) (suspension in the IV degree instead of the V), which is actually an htonal resolution of M3.

Some of the **half cadences**, such as the so-called Phrygian one (figure 6b), are cadential progressions helped by Phrygian resolutions of M3 (CE→B). The half cadence that comes from the "dominant of the dominant" forms a htonal relaxion. Nevertheless, in a context in which tonality is well established, it is better to interpret a half cadence as a kind of suspension or "expectation" in the tension of a very familiar chord (the dominant chord), although the tension is not fully resolved. We could say that the ear is so much used to the authentic cadence that it regards it as being assimilated in the chord previous to "consummation" and allows the tension to remain suspended and not resolved, at least momentarily.

The **deceptive cadence** (figure 6c) is similar to half cadence in the sense that it is not a complete resolution of the tension, but the cadential progression is helped by the htonal resolution of M3 to two of the three notes of the 6th degree chord (in F major CE to (d)FA and in F minor CE to (D♭)FA♭). The deceptive cadence in the major mode is in fact an htonal succession of functional fundamentals C→F, if we consider F as being one of the two fundamentals that represent the D minor chord (in the following sections of this chapter we will explain the true functional fundamentals of the chords, as happens in the case of the minor chord).

The **Neapolitan sixth chord** is used before the dominant chord in a cadence, and the most frequent resolution consists in placing the $^6_4$ cadential chord (the tonic chord in 2nd inversion) between the two chords (figure 6d). In the first case (without the cadential $^6_4$), we find two chords that, together, contain the cadential 7M3 structure (F♯A♯E), as we have seen in figure 5. In the second, more frequent, case, there is an explanation to be added: the tension of M3 of the Neapolitan sixth chord is resolved in a Phrygian way with the fundamental B of the tonic chord (it is a well-known fact that the cadential $^6_4$ is perceived both as a tonic chord and a dominant chord with two appoggiaturas).[6]

There are also cases of chords with a structure of Neapolitan sixth that resolve directly on the tonic by taking benefit of the Phrygian resolutive power.

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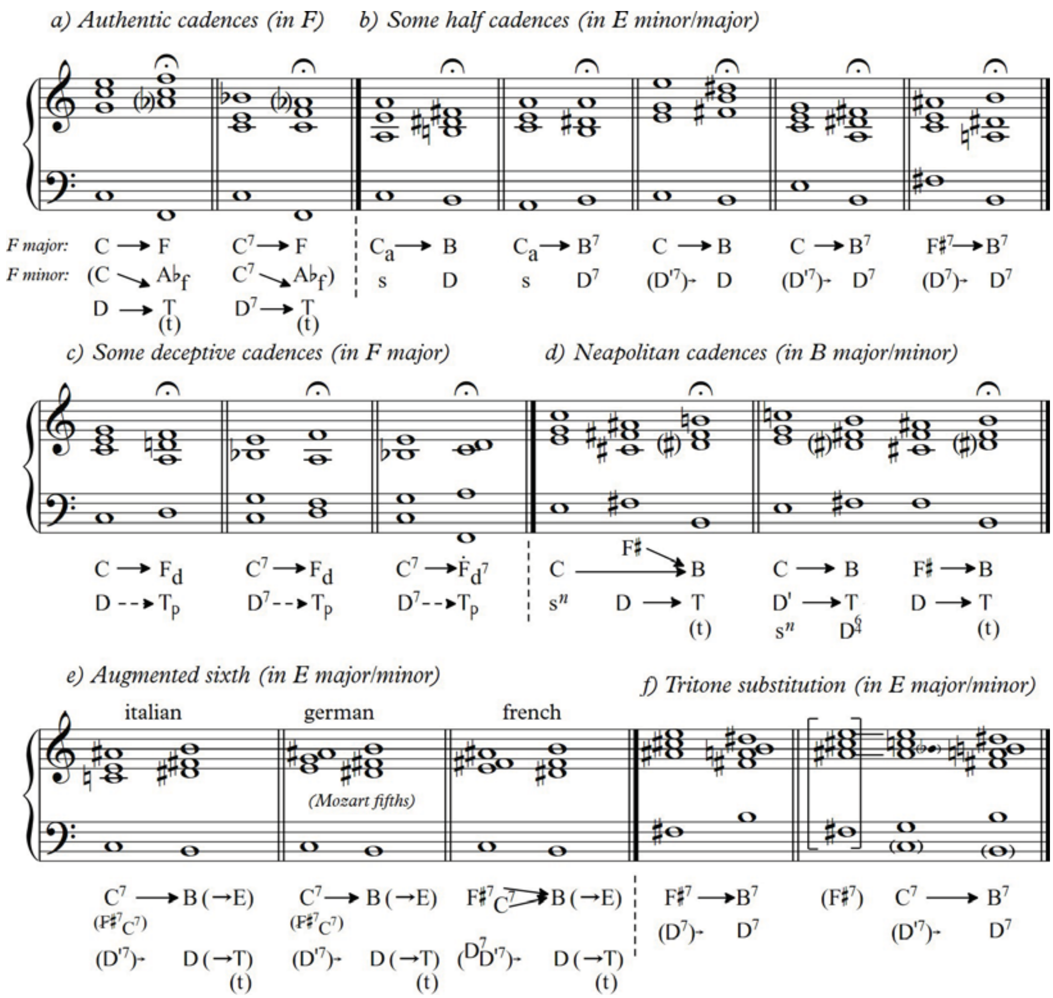
[6] In the major mode if we consider the cadential $^6_4$ as a pedal of the dominant in the bass (C in F major) and above the succession of chords (FA)-(EG); in this case note that (EG) is a Phrygian resolution of (FA).

Fig. 6



The chords of the lowered 2nd degree (as in the case of the Neapolitan sixth) may be regarded as having a subdominant function if they have the subdominant in the bass (s^n) and precede the dominant, or they may also be considered Phrygian dominants (D'), especially if they have the fundamental in the bass and precede the tonic. See more details in chapters 5 and 7.

There are several **chords of augmented sixth** (figure 6e). The Italian augmented sixth chord and the German sixth are chords with a structure of dominant seventh (that is to say, they contain the 7M3 structure: CEB♭), but with B♭ enharmonised in A♯ because it plays the role of a melodic leading-note. In effect, this chord benefits from the slight virtual fundamental that is located at a tritone distance of the fundamental of every dominant seventh chord due to the two virtual fundamentals that every interval of tritone has; for instance, (C)EB♭ and (F♯)A♯E.

The difference between the German and the Italian augmented sixth chords is found in the fact that the German has the fifth of the fundamental and the Italian does not (most certainly to avoid "Mozart's fifths"). Both make use of the Phrygian resolution of CEB♭ to resolve to the dominant (C→B in E) —or an htonal resolution from the point of view of the other hidden virtual fundamental (F♯→B).

The French augmented sixth chord makes the hidden virtual fundamental F♯ become real and, therefore, it is a chord with two 7M3 structures at a tritone's distance. The resolution of the French augmented sixth to the dominant is a curious case of both Phrygian and htonal resolution at the same time. One of the two structures resolves in a Phrygian way (CEB♭(A♯)→B) and the other does it htonally (F♯A♯E→B). The French augmented sixth chord also seems to be a solution to avoid "Mozart's fifths" by adding an anticipation of the fifth of the dominant.

In his *Treatise on Harmony*, Tchaikovsky regards augmented sixth chords as "dominant" chords that resolve to the tonic (from the lowered 2nd degree), not to the dominant (or else to the dominant as a local modulation); therefore, he considers Phrygian dominants (D') in the Italian and German cases, and a mixture of dominant Phrygian (D') and tonal (D) in the French case.

A dominant 7th chord may become another 7th dominant chord if we raise the fundamental half a tone, we lower the fifth half a tone and we leave the notes making up the tritone unchanged. This is what in jazz theory is called **tritone substitution**. This theory posits that this change of dominant sevenths can be made without it affecting the chord's harmonic function. Actually, it is similar to the use of the classic augmented sixth according to Tchaikovsky's views. That is to say, this substitution, which provides another fundamental at a tritone's distance, changes an htonal resolution in Phrygian resolution and a Phrygian resolution in htonal resolution (figure 6f).

All these examples of cadences and secondary dominants that appear in figure 6 only make use of the resolutions C→F, C→B and F♯→B (as functional fundamentals of the M3 and tritone tensions), that is to say, htonal or Phrygian resolutions. Moreover, the theory of the tritone substitution proves that these two types of resolutions have a tonal function similar to that of the dominant (understanding the dominant as a chord that resolves its tensions to a tonic or tonicized chord). That is to say, the major or dominant chords (see 3.1) whose fundamentals are a 5th above / 4th below or a semitone above the fundamental of the chord to be resolved can be used as a dominants chords (secondary or not). In other less complex words and making use of the terms employed in our research: a (secondary) dominant uses an htonal (D) or a Phrygian (D') resolution above the (passing) tonic chord. Most cadential successions used in tonal music may be summarised with these two kinds of relaxions.

Given the fact that M3 and tritone's intervals and their two main resolutions (htonal and Phrygian) are so important and help us understand a significant part of the tension resolutions of harmonic progressions in music analysis and in composition, would it not be possible to identify in a quick way these intervals within the chords, despite the degree of their complexity? The answer is "yes", and we will be able to find the solution thanks to a stunning coincidence.

1.6 A fundamental and its main harmonics also have a 7M3 structure.

In 2.1 we prove that the harmonics that are really important for the auditory system include at the most the first seven ones: 2, 3, 5 and 7 (4 and 6 are octaves of the first ones). See the discussion on the importance of each one of these harmonics when establishing the harmonic and tonal laws in section 2.1. That is to say, based on a fundamental C, the main harmonics are those corresponding to the notes C (2), G (3), E (5) and B♭ (7) (!!) (this E natural being a bit low in respect to the tempered E or the Pythagorean E; and B♭, even lower in respect to the tempered B♭ or the Pythagorean B♭ [see 2.8]). In other words, the 7M3 (CE(G)B♭) structure is included (with the fifth of the fundamental added).

Does this mean that when we hear only one note in effect we are hearing a sort of hidden dominant seventh chord? Somehow this is true, and a proof of it is that we have a stronger sensation of resolution when after a note we hear the lower fifth than when we hear the lower eighth, although, according to the logic of the harmonics weight, the opposite should happen, since the 8th is the 2nd harmonic, and the 5th is the 3rd.

Therefore, the significant harmonics of a sound include the 7M3 structure, always taking into account that we are not referring to a real chord because the notes corresponding to the harmonics are sinusoidal waves (fluted timbre) (see 2.1 and 2.2), and that there are important differences in tuning between the tempered minor seventh and the "natural" seventh (see 2.8), and also slight differences with the M3.

That said, we could ask ourselves: if the 7M3 structure is found in the harmonics and is therefore a natural phenomenon, why does such a natural structure, despite being real notes, create tension?

The main answer is that the harmonics that build up the musical "laws" in our brain are the octave and the fifth (see chapter 2). The harmonics 5 and 7 are too structurally weak, and not because, occasionally —timbrically in some instruments—, they can even be more strongly heard than the harmonics 2 and 3, but because the perception of the harmonics that the brain processes from childhood follows a formal and mathematical structure where the two first prime numbers have a fundamental importance (see Fourier transform and figure 14 in chapter 2). The harmonics 5 and 7 contribute to the «consonance» of the dominant seventh chord made up of real notes but do not resolve the more powerful harmonic tension that is produced by the two real “quasi-fifths” they contain. We should add to it the supplementary tension that is produced by the equal-tempered tuning, which is unnatural.

This surprising fact allows us to elaborate a notation (actually a symbology) of the chords so that at first sight we know whose M3 intervals and whose tritone’s ones are included in them, and, at the same time, show the authentic fundamentals of the chords according to the harmonics structure.

The method is very simple:

The CE or CEG chord will be simply notated as C (for E and G are its two main harmonics [after the octave]).

Optionally, if the fundamental has its fifth, we may write a dot above (Ċ).

We will notate the CEB♭ or CEGB♭ chord as C^7 (since B♭ is the next harmonic —and the less— important one).[7]

We will notate the EB♭ or EGB♭ chord as Ȼ^7 (because C is the virtual fundamental of the EGB♭ harmonics). In the case of the tritone we have seen above that it has another virtual fundamental at a tritone’s distance (F♯) (very weak when the chord contains G).

When a fundamental only has the octave or the fifth (that is to say, neither has its M3 nor the tritone), we will spell it in a lower-case letter. For instance, the two-note chord CG will be notated as **c** or **ċ**.

As previously mentioned and if we prefer, in order to differentiate the chords with the fifth of the fundamental from those which do not have it, we may write a dot above the notation (or else write it in bold when we use some word processor). It might be useful in the analyses of some works with a very simple harmony; however, nothing happens if they are not differentiated in practice, for we have seen in 1.3 that the chord’s harmonic function scarcely changes whether the fundamental’s fifth is added or not; however, it is very important to note that this only happens as long as adding the fifth entails that no other M3 or tritone interval is formed (!!) (do not confuse harmonic function with sonance —chord’s consonance/ dissonance— because adding a fundamental’s fifth or not may really alter its sonance significantly).

This notation of the chords’ fundamental structures according to the harmonics’ structure appears in figure 7.

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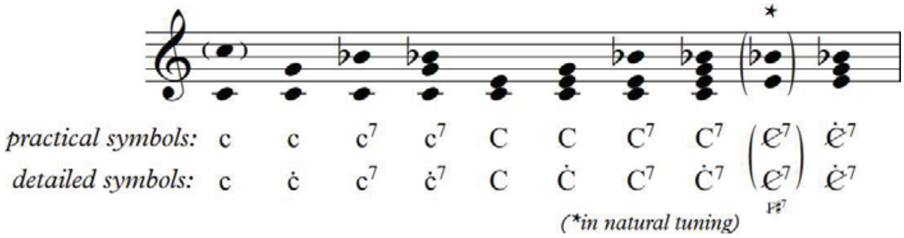
[7] In my book, *La convergència harmònica* (1994), I used the symbol ° instead of 7, but I prefer to change it here due to the use made of symbol 7 to indicate the minor 7th, especially in jazz and modern music treatises.

Fig. 7



Therefore, to know if a chord has the tension of M3 or that of the tritone, we will simply have to make sure that there is some upper-case letter in the notation (if a 7 appears, we will know that it further has the tritone tension), whether crossed or not. We will also learn that its homotonic local resolutions will be —in the case of capital C— towards the fundamentals F or B. The resolution of C may be to F or B in upper-case letters, or to **f** or **b** in lower-case letters (!!), because here we are referring to the resolutive chord and, therefore, it may have or not have a new tension. We will call the fundamental, which represents the M3 and/or triton's tensions, spelled in capital letters, **functional fundamental** in order to differentiate it from the fundamental in lower case.

However, we see that in figure 7 only a small portion of the chords appears. What happens with the minor chord and all the others?

We will now deal with the next important point.

1.7 Most chords may be separated into one or two chords having the harmonic CEGB♭ complete or partial structure.

I call this harmonic CEGB♭ structure, whether complete or partial, convergent structure (its notes converge towards the fundamental C). That is to say, according to this formulation, most chords may be represented with one or two fundamentals, each one representing its part of the structure —therefore representing its tensions, as we have seen in figure 7.

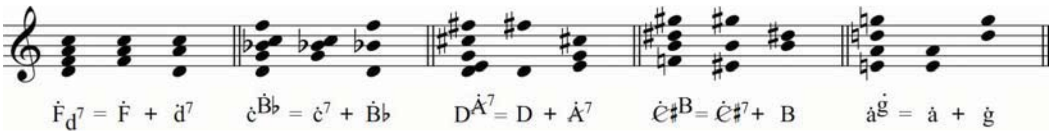
This has been demonstrated in annex 1, where we may see a complete list of all possible chord classes represented by these fundamentals.

This is the reason for the title of the following section.

1.7.1 We may summarise the harmonic tension of most chords with one or two fundamentals, which results in eight large families of chords.

Some examples of this separation or breaking down into CEGB♭ convergent structures of several chords may be seen in figure 8 (see 8.4 as well).

Fig. 8



The symbology of these chords will provide us with some information on the resolutive tendencies of the chords on a local level. For instance, if we find a capital G (even if it has been crossed), it means that the chord has a tendency to resolve locally or to become linked to other chords that have C or F♯ as fundamentals (with capital o lower-case letters; the heightened sense of relaxion or resolution will depend on what the other notes of the chords and the other fundamentals in case they have more than just one will be, and, obviously, on the tonal context in which they are immersed: the tonal field). If, in addition, this G has a 7, the tendency to resolution —to the same notes— will be strengthened. In this case, since G is a capital letter, it is a functional fundamental. In principle, this symbology does not distinguish between inversions (to distinguish inversions, see 3.4).

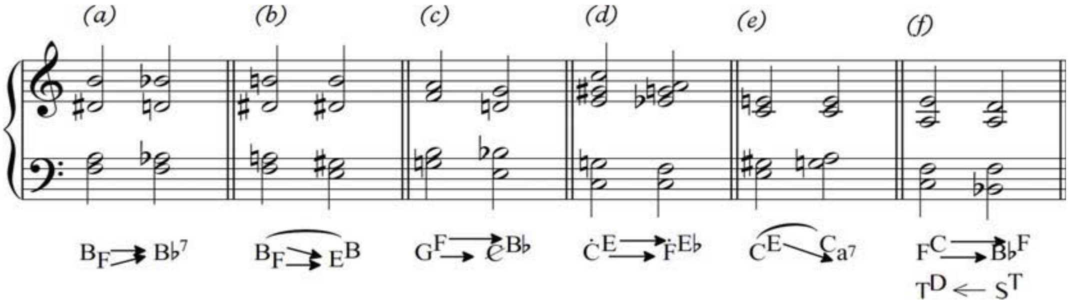
When we find one M2 among the fundamentals, it means that we have a powerful tonal vector (the fundamental of the higher M2 must be functional, that is to say, notated with capital letters), for it will mean that, together, both chords contain the 7M3 structure (see figure 10). For instance, the fundamentals B♭/b♭ and C determine a tonal vector towards F; C must be spelled in capital letter (it contains the CE interval) and B♭ may be spelled in capital or in lower-case letter.

For further details on the meaning of the notation or symbology that we make use of, and the building and characteristics of chords' families, see chapter 3. We call this notation **fundamental symbology**, for it is made up of the chords' real or virtual fundamentals, according to the convergent separation that we have previously explained.

1.7.2 We can apply the fundamental symbology of chords to create or find local relaxed progressions of chords.

Regardless of the tonal forces that appear in a musical fragment we can use combinations of htonal and phrygian resolutions between fundamentals to link chords in a relaxed or fluid manner. In Figure 8bis we have some examples of this, using chords that have more than one fundamental. These examples are only local resolutions because if we are immersed in a tonal field we must always take into account the function of these fundamentals in the tonality. For example, in Figure 8bis (f) we have left-right homotonic relaxions; but we may notice that we have an M2 between fundamentals (B♭-C). As we have seen, this implies that with the notes of the two chords we have a 7M3 structure (CEB♭), which creates a tonic F, which is precisely the main fundamental of the first chord, that is, we also have a relaxion, in this case tonal, in the opposite direction, from right to left. There is relaxion in the two directional senses, one with homotonic color and the other with tonal color.

Fig. 8bis



We have devoted chapter 6 to see some examples of these local homotonic relaxions.

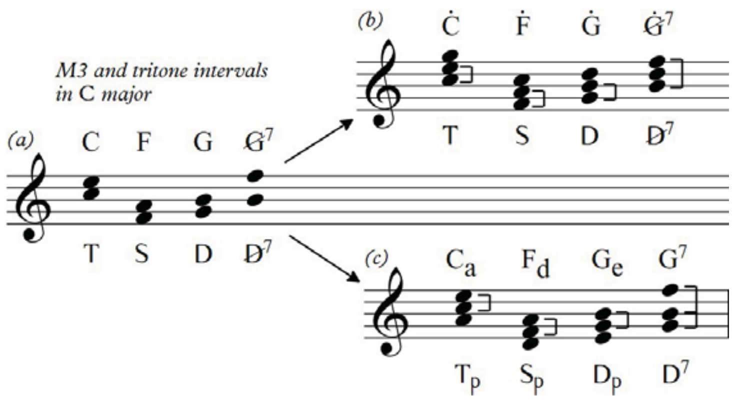
1.8 The reduction of tonal functions to three (tonic, subdominant and dominant) may be explained by means of the tensions of M3 and tritone intervals.

It is a well-known fact that at the end of the 19th century, Hugo Riemann reduced the chords' tonal functions (at least within the diatonic scales) to three different «types»: the functions of tonic, subdominant and dominant (most probably, as an evolution of Rameau, Daube and Jones' previous works). Riemann regarded the major triads of tonic, subdominant and dominant as the primary «harmonies», and maintained that every other secondary chord in major or minor modes also (and only) adopted a tonic, subdominant or dominant, «meaning». This theory was rather well accepted, especially in Germany, so that many treatises on harmony, in the tonal analyses, replaced the Roman numerals of the theory of scale degrees (I, II, III, IV, V, VI, VII) with the symbols based on T, S and D (T_p, S_p, D_p, $\not{D}^7$, etc.).

We will see that this is a straight consequence of considering the really «functional» intervals: those of M3 and tritone.

In effect, if we take the M3 and tritone intervals that appear in the major mode, we will obtain figure 9 (a). In it, above the pentagrams we have written the fundamental symbology, as we have seen in 1.7, and below, the symbols used by Riemann and his adherents. Note that only three functional fundamentals (C, F y G) appear, those corresponding to the tonic, the subdominant and the dominant. If in (a) we complete the triad on the upper part, that is to say, if we add the 5th in the first three chords and the minor 3rd in the fourth, we obtain the chords that appear in (b), which have the same function as those appearing in (a) because, as we have seen above, the 5th of the fundamental does not have a great influence on the variation of the chord's "tension" and therefore on its function. The symbols representing the tonal function (T, S and D) keep on being the same.

Fig. 9



On the other hand, if we complete the triad in its lower part (we add a lower minor 3rd to the first three chords), the situation changes: the major chords become minor but, by keeping the M3 interval, they still keep its tension, and therefore keep a large part of their function; Riemann's school writes the symbols that appear in the lower part in (c), keeping the symbols of T, S and D, and adding below a lower-case **p** (in the upper part of the pentagram we find again the separation of the chords, in this case, the first three ones are minor chords that break down into two fundamentals, although the functional fundamental is still the same.[8]

In the minor mode, the symbology used by Riemann and his adherents (Grabner, Maler, Motte) is much more complicated and has actually undergone continuous revisions, its global result being a labyrinth of symbols; however, the base is still the same: the use of the T, S and D in capital or lower-case letters (D^+, $°D_p$, $°S_p$, S_g, Dl, T_g, dP, sP, dL, tG…) to represent the tonic, dominant or subdominant functions.

In this book, we will write the symbols of the tonal functions based on Riemann's notation; however, in order to maintain the coherence with the fundamental symbology that we have seen in 1.7, a new revision of the Riemannian symbology will be unavoidable for the more complex and chromatic chords (together with the contributions of Ernö Lendvai's tonal axes theory). As a sample in figure 10 we show the skeleton of a typical T-S-D-T cadence both in the major and minor modes.

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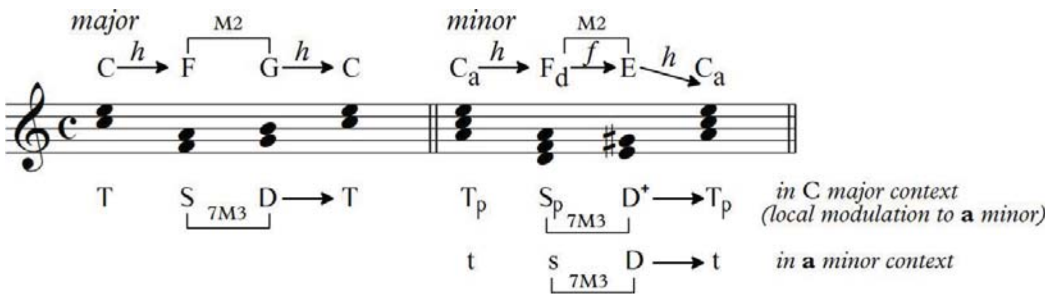
[8] Let us observe that an m3 interval also has a very weak virtual fundamental in the note corresponding to the lower m3, that is to say, (C)EG(B), (F)AC(E). Riemann, who even changed the whole symbology when these chords adopted this function in some harmonic progression, also accepted this.

Fig. 10



In the case of the minor mode, in figure 10 we write the functional symbols first from the point of view of C major (local modulation in A minor), and below, the symbols in a context in A minor: ***h*** means htonal resolution and ***f*** the Phrygian resolution. The chords based on S and D contain a 7M3 structure and establish the key (tonality) of C. Likewise, s and D (also S_p y D^+) contain a 7M3 structure, in this case of A, creating a tonal vector towards A minor.

Obviously, figures 9 and 10 are only schemes of intervals and chords , and not an example of voice conduction.

The M3 interval thus defines the primary structures of T, S and D, and the tritone, by means of its virtual fundamental, the structure of Đ virtual dominant.

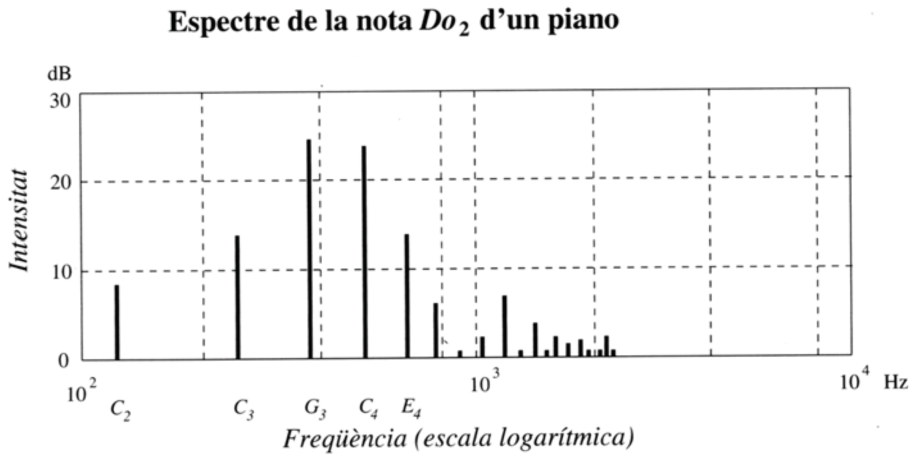
For further details on tonal functionalities, see chapter 5 devoted to tonality; for a summarised general overview of the whole theory, see the recapitulation in 5.8.

2. The harmonics of a sound as the basis of musical perception

2.1 Significant harmonics for the human auditory system

If we play C_2 on the piano (corresponding to a frequency of 131 Hz), the acoustic spectrum is approximately as follows:

Fig. 11



The vertical lines on the graph show the intensity of each of the harmonics of the note played.[1] This sound structure, a combination of simultaneous sounds with the same basic timbre (sinusoidal waves) but of differing intensity, enters the cochlea and excites a number of hair cells, each of which is sensitive to certain different frequencies. These frequencies travel along nervous fibres to the auditive cortex of the brain. This is where we identify the combination of sounds in figure 11 as "one" note on the "piano". That is, each sound is the sum of a collection of sounds of elemental timbre. This elemental timbre is always the same for all sounds; is the sound corresponding to a sinusoidal wave. [2]

Although we have a collection of frequencies, the brain summarizes them all in one (which has the greatest common divisor) and is a frequency that may not exist in the collection of sinusoidal frequencies we have (!) (later on we will see the importance of this when we study the virtual fundamental). The information we

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[1] H.F.Olson (1967). This spectrum changes with time, from the moment the key is pressed until silence returns.

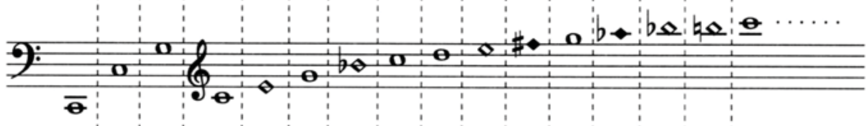
[2] According to the Fourier's theorem, any periodic wave (sound is a periodic wave) can be decomposed as the sum of sinusoidal waves.

receive from the "volume" of each harmonic the brain makes it a concept we know as a *timbre* and it is very useful to recognize what produces the sound and distinguish, for example, between the voices of friends or different musical instruments. The fundamental note, as shown in figure 11, does not have to be the most powerful one, even, as we say, it may not exist. What matters is the pattern, the relation that the frequencies of the harmonics establish with each other.

From birth, the brain is familiar with these frequency relations of the harmonics that form a sound, which are "intervallic" relations that always follow the same pattern and at first sight it is very simple: if *f* is the frequency of the key of the piano (the fundamental sound), the harmonics series have frequencies *2f, 3f, 4f, 5f, 6f,* etc. (the double, triple, quadruple, quintuple and so on of frequency *f*).

But translate this pattern of frequencies into musical notes is not so simple. In figure 12 we have represented, in arpeggio form (from a C_0), this pattern-chord translated to musical notes —with its divergence with respect to the three main tuning systems—. The curved drawing of figure 12, instead of the straight line to be expected from the function *nf*, is due to Weber-Fechner's law which states that sensations are in logarithmic relation to the stimuli. Thus, for example, a frequency which is 4 times higher (*4f*) than another (*f*) does not involve musically 4 octaves but only 2. The same thing happens with the sound intensity (thanks to that in the orchestras there can be many more strings than other instruments). A negative number indicates that the *pitch* of the harmonic is lower than the *pitch* of the note in the tuning system. All numbers are rounded to integers.

Fig. 12



	f	2f	3f	4f	5f	6f	7f	8f	9f	10f	11f	12f	13f	14f	15f	16f	……
(equi)Temperada	0	0	+2	0	-14	+2	-31	0	+4	-14	-49	+2	+41	-31	-12	0	cents
Pitagòrica	0	0	0	0	-22	0	-27	0	0	-22	-37	0	+25	-27	-22	0	cents
Zarliniana	0	0	0	0	0	0	-15	0	0	0	-17	0	+27	-15	0	0	cents

100 cents= mig to (equi)temperat (1 octava= 1200 cents)

Of these collections of notes and intervals, which are those that the brain «takes into account» to construct «musical laws»: scales, consonance, chords, sensations of tension and relaxion, tonality, etc.?

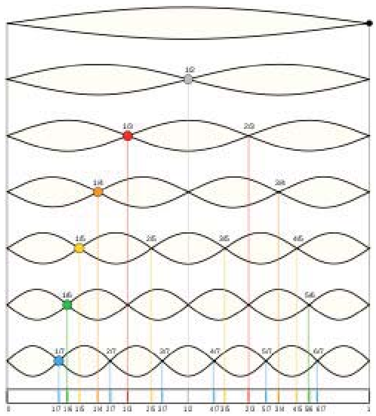
This is a question that does not have a unanimous answer. And it goes from considering the first three harmonics to considering the first 16.

In this chapter we will see that all harmonics can be considered, but at the same time they **can be summarized in the *2f, 3f, 5f* and very slightly the *7f*** (it is to say, the first 4 prime numbers [after *f*] of the harmonic series) and each one of them serves to establish different psycho-acoustic processes that generate the perception that we humans have of the musical tensions.

But before doing the demonstration let us first do a historical review:

Before the phenomenon of harmonics was known (discovered between the 17th and 18th centuries)[3] theorists considered musical intervals as relations between natural numbers (2, 3, 4, 5 ...). For example, the octave as the 2/1 ratio, the fifth as the 3/2 ratio, the fourth as the 4/3 ratio, etc. Since, for example, using a monochord or a violin string, this was the ratio of the position of the fingers to the notes of the scales they used (figure 13).

Fig. 13



But note that to consider these vibrational relations between natural numbers in a string is the same thing as considering the harmonics of a sound. Taking a look at figures 12 and 13 and applying simple mathematical concepts it is seen that the relation 2/1 is the same as the relation *2f/1f*, the 3/2 that the *3f/2f*, the 4/3 that the *4f/3f*, etc. That is to say, for example, the frequency corresponding to an upper fifth of a fundamental note is obtained by multiplying its frequency by 3/2 (the interval between *2f* and *3f*), the fourth by 4/3 (the interval between *3f* and *4f*), etc.

Pythagoras and Plato[4] believed that every intervallic relations in music (and in

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[3] Foster attributes it to Marin Mersenne (1588-1648) (Foster 2010) although Rameau did not speak about it until 1750 (Rameau 1750).

[4] In the Timaeus (Dialogs) (35b-36b) Plato relates the creation of the "soul of the world" according to the relations of numbers 2 and 3 (he establishes the Pythagorean semitone [limma] with total accuracy [256/243]).

other natural principles) came from the relationship between numbers 2 and 3. That is, they had enough with the octave and the fifth —the first two harmonics— to build all the musical scales.

Years later Aristoxenus doubted about this and introduced, among others, the 5/4 relationship (harmonically the interval between *4f* and *5f*) within complex divisions of the tetrachord, although he said that the true musical scales were not mathematical but "felt".[5]

During the Middle Ages the Pythagoric school was prevailing although in England there was a style of composition that used the natural third (5/4) as a consonance.

In the 16th century Zarlino introduced the *senario*, composed of numbers 1, 2, 3, 4, 5 and 6 as generators of all intervals and musical laws. In fact, to establish the *senario* is the same as considering the relations until the number 5 since 6 = 3 x 2. Scientists and musical theorists like Kepler[6], Descartes or Rameau[7] agreed to consider only the relations between numbers 2, 3 and 5 as generators of musical scales and intervals and in fact this relation is the one that is still explained nowadays in many treatises of harmony.

Some theorists also introduce or share the ratio 7/4 (*4f-7f* interval) as the minor seventh, although this interval is more accepted as two fourths (4/3 x 4/3) from the fundamental (or two lower fifths). This is the case of Leibniz[8] , Tartini[9] , Euler[10], Kirnberger and Vogel[11]. Even the father of functional harmony, Hugo Riemann, although he favored the *senario* ratios for tonal relations, admitted that the interval of the minor seventh was an interval "given directly by nature" (Elementar-Musiklehre, Hamburg, 1883).

In fact, in the daily practice of music, with our equal-tempered system these

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5 See: The measurement of Aristoxenus's Division of the Tetrachord, by Joe Monzo, http://www.tonalsoft.com/monzo/aristoxenus/aristoxenus.aspx.

6 See Cohen, 1984.

7 In the seventeenth century Descartes, in the *Compendium musicae* (1618) writes: "All the variety of sounds, relative to the treble and the bass, is born in the music only of these numbers: 2, 3 and 5". In his book *Demonstration du principe de l'harmonie* (1750) Rameau explains the C diatonic scale taking the first 5 harmonics of F, C, and G.

8 In a letter to Christian Goldbach (Luppi, 1989) Leibniz writes : "Nam our Intervalle vstinata omnia sunt rationum compositarum ex rationibus inter binos ex numeris primitivis 1,2,3,5. Si paulo plus nobis subtitiltatis daretur, possemus procedere ad numerum primitinum 7").

9 Tartini (1754 and 1767).

10 Partch (1974).

11 Vogel (1975).

ratios never occur in an exact way, but studies like those of Fransson, Sundberg & Tjernlund (1974) seem to show that considerable variations can occur in the tuning of a piece of music without it meaning that the notion of tonal and harmonic structure determined by these numerical relations is lost.

Thus, throughout history, and in order to demonstrate musical laws, musical theorists considered mainly the relations between numbers 2 and 3; also many of them incorporated the 5, and some few the 7 (that is, the relations corresponding to the harmonics 2, 3, 5 and 7: C, G, E, B♭).

Note that in the harmonic series, as we move forward, the notes are getting closer and closer together. Thus, for example, between harmonics 1 and 4 there have two octaves, but instead, between harmonics 24 and 30 we have 7 notes within a third, that is, approximately a quarter tone of difference between them; between harmonics 32 and 36 we have only one tone of difference, and so on. They are approaching logarithmically.

When sounds are very close in frequency, it operates what in acoustics is known as "auditory masking": when a set of sounds sound simultaneously at very close frequencies, the auditory system is unable to appreciate all the frequencies of the sounds; and the most potent ones completely nullify the perception of the weakest, making themselves unintelligible [12]

This is what begins to happen from the harmonic 16, and from the 24 the sounds are already masked and indistinguishable between them.

Our thesis is that we can consider a sufficiently high number of the harmonic series (for example, up to harmonic 24) without this being an obstacle so that we can determine that the harmonics really significant for the auditory system can be reduced to the prime ones within the first seven (*2f, 3f, 5f* and *7f*).

The reason is that harmonics that are multiples of a prime harmonic have less importance to the auditory system than those who occupy a prime position, since they are heard and understood as generated by them. Just as the fundamental is the synthesis of all its harmonics, the prime harmonics are the synthesis of the rest of non prime harmonics (their own harmonics). In the harmonic structure we could

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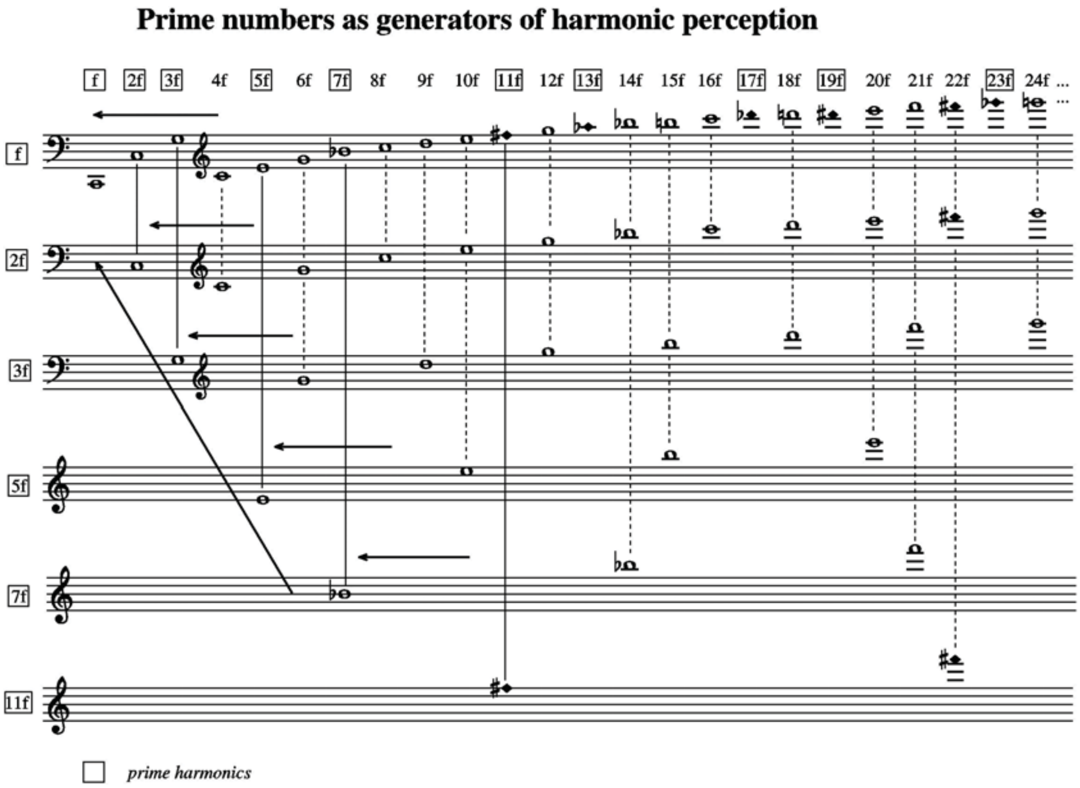
12 Calvo-Manzano, 1991, and Moore, 1986: «The ability to hear frecuencies separately is known as frequency resolution or frequency selectivity. When signals are perceived as a combination tone, they are said to reside in the same critical bandwidth. This effect is thought to occur due to the filtering within the cochlea, the hearing organ in the inner ear. A complex sound is split into different frequency components and these components cause a peak in the pattern of vibration at a specific place on the cilia inside the basilar membrane within the cochlea. These componentes are then coded independently on the auditory nerve which transmits sound information to the brain. This individual coding only occurs if the frequency componentes are different enough in frequency, otherwise they are in the same critical band and are coded at the same place and are perceived as one sound instead of two».

consider two levels of auditory perception (see figure 14):

-The fundamental as the perception of all the (prime) harmonics.

-The prime harmonics as perception of all the multiple harmonics.

Fig. 14



What really gives importance to each prime harmonic is the sum of its audible multiple harmonics, not masked and distinguishable by the auditory system.

Each prime harmonic *p* has its own harmonic series *2p, 3p, 4p, 5p, 6p ...* (8th, 5th, 8th, major 3rd, 5th ...)

Within the first 24 harmonics we see that G (*3f*) has 8 harmonics, E (*5f*) has 4 and B (*7f*) has 3. They are the most important for the auditory system.

The next prime harmonic, the *11f*, does not intervene at all in establishing harmonic relationships that mankind has used to make music (except for certain cases in electroacoustic music). The proof is that it falls almost exactly in the middle between F and F# (on a equal-tempered piano, only one cent [one hundredth part of semitone] in favor of F#). Something similar happens to the next prime harmonic, the *13f*, which is placed between A♭ and A♮ (9 cents in favor of A♭).

It is not surprising that the situation of these two prime harmonics, equidistant between two notes of the equal-tempered scale, has produced and produces confusion. Thus, for example, in the harmony books of Schoenberg (1922) and Piston (1941) the harmonic 11 appears as F♮ instead of F♯. In the treatise on harmony of Schenker (1906) and in the compositions of Albrecht and Scriabin based on the harmonic series appears the harmonic 13 as A♮ instead of A♭. In the treatises of Helmholtz (1863) and Piston both F♮ and A♮ appear, although Helmholtz already warns that they can not properly be considered as musical notes. And this confusion with harmonics 11 and 13 continues today if one takes a look at the correspondences between notes and harmonics that can be seen on the internet. Prime harmonics 11 and 13 have had no influence on the establishment of musical laws, nor does it make any sense to assign them a musical note belonging to any of the known tuning systems.

This way so, in agreement with the history of musical theory from the Hellenic culture and from what we have seen on the prime harmonics, it is sufficiently demonstrated that, of all the harmonics of a sound, the only ones that have had a functional meaning for the the human auditory system have been *2f* (C), *3f* (G), *5f* (E) and, very slightly, *7f* (B♭).

The results shown in figure 14 reflect the history of harmonic theory in regarding the first five to seven harmonics to be those that are functionally perceived by the auditory system.

It remains to be seen whether in the future, perhaps by means of musical robots, the use of the following prime harmonics 11 and 13 through a coherent creative construction and microtonally tuned interpretation will be able to produce in our brain a new level of harmonic perception. Although I am rather skeptical in this regard. I have tested how a small composition might sound by using these harmonics (see https://www.youtube.com/watch?v=aHb2hovf95o).

2.2 Virtual fundamental

Before continuing to study the relationship of harmonics with musical harmony we will explain a psychoacoustic phenomenon of great importance in music.

It is the virtual or missing fundamental (known as 'virtual pitch' or 'missing fundamental' in acoustics): a frequency that does not exist in a sound, but that our ear tells us that it is precisely the frequency, the height, the pitch we hear. This occurs if we filter (we remove) the first harmonic(s) of a sound. In this case we continue hearing the fundamental frequency.

To prove it I have done a video taking the harmonics of C. In this video we may see that if we delete the frequency corresponding to C_2 and even the first seven harmonics, we still hear the note C_2 (!). If we take only the harmonics corresponding to the notes E_4, G_4 and $B\flat_4$, we continue to hear C. The reason is that we eliminate the first harmonics, but not their multiple harmonics, that is, the harmonic structure remains the same, that of figure 14. This video can be seen in: https://www.youtube.com/watch?v=0Y4-NQQ6hAY

To see a mathematical proof of this phenomenon see also: https://www.youtube.com/watch?v=p3iWLkXAePM6

If instead of sinusoidal sounds we use timbred notes, for example on the piano, the effect is greatly diluted, but it still works (!). For example, let us go to the piano and play the note C_2 and immediately the chord of figure 15 several times. We will hear that when playing the chord, if we make the effort of remembering the C_2 that we have played before, this note magically appears as if it were actually sounding, it is psychologically perceptible (!) but does not actually exist in the chord.

Fig. 15

This phenomenon —in the history of musical theory (although surely ignoring its scientific basis)— has also taken part when constructing various harmonic laws; from concepts such as the *basse fondamentale* by Rameau, the *terzo suono* of Tartini (Tartini regards B♭ as the virtual fundamental of the diminished fifth D-A♭)[13] or the consideration of the diminished triad chord EGB♭ (or, as Tartini, simply the diminished fifth: EB♭)[14] as a dominant chord, usable to make cadences, that is, a chord with C as virtual fundamental (C as dominant of tonic F). It is a fact formally consolidated by Riemann by coding the diminished triad chord with the symbol Đ^{7} (D of Dominant).

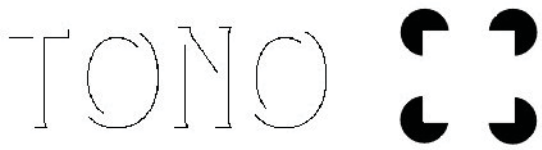
Although the missing fundamental phenomenon was already known in acoustics, Ernst Terhardt introduced the term virtual pitch in 1974 and exemplified it with the visual analogy shown in figure 15bis. In visual perception we see the white square, although it does not really exist; or we see the relief of the letters by putting only a

13 Tartini, 1767, p. 85

14 According to Tartini, with the B♭ somewhat low to fit with the «natural» 7th of C. Tartini could do this because he was a violinist.

few lines. The virtual fundamental would be an equivalent effect —acoustically speaking— to the “finish” rendered by the brain of the contours of figure 15bis.

Fig. 15bis



Visual analogies of the concept of virtual pitch [Terhardt (1974)].
[from S. Coren, Psychol. Rev. 79, 359-367 (1972)].

This acoustic phenomenon will help us in knowing the true fundamentals of complex chords and confirms us that the «true» fundamental (virtual) of the EGB♭ chord is C, it is not E.

In the following sections we will explain, one by one, the effect that, in our opinion, have the first four prime harmonics in the human auditory system that have allowed to develop many of the musical laws. That is, we will study separately the prime harmonics *2f, 3f, 5f* and *7f.*

2.3 Harmonic *2f*

It is the octave of the fundamental.

If we look at figure 14 and if we consider the harmonic *2f* as a fundamental sound, we see that the harmonics of *2f* coincide in 50% with the harmonics of the fundamental sound *f.* That is, the harmonic *2f* has half the harmonics of *f* (in any given neighbourhood).

From what we have explained previously about the virtual fundamental —we have seen that we can eliminate harmonics of *f* without this affecting too much the feeling of being still listening *f*— we could say that the auditory system considers *f* and *2f* as the “same” note but with a different “timbre”, since they “almost” have the same harmonics.

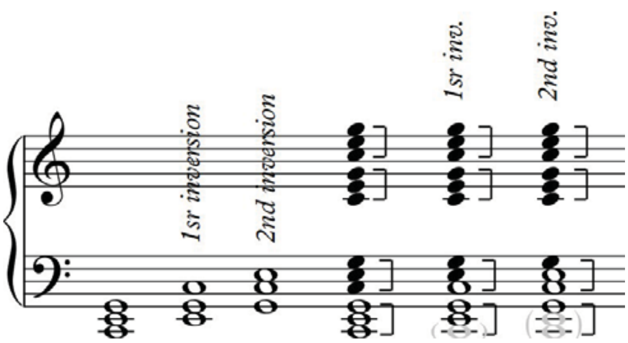
The great amount of harmonics in common between a sound and its octave and the acoustic effect of the virtual fundamental have made so that in music (in the human brain) these sounds have been considered, indeed, the same note. This fact is very useful both for the brain and for music theory because it has allowed to simplify information drastically, reducing the whole range of notes on a diatonic scale of only 7 notes (and their modes) (see in 2.6 the construction of the diatonic scale) and in a chromatic one of only 12 (once the equal-temperament is applied).

That is to say, **the strong and powerful perception by the auditory system of harmonic *2f* with its common harmonics associated with *f* has allowed to identify the sounds and their octaves as the "same" note** and speak of notes and chords without continually specifying the actual pitch of them. However, it is the "same" note in quotation marks. We all know that, in musical practice, it is not the same to use a note or its lower or upper octave, especially if this changes the inversion of a chord, although "functionally" or "harmonically" it plays the same role.

Harmonic *2f* is also the "culprit" that, functionally, the chords can be identified with their inversions.

In figure 16 we have a chord (it might be anyone) with its inversions. If we add the octaves to the chord notes (which really can be heard as harmonics), we see that the fundamental triadic position of the chord is repeated in the upper layers. The ratios between intervals remain the same and by means of the virtual fundamental effect the ear functionally matches the chords assigning to them the same fundamentals. However, *sonance* (consonance-dissonance) can be quite, if not very, different. But, as we said in the previous chapter, we should be able to distinguish the "sonance of the chord" from its harmonic function.

Fig. 16



We must do a parenthesis and comment on the particular case of the 2nd inversion, with a sixth and a fourth from the bass: if we consider the following significant harmonics after the octave, which are the fifth and M3 and the trend of the notes in solving dissonances towards notes to a lower second, we see that the ear can also understand that the bass is the real fundamental (we hear its fifth and its M3 as harmonics) and can understand the 6th and 4th as notes close to the 5th and 3rd, notes that "collide" with the latter (and need a resolution) and consider them as appoggiaturas that must resolve a second lower —with increased effect if the bass is an important note in the tonality—. This is the case of the known cadential $^6_4$. The second inversion of a major or minor triad can then be interpreted by the ear in two different ways, or both at once, since we can understand the cadential $^6_4$ as T($^6_4$)-D-T or D$^6_4$-$^5_3$-T.

Thus, harmonic *2f* is responsible for identifying the octaves as a 'same' note and for the functional equivalence between the inversions of the chords.

2.4 Harmonic *3f*

It is the fifth (+ octave) of the fundamental.

If we look again at figure 14 and if we consider the harmonic *3f* as a fundamental sound, we see that the harmonics of *3f* match on a 33% with the harmonics of the fundamental sound *f*. That is to say, the harmonic *3f* has 1/3 of the harmonics of *f* (in any neighbourhood).

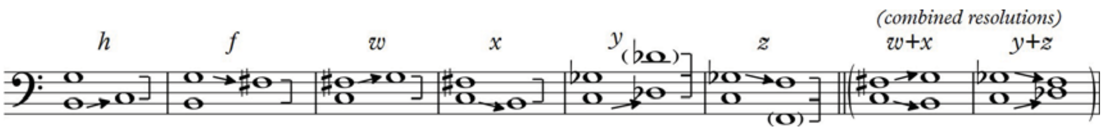
The amount of harmonics that we should remove to the C to become G would be too many and the auditory system can no longer consider the G as a "timbre variation" of C, as it did with the harmonic *2f*, its octave. But note that the harmonics of G (*3f*) fit perfectly with those of C (*f*&*2f*). For this reason, if the two notes sound simultaneously, we say that they form a perfect consonance.

If instead of being a 13th (5th + 8th) is a perfect fifth, the harmonics do not fit so perfectly, but the difference is only of an 8th in the first harmonics. The notes corresponding to the harmonics are the same. Something similar happens if we lower a further 8th and get the 4th, which is the inversion of the interval of 5th. We are losing consonance, but the upper harmonics continue to fit.

In fact the fifth (P5) is the only perfect consonance between two different notes. The consonance par excellence.

If we hear an interval near the fifth such as the minor sixth or the diminished fifth, the ear hears it as a "quasi-fifth" or "false-fifth", thus creating a harmonic tension that is resolved if the "quasi-fifth" is "tuned" to a perfect fifth (in the case of m6/M3 the tension is lesser because it contrasts, as we will see, with the imperfect consonance justified by the harmonic *5f*). See figure 17 (and also figures 1, 2 and 3 of the first chapter).

Fig. 17



These simple harmonic tension-relaxions of figure 17 (produced by the intervals close to P5 and their resolutions), although it may be hard to believe, are the basis of most harmonic tensions of tonal music as we have seen in the previous chapter.

The fifth, as we shall see in 2.6, also intervenes in the construction of the diatonic scale in the major mode.

This way so, we could say that the harmonic *3f* is the principal causer and generator of the functional laws (cadences, tonality, secondary dominants, etc.) of harmony. The re-reading of the previous chapter and the reading of the rest of the book will reinforce this fact, which might be summarized in the importance that the tension of the "quasi-fifths" and the tendency of the notes towards their lower fifth have for harmonic laws.

2.5 Harmonic *5f*

It is the major third (+ two octaves) of the fundamental.

If we look again at figure 14 and if we consider the harmonic *5f* as a fundamental sound, we see that the harmonics of *5f* coincide on a 20% with the harmonics of the fundamental sound *f*. That is, the harmonic *5f* has 1/5 of the harmonics of *f* (in any neighbourhood).

It is a weak linkage with the fundamental, but still perfectly perceptible by the auditory system.

This setting helps the M3 interval/chord to be relatively stable despite having the upper harmonics forming minor sixths, an interval close to the attractive fifth, stability that is definitely reinforced if we we confirm the fundamental adding also its fifth (that is, forming the major triad chord); we still reinforce it more if we have instruments of variable tuning or voices that drop a bit this M3 (in the resolutive chords) to make it correspond with the natural tuning of the harmonic *5f*, which is a bit lower than the tempered M3 of keyboard instruments (see figure 12).

Before the 15th century the M3 interval was considered dissonant —or imperfect consonance with tension to be solved[15] —because it was usual the Pythagorean tuning and vertically (heard as chord) the Pythagorean ditone is really dissonant. Keep in mind that between the ditone and the natural 3rd (*5f*) there is almost a quarter tone difference (22 cents). In fact in classical music of India they are considered (or were considered) two different intervals. The *5f* as a *shruti* named *Raktikâ*, which expresses pleasure, sensuality..., and the Pythagorean 3rd a *shruti* named *Raudrî*, which has an antonym meaning: warrior, terrible (Danielou 1943 and 1967).

The harmonic *5f* is the causer that the m6/M3 "dissonance" —due to its proximity to the P5/P4— is accepted by the ear as semi-consonance, something that does not

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15 The *regola delle terze e seste* said that these imperfect consonances had to resolve its tension with a perfect consonance (Dalhaus, 1990) and Gafurius, in *Practica musicae* (1496), describes the thirds and sixths as irrational consonances (Tenney [1936]).

happen to the tritone that is also separated from the perfect consonance by half tone of difference.

The harmonic *5f* establishes the imperfect consonance of the M3 (and by bounce that of the m3, since it is the interval that is formed between *5f* and *3f*). That is to say, it is a harmonic that establishes some laws of music sonance, but does not affect to the functional laws.

Broadly speaking, we could say that the harmonic *3f* (the P5) is the causer of the functional laws of harmony and the harmonic *5f* (the M3) to extend the perceptions of sonance.

Before continuing with the harmonic *7f*, with these two harmonics we have studied (*3f* and *5f*) we can already demonstrate how the diatonic scales have been formed from the perception of these harmonics.

2.6 Formation of music scales

The (major) diatonic scale can be formed in a very simple way considering only the harmonics *3f* and *5f*.

In fact, only with the *3f* we can form the diatonic scale, but with Pythagorean tuning.

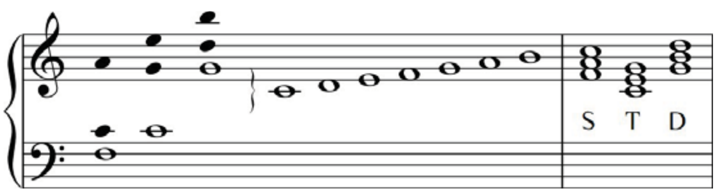
For example, let us take F and let us construct a scale formed by the *3f* harmonics of the harmonics *3f*, that is, the fifths of the fifths.

We will obtain F, C, G, D, A, E, B, that is, C-D-E-F-G-A-B.

There is another way to build this scale, adding the use of the *5f*:

We take the harmonics *3f* and *5f* from the first two fifths (*3f*) of F (figure 18).

Fig. 18



We have not made a discovery. This second construction is the same that Rameau gave in his book *Démonstration du principe d'harmonie* (1750).[16]

[16] In the summary of the *Academie Royale des Sciences* at the end of the book: «Ainsi l'Auteur exprime fa, ut, sol par les nombres 1, 3, 9, & la proportion qu'ils forment, est ce que M. Rameau appelle Basse fondamentale d'ut en proportion triple, ou simplement Basse fondamentale. Les trois sons qui forment cette Basse, & les harmoniques de chacun de ces trois sons, composent ce qu'on appelle le Mode major d'ut».

Which of the two constructions from the harmonics is the one that the auditory system really considers when listening to this scale? Most probably, both at the same time (the Pythagorean and the so-called «natural») although the tuning is not the same; in fact equal-tempered system comes to make an intermediate tuning of the two (though closer to the Pythagorean).

The reader might have noticed that the two constructions start from the tone F. So, if F generates the seven notes of the scale, would not it be more logical to put F at the beginning instead of C and become the scale of F rather than C? The answer is that these seven notes create a tonal vector towards C and not to F and therefore they rest in C. Why do they rest in C and not in F? Due to the laws of "quasi-fifths" we have seen in 1.1 and 2.4, that is, the tensions that the intervals of M3 (GB) and tritone (BF) create and which form the 7M3 structure that we have seen in 1.4 (in the case of C major, the structure formed by notes GBF). A 7M3 structure creates a powerful tonal vector. The major mode has a unique 7M3 structure that determines which of the seven notes is the "tonic", the note that provides more rest.

Briefly explained, before the 16th century, and more concretely before the formal change of modal theory in Glareanus' Dodecachordon (1547), the theoretical modes, in order to avoid tritone, were based on the hexachord (*cantus naturalis*) of Guido de Arezzo (11th century), that is, the diatonic scale without B: *Do (Ut), Re, Mi, Fa, Sol, La*. The hexachord does not have the 7M3 structure and the resting note is not so clear and depends on the melodic drawing. Therefore, in those times, music was easily modal because, apart from being basically horizontal, many notes could be the *finalis* note. According to the theory of the time, these final notes could be *Re* (D), *Mi* (E), *Fa* (F) or *Sol* (G). Although the modal theory is more complex since — in a same chant— hexachords could also be used from the notes F (using B♭) and G (as a kind of transport or modal modulation). If the notes F and B♮ were used in the same melody, they were notes that were not close (in time); if they appeared, between them it was customary to put the *finalis* note, but, for example, F-G-A-B♮ was almost never used, in these cases B♭ was used.

When the palette of notes gradually passed from hexachord to heptachord (the diatonic scale), the *Do* (C) and *La* (A) notes were added as *finalis* notes (Glareanus, 1547), which ended up being the predominant modes because then it is easier to incorporate the tritone; and the heptachord already has the 7M3 structure (GBF) that places tone C as *finalis*, as the most "acoustically" resolutive note. And the eolian mode (*finalis* A) was becoming our current minor mode when, little by little, the leading-tone G♯ was joining into the mode, since, in this way, the 7M3 structure (EG♯D) were included in it.

2.7 Minor chord and minor mode

Is there an explanation of the minor chord from the harmonics? There have been (and there are) many discussions about it. For me, the explanation of the consonance of the minor chord is quite simple: the major chord and the minor chord are the two single chords formed by the intervals that are created with the notes corresponding to the harmonics *2f, 3f* and *5f* (and their octaves). If we take a look at figure 14, we will see that the intervals created between the notes are: 5th (CG), 4th (GC), major 3rd (CE), minor 6th (EC), minor 3rd (EG) and major 6th (GE). There are only two chords that, with their octaves, have ALL and ONLY these intervals (figure 19): they are the major and minor triads. In fact, as "interval classes", they are only combinations of P5, M3 and m3.

Fig. 19

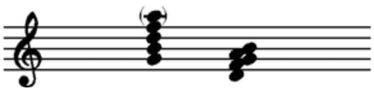
This is why these two chords are the most consonant of all those who can be formed with three different notes.

Next a question would come with which music theorists do not seem to agree on the answer: what is the fundamental of the minor chord? (fundamental in the sense of functional representative of the chord).

Conventional musical theory often fails to make an adequate distinction between two very different concepts —(con)sonance of a chord and a chord's function as a generator of harmonic tension. (Con)Sonance refers to the greater or lesser sensation of smoothness or roughness we experience on hearing the chord, while the function of a chord refers to the harmonic tension which that chord creates.

For example, the two chords shown in figure 20 share the same harmonic function as they both create a tonal vector towards C. Their degree of sonance, however, is totally different. In this example the root is the same whether we define it as the bass of the "root state" (ordering the notes in thirds), or as the representative of the function of the chord. But the bass of a chord ordered in thirds is not always the same as the root defined representative of the function of the chord. This is true in the case of minor chords.

Fig. 20



The chord ACE probably has its greatest consonance when A is the bass note (and is the bass ordering the notes in thirds). For conventional theory A is therefore the root of the chord. But students of harmony soon learn that they cannot use this chord as the dominant of D and so A, which is the dominant of D, cannot represent the function of the chord. For functional harmony A is not the fundamental of the chord. This confusion concerning the root of the minor chord is due to the fact that we are talking about two different ideas —on one hand the (con)sonance of a chord and on the other its function.

In the 18th century Rameau had already defined the chord ACEG as having a double function with A or C as the "*basse fondamentale*", depending on the harmonic progression or the inversion (A as a "*sixte ajoutée*"). When Riemann classified the functions of all chords into just three categories, (subdominant, tonic and dominant)[17] he made the D minor chord in the key of C major into a subdominant (he anotated it Sp) (fundamental F); the chord of E minor became a dominant (Dp) (fundamental G); the A minor chord became a tonic (Tp) (fundamental C), and the diminished chord BDF was a dominant with G as its virtual fundamental. Riemann was in agreement with Tartini, many years before this concept was defined acoustically by Terhardt.

All this tends to indicate that from the point of view of functional harmony, the fundamental of chord ACE is C.

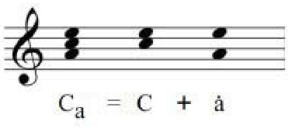
In this way musical theory seems to give A as bass to underpin the most consonant chords and C to represent the function of the A minor chord.

This is in agreement with our separation of chords (scientists would use the word *chromatography*) according to their convergent structures that we explain in 1.7 and 3.1.

The ACE chord can be separated in its P5 (AE) (*3f* of A) and its M3 (CE) (*5f* of C) (figure 21).

The interval that creates tension in the chord is that of M3 (CE), which tends to resolve or continue locally relaxed with chords containing the fundamental F or B.

Fig. 21



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[17] Rameau had already said something similar in *Génération harmonique* (1737): «Il n'y a que trois Sons fondamentaux, la Tonique, sa Dominante, qui est sa Quinte au-dessus, & sa Sous-dominnate, qui est sa Quinte au-dessous, ou simplement sa Quarte».

Concerning the minor scale(s), as we know, in music practice, three different types of them coexist, the so-called natural, harmonic and melodic ones, which vary according to different combinations of VI and VII degrees (at the end of 2.6 we have already seen how the minor mode was formed from the eolian mode). In any case, it is possible to show, as with the major chord, its internal harmonic coherence:

Let's choose F again. Let us take the three upper fifths and the three lower fifths: F: C, G, D and A♭, E♭, B♭: F, that is, C-D-E♭-F-G-A♭-B♭, we obtain the natural minor scale.

As with the major diatonic scale, we can also construct it in another way: let us take the three minor chords formed on the first two fifths of F (FA♭C) (CE♭G) (GB♭D), we return to have the natural minor scale.

However, when chords are used, the natural minor mode has a problem (problem not so obvious when it is only melody or horizontal polyphony). This mode has the tonal tension of the 7M3 structure of another tone, in case of C natural minor would be the structure B♭DA♭ that causes that the rest note become E♭ instead of C (remember that E♭ major and C minor share the same key signature). So what composers do is try to change the 7M3 structure, at least in cadential processes. They change the structure B♭DA♭ by the GBF structure, that is, they simply use B♮ instead of B♭. The scale thus formed is precisely the C harmonic minor scale.

In order for the harmonic minor scale not to sound so "exotic" with the appeared augmented 2nd (A♭-B♮), often for ascending melodies A♮ is used instead of A♭ (G-A♮-B♮-C) and for the descending ones B♭ is used to take advantage of the melodic phrygian resolution towards the dominant (C-B♭-A♭-G), the so-called *basso di lamento* (which coincides with the Greek Dorian tetrachord[18]). In this way we obtain the so-called melodic minor scale, consisting of an ascending htonal resolution (GB)→C and a descending phrygian resolution (A♭C)→G (figure 22).

Fig. 22

[18] Plato considered the Dorian [diatonic] mode (the medieval Phrygian) as a model of order for the Republic (Levarie, 1992).

2.8 Harmonic *7f*

It is a sound very close to the minor 7th as seventh degree of the diatonic scale or very close, in other words, of the tempered minor 7th of the piano.

As we see in figure 14 the weight of this prime harmonic is quite small due to the few and distant harmonics they share in common with the fundamental. In 2.1 we have seen that some theorists, such as Leibniz, Tartini, Euler, Kirnberger and Vogel, considered this sound (at least) as Riemann considered it, that is, an interval «given directly by nature» although its natural tuning, due to the current use of the equal-tempered tuning is rarely put into practice.

The "blue notes" used in jazz surely have their origin in the perception of this seventh harmonic. For example, in the blues it softens the clash that occurs in a chord between the tempered major third of the tonic/root (E♮) (piano or guitar) and the minor seventh of the subdominant F (E♭) (usually in the voice or instruments with variable tuning) as the distance between the E♮ and the E♭ (blue note [natural minor 7^{th}] in this case) is widened (50 to 81 cents) and the clash is more "sweet" when the E♭ sounds above the C^7 (E♭ sounds a bit lower than the temperate one and softens the dissonance E♮-E♭). Also the barbershop quartets usually sing the harmonic seventh in 7^{th} chords.

I have done a test (https://www.youtube.com/watch?v=FU2ov_r9Li0) which consists in seeing the tuning difference of a dominant seventh chord according to the equal-tempered tuning and according to the harmonic tuning following the frequencies that correspond to G, E and B♭ according to the harmonics *3f, 5f* and *7f* (disregarding octaves), using sampled voices. In this example, we hear, alternately, the equal-tempered chord and the "harmonic" chord. First we hear the tuning difference of the two B♭, which is clear (the harmonic B♭ sounds lower than the tempered one). But when we hear the whole chords, this difference of tuning decreases and it is not clear which is the chord that sounds more "tuned". Perhaps our habit of listening to the equal-tempered system makes the tempered one, in the first instance, that seems more "tuned", but the second has a "harmonic sweetness" that does not have the first.

Note that although the harmonic *7f* is weak, considering the first 4 prime harmonics of a sound (*2f, 3f, 5f, 7f,* which are the only really significant ones for the auditory system) we are hearing a chord very similar to that of dominant seventh of the fundamental, which helps that, hearing only a single note, we have the sense of resolution upon hearing a note a 5th lower or a minor 2nd lower (and so on) (htonal and phrygian resolutions).

The *7f* harmonic will also help us to find the true fundamentals of chords from which we will have guidance on the real tensions and local relaxions between chords (see 1.6, 1.7 and the next chapter) as it allows us to summarize the 7M3 structure (which coincides with the significant harmonics of a fundamental note) with a single symbol. For example, in the case of GB(D)F, using G^7.

But, as in the case of the harmonic *5f*, it has not influenced the construction of the relations and functional laws of harmony and tonality.

3. Functional study of chords

I use the term *chord* in the title, but when I speak of chords I mean also arpeggios and in general the set of harmoniously significant notes that occur in a given time neighbourhood, although the graphical representation is in chord form.

We have already seen in these first two chapters which are the main intervals that cause harmonic tension and in 1.6 and 1.7 we have introduced the way to encode the chords so that this symbology gives us information about these tensions (the tensions of M3 and tritone). Next we will establish the eight large families of chords that can be constructed considering their internal harmonic tensions, determined mainly by the interval formed by their main functional fundamentals.

3.1 Classification of chords according to their harmonic tensions. The fundamental symbology

Regardless of their consonance or dissonance (sonance) we could make a classification of the chords according to their harmonic function represented by their functional fundamentals. I call fundamental symbology the system used to represent the fundamentals of chords. Let's summarize how it works:

First recall here figures 7 and 8. A capital letter means that we have a tension of M3 in the chord and a capital letter with a 7 or a crossed one means that in the chord there is a tritone tension (in the second case with a virtual fundamental). We call functional fundamentals to the fundamentals in capital letter since they represent a chord with "quasi-fifth" tension. The combinations of notes corresponding to the first seven harmonics are thus represented by a single letter (representing the fundamental), with different symbols depending on the combination of these notes (figure 7), although in everyday practice, for the most common chords, we do not usually distinguish whether the fundamental has the fifth or not.

Fig. 7

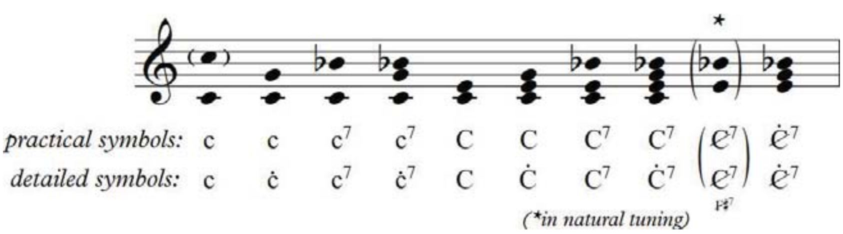


Fig. 8

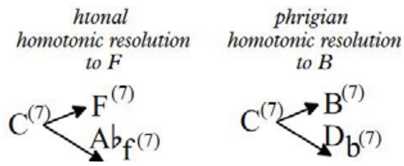
If we have two fundamentals, we put in the upper part (superscript) the fundamental that is harmonic of the other. If they are on the same tonal axis (see 4.5), we put in the lower part (subscript) the lower minor 3rd or the tritone. In figure 8 we have some examples.

That is, if the symbol has the form X^Y means that Y is a fundamental a fifth, a major third or a minor seventh from the fundamental X. If the symbol has the form X_Z (X, Y and Z may be in uppercase or lowercase) means that Z is a fundamental a lower minor third or a tritone with respect to X. If the two fundamentals are separated by a semitone, we usually put it in the form X^Y, where Y is the major seventh, although sometimes we also put the minor second above (usually when there is a Z below, for more details see annex 1).

When the fundamental has its minor seventh in the chord, we place a 7 above (except when Y is precisely the minor seventh of X). When we have two fundamentals, we put a dot on those that have their fifth in the chord, with two exceptions: when Y is precisely the fifth of X (we do not put a point on the X) and when Z is the minor third of an uppercase X (we do not add a point to Z because it is already implicit —X in uppercase means that it has the M3, which is the fifth of Z—, this saves us work by symbolizing the minor chords). As you look in this chapter at the examples of the different kinds of chords it will be easier to understand how the fundamental symbology works, which I think is quite simple.

In any case what is really important is to know the fundamentals of the chords. The way they are placed is secondary.

Fig. 23



This fundamental symbology informs us that these functional fundamentals (as we have seen in 1.2 and 1.7.2) tend to «resolve» in other fundamentals that are a lower fifth/upper fourth (htonal resolution) or a lower minor second/upper major seventh (Phrygian resolution) (figure 23), aside the secondary resolutions that will be seen in chapter 4. I use the term "resolution" in a broad sense, when I use it, I mean in general a progression or succession in which the passage to the next chord is harmonically and locally relaxed, either as the end of a musical section or not. We will devote the entire chapter 6 to place examples of different combinations of these local homotonic relaxions/resolutions with different types of chords.

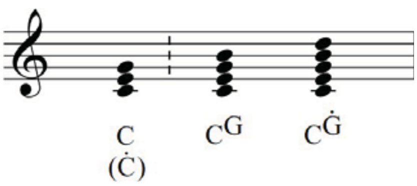
3.1.1 Major chord family

They are chords of genre C or C^{G}.

They are made up of notes corresponding to the harmonics *5f* and *3f* of C and the harmonics *5f* and *3f* of the fifth of C.

We have represented some of them (ordered by consonance) in figure 24.

Fig. 24



A subdivision can be made between those with a single fundamental (the major triad) —a single tension of M3— and those with two fundamentals —two tension of M3— at a distance of fifth.

They are tonally stable chords and can be used as endings of compositions, preferably with the fundamental on the bass. His notes form combinations of *2f, 3f* and *5f* (they have some or all of the following notes corresponding to the harmonics: *2f* [C], *3f* [G], *5f* [E], *3f* of *3f* [D], *5f* of *3f* [B]). In case of C^{G}, the tension GB towards C is «satisfied» if in the bass we hear C, although it can also resolve in another chord based on C. The C (as fundamental) can be kept stable or continue a progression in chords based on F or B (if we want it relaxed). Also secondarily we have homotonic relaxion in chords based on D and E (as will be seen in 4.1 and 4.2).

The different chords of this family may have a very different *sonance* but all have a similar harmonic function and in this chapter we are classifying the chords by their harmonic function, not by their consonance or dissonance.

The chords $C^{\dot{g}}$ (CEGD), $C^{\dot{e}}$ (CEB), $\dot{d}^{C}$ (CEDA) and $\dot{d}^{\dot{C}}$ (CEGDA) can also be considered within the major chord family (of C) since the fifths ġ, ė y ḋ do not change the global function of the family. They are chords that could also substitute relaxed tonic chords (preferably placing C on the bass). In fact, $C^{\dot{e}}$ chord could also be coded C^{G}, since the notes C, E, B cause the G to be virtually heard as harmonic (fifth) of C.

In general, adding a lowercase does not alter the chord function too much as long as new functional fundamentals are not created, that is, new intervals of M3 or tritone between the notes of the chord. In this case we would have a new functional fundamental with new tension.

Located at the border of the functional change, the chord formed by three major triads at a fifth distance (CEGBDF♯A), with C on the bass, although it has three functional fundamentals (C, G and D), we could also consider it as belonging to this family.

For more information on the resolutions, successions or progressions of this chord family see also 6.1 and 6.2.3.

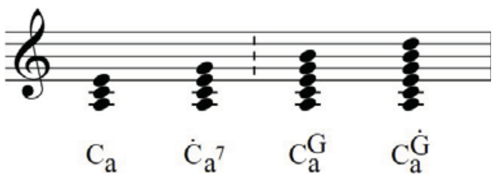
3.1.2 Minor chord family

They are chords of genre C_a or $C^G{}_a$.

That is, they have the functional fundamentals of major chord family with a lower third of the main functional fundamental, which may be in the bass or not.

We have represented some of them (ordered by consonance) in figure 25. The C, as a capital letter, implies that it has the note E, so A always has its fifth E, so it is not necessary to put a point on *a* ($C_{\dot{a}}$) nor to put the ^{7} if its seventh minor G already appears in the code.

Fig. 25



For more details on the minor triad, see 2.7.

Minor chord family can also be used to finalize a composition or a musical section.

As in the case of major chord family, a subdivision can be made between those having a single tension of M3 and those having two M3 at a distance from P5 (and also, of course, distinguishing between the minor triad and the others).

They have A (*a*) as weak fundamental due to the convergent structure AE(G) contained (harmonics *5f* and *7f*), but, as we have seen, functionally the main fundamental, as representative of the function of the chord, is C.

The resolutions or relaxed progressions are similar to those of the major chords. In fact we could have put them in a single family, but in this case, the different characteristic sonance of the major and minor chords has made me choose to separate them into two families since, as we say, functionally could have been included in a single one.

For more information on links to these chords see also 6.1.

3.1.3 Dominant chords (unitonal)

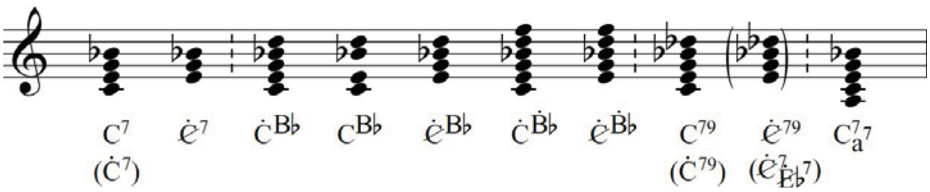
They are the chords that contain a single 7M3 structure (in real or virtual form) and therefore they form (by themselves) a very strong tonal vector. Hence the name of dominant, by its cadential force towards chords based on the tonic established by 7M3 (F in the examples).

We have placed in other families chords that contain more than one 7M3 structure, such as, for example, symmetric dominant chords (3.1.5); however, they must also be considered dominant chords, what happens is that they are dominant of two or more tonics. In fact the unitonal dominant chords and the symmetric dominant ones have many similarities and could also have been put into a single large family of dominant chords.

They are chords of genre C^7 or $C^{B\flat}$ (and $C^7{}_a$ or $C^{B\flat}{}_a$).

We have represented some of them in figure 26.

Fig. 26



It is also possible to establish a subdivision between those who have a single functional fundamental and those who have two, but all have a clear cadential force towards F as they contain the subdominant, the dominant (real or virtual) and the leading-tone of the tonality (the structure 7M3). Those with two functional fundamentals these are precisely (as fundamentals) the dominant (C) and subdominant (B♭) of the theoretical tonic (F).

We have placed in this family also the dominant seventh chord plus a minor ninth. This chord, from another point of view, is a diminished seventh chord (which has four virtual fundamentals at a distance of m3) in which one of the four possible virtual fundamentals becomes real, therefore dominates over the others (although we have to take into account the small tension that also create the other virtual ones). This chord will be symbolized with 9, so when we see a 9, just like when we see a 7, it refers to the interval of minor 9th (in jazz normally the symbol ♭9 is used). It is important not to confuse it with the dominant 9th chord (with the major 9th).

This chord is coded $C^{B\flat}$ (since it includes the major thirds CE and B♭D). 7 and 9 are the only numbers we use in the fundamental symbology and always refer to the minor 7th and minor 9th intervals.

The diminished seventh chord, if the tonality is well established, can also be placed in this family and symbolized as a ninth chord we have seen before but with virtual fundamental, wich plays the dominant function ($\dot{C}^{79}$); but only in the case that there are no enharmonic tonal confusions since it structurally does not have a single fundamental and therefore is not strictly unitonal dominant. When the tonality is not defined this chord has four symmetrical fundamentals in competition with each other and therefore we have placed it also in the family of the symmetric dominant chords (3.1.5). This chord has one foot in each of the two families.

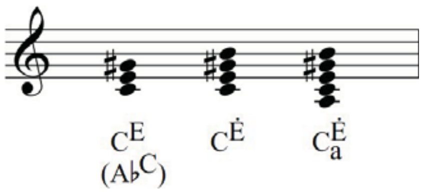
For more information on the unitonal dominant chords see 6.2.2.

3.1.4 Augmented chords

They are chords of genre C^E (and $C_a{}^E$).

They have the fifth of the functional fundamentals augmented (or the m6 from another point of view). Some examples are shown in figure 27.

Fig. 27



These chords have always a third hidden fundamental since there is another M3 (enharmonic): A♭(G♯)-C. It is not necessary to put this third fundamental in the symbology, unless it is in the bass (then we would write $A\flat^C$ instead of C^E), but we have to consider its existence.

Like the diminished seventh chord, the three-note version of the family is a chord dividing the octave into equal parts.

For more information on links to this chord family see 6.2.4.

So far we have seen families of chords with one or two functional fundamentals whose intervals (between the two fundamental) are of P5, M3 or m7, that is, correspond to the harmonic intervals *3f*, *5f* and *7f* from the main fundamental.

The two families that will be discussed below also have two functional fundamentals, but separated by m3 or tritone intervals. That is, they are on the same tonal axis (see 4.5). To distinguish them from the other families we will put the second fundamental in the lower part (instead of superscript) (also for maintaining the coherence with the minor chords where we have written the lower minor 3rd in lowercase as subscript of the functional fundamental).

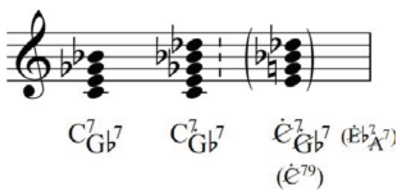
3.1.5 Symmetric dominant chords

They are chords of genre $C_{G\flat}$ o $C_{F\sharp}$.

They are chords that have two tensions of M3 (and also two of 7M3). The fundamentals are separated by a tritone interval.

We have represented some of them in figure 28.

Fig. 28



The two fundamentals of these chords divide the octave into two equal parts. That is to say, we have two M3 in symmetry and the order of the two fundamentals, in the symbology, could be the other way around. This distance of tritone automatically implies that the two fundamental also have their minor 7th (enharmonized or not). Which means that the two fundamentals have the 7M3 structure, but in competition between them in the sense that the two tonal vectors are in the opposite direction in the circle of fifths. This, and the fact that it is not a very dissonant chord, gives it a very characteristic sonance. Therefore, they are chords with (double) dominant function. They could have been placed in the dominant family, but we preferred to separate them due to the difference between the functional fundamentals.

We have also placed here the diminished seventh chord, since if we take its virtual fundamentals, it could also be considered belonging to this family, but we would have to add two more virtual fundamentals, all four separated by minor thirds. As we said in 3.1.3, when the diminished seventh chord is inserted in a clear tonal neighbourhood and there are no possible harmonic confusions —for example, the diminished 7th chord on the 7th degree (leading-tone) of the minor mode—, then we will symbolize it as $\dot{G}^{79}$ (in C minor) and it will be a chord that will be between this family and the unitonal dominant chord family, although structurally it still has four symmetric virtual fundamentals. Of the four we usually write at most two, the most significant ones according to the harmonic context in which they are laid (separated by a tritone or a m3).

When the m7 of C (B♭) is enharmonized as M3 of the other fundamental F♯ (A♯) we find the much used french augmented sixth chord (CEF♯A♯).

For more information on these types of chords and their links see 6.2.1.

3.1.6 Major-minor chords

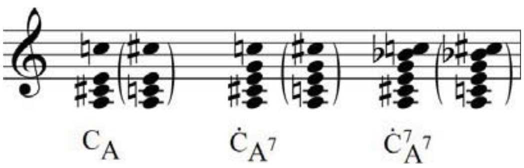
They are chords of genre C_A.

They are chords in which a fundamental forms, simultaneously, a major triad and a minor triad (apart other possible added notes). Automatically, a second functional fundamental (a minor 3rd up) appears. There is a “collision” of m2 between its major 3rd and its minor 3rd, which is the other fundamental. They are rather dissonant chords.

We have represented some of them in figure 29.

If we include the fifth of C, automatically A acquires its m7 and therefore we obtain the 7M3 structure (AC♯G) and it gain a dominant flavor (tonal vector towards D) (this chord is pretty used in blues).

Fig. 29



We thus have two tensions of M3 and two tonal vectors separated by a distance of m3. Tonal vectors that acquire more force if they become 7M3 structures.

See also 6.2.5.

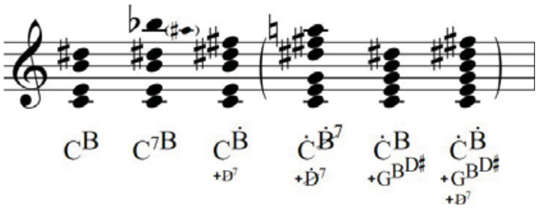
3.1.7 Cluster chords

They are chords of genre C^B (o $C^{D\flat}$).

It is the most dissonant chord family since the two functional fundamentals (the two M3) are separated by an interval of m2.

Figure 30 shows some chords of this family. Note that if we complete the triads of C and/or B, we add tensions that appear in other families. If we put the note G, we add a new tension of M3 ($+G^B$, then it also acquires the augmented chord character) and if F♯ appears, we add a new tritone tension ($+\text{Đ}^C$). I use the word «gene» which I believe it works to exemplify this fact: they have «genes» from other families. In the same way that the diminished seventh chord has genes from the dominant chord families and the symmetrical chord families, or chord $\dot{C}^7_{\dot{A}}$ has genes from the major-minor family and the dominant family, cluster chords soon acquire genes from other families when the convergent structures of C and B are completed.

Fig. 30



In the symbology we put B above because it is the harmonic *5f* of *3f* of C, but we could also put it below (B^C) as main fundamental since, as we will see, the chord better resolves on a chord based on E (B→E) than on one based in F (C→F) (see 6.2.5).

With the chord families studied so far we have all possible combinations between two functional fundamentals.

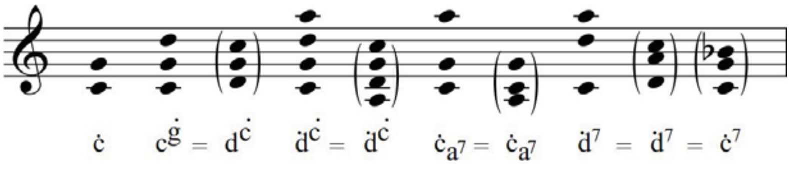
3.1.8 Suspended chords

They are chords that do not have functional tensions. That is to say, they do not have intervals of M3 nor intervals of tritone. They are basically quartal and quintal chords. Therefore in their symblogy no capital letters appear (they have no functional fundamentals). In jazz theory are represented as C^{sus2} or C^{sus4} although our family has a wider range.

They are chords of genre $c^{(7)}$, c_a, c^g o d^c ($c^{b\flat}$) (in lowercase). Some of them, according to the note they have in the bass, can be coded in two different ways (c^g or d^c).

We have represented some of them in figure 31.

Fig. 31



I use the term "suspended" not as a synonym of nonchord suspension tone (*ritardo* in italian, although some of these chords may be considered with suspensions in certain harmonic progressions), but because they do not have harmonic tension (although can have *sonance* tension) and therefore we could say that they are like «suspended» in the air. Most are not very consonant, but have almost no tonal tension. See also 6.2.5.

3.1.9 Other chords

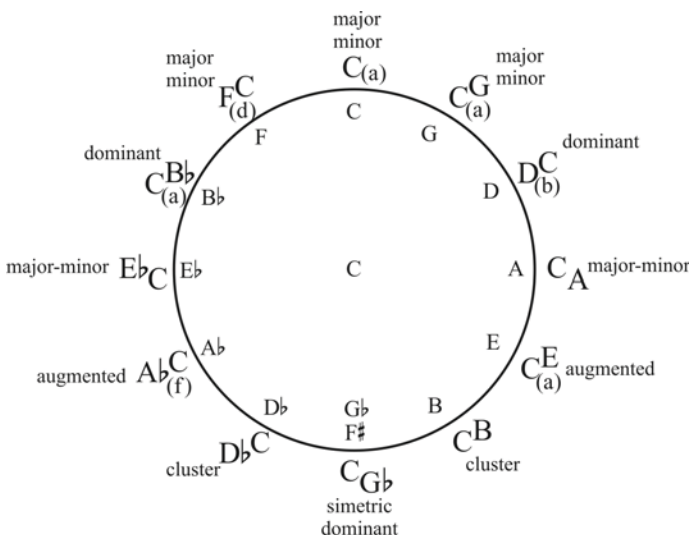
Most three- and four-note chords can be included in any of these families we have seen so far, as well as pretty five- and six-note chords. From five notes many chords enjoy the properties of two or more families at a time (see annex 1). In 3.3 we can see a correspondence of the most used and known chords with their fundamental symbology (and therefore seeing the family to which they belong) that informs us of their internal harmonic tensions.

We have already seen that in these families there are chords, such as the diminished seventh, that have genes from several of them, even an independent family could have been made only for this chord. Also, for example, all chords that include the 7M3 structure (real or virtual) have genes from the dominant chords families. What is really useful and important is to know what are the functional fundamentals of chords and with this information know what types of tensions occur between them, since, looking at the fundamentals of the contiguous chords, we know their global tensions and their homotonic (local) and tonal relaxions or resolutions. As we will see in detail in chapters 6 and 7.

3.2 The functionality of chord families

We can summarize the chord families we have just seen, those that have one or two functional fundamentals (all of them except except the suspended family) using the circle of fifths (figure 32).

Fig. 32



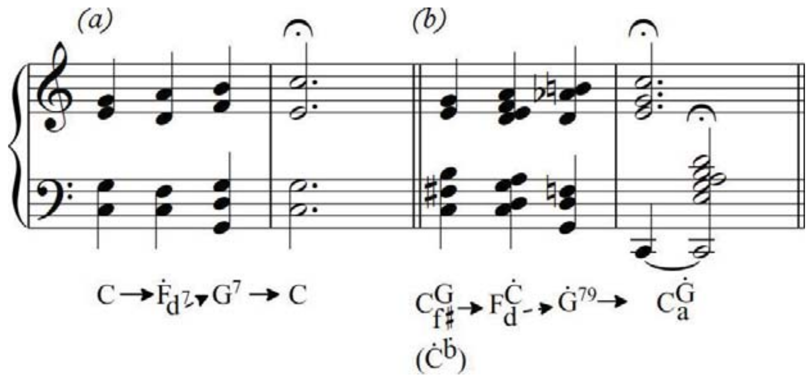
If we add notes to these chords in a way that does not create new functional fundamentals, then this does not affect its harmonic tension (although it can affect the *sonance* tension). For example, adding note A to chords C or C^{G} to convert them into chords of the minor chord family. As I said I have preferred, however, to put the major and minor chords into two distinct families (they could have been placed into one) because of their characteristic sonority and the great literature that exists in the history of music about the chords with the major third and those who have the minor third. They have a different sonority, but their harmonic function, in the sense of the notes to which they want to resolve homotonically (locally, independently of the tonality), is similar (determined by the M3 intervals they contain). As we have already seen, proof of this is the functional symbology applied by Riemann to minor chords (see 1.8 and 5.3) or the use of Rameau's *sixte ajoutée* or the use of C^{6} or C^{6}_{5} symbols for chords in which A is added to the major triad and, in both cases, maintain the function of C (as long as the bass is C).

Another added note that usually does not vary the function of a chord is the major second major of the fundamental (D for chords with fundamental C —the fifth of his fifth).

In any harmonization, if chords are changed by other chords of the same family (having the same functional fundamentals), the color and con(sonance) of the harmonization vary, but its functional structure remains similar. Or, in other words, if we want to color chords without varying their harmonic function, we can add or change notes as long as we do not create new uppercase fundamentals (functional fundamentals) in the functional symbols.

In Example 3.1 we have two cadential progressions. In (a) a simple version is exposed and in (b) the chords are colored by changing or adding notes, which do not substantially modify the functional structure of the chords. C is converted to $C^{G}{}_{f\sharp}$ (it would also be possible to code it as $\dot{C}^{b}$), C^{G} comes from the major chord family of C and the added F♯ creates a tritone (CF♯) tension, but since there are no notes A or E♭ in the chord the virtual fundamentals that are created (Ɖ y A̶♭) are very weak. F_{d} becomes $F_{d}{}^{C}$, of the same family of minor chords. G^{7} becomes G^{79}, of the same family of dominant chords. The final triad C becomes $C_{a}{}^{G}$. C^{G} is from the family of C and the added A gives us another example of what we have discussed above regarding the true functional fundamental of the minor chord. In fact this last chord is not heard as an enriched inversion of A minor but, being C in the bass and having the added note D, A is felt like a Pythagorean harmonic of C (C:G:D:A).

Ej. 3-1



Example (b) is a re-harmonization of (a) using chords from the same family or maintaining the functional fundamentals of the four chords, but a more aggressive harmonization could have been done, adding new functional fundamentals without necessarily losing the cadential sense of the progression. We should only be careful not to lose the 7M3 structure (of C major) in the two chords prior to the final chord (although in certain cases the 7M3 structure of this tone could be shared with other 7M3 structures).

3.3 Correspondence between the main known chords and the fundamental symbology

It will help to clarify the meaning of chord families and the fundamental symbology if we draw a correspondence between the main chords known in the history of music and the fundamental symbols according to the harmonic theory of this book.

In figure 33, below each chord, we place in the first line our fundamental symbology and, below, other commonly used symbols, although the symbology to represent chords are not currently standardized. For the case of augmented sixth chords see figure 6.

Figure 34 shows the fundamental symbology of known chords that have been baptized with a name. We put the symbols independently of the tonal context in which the chords are immersed. For example, the Tristan chord, if it is isolated (without tonal context) creates a clear tonal vector towards F♯; if the chord were to be resolved in this tonic, the F should be enharmonized to E♯, leading-tone of the tonality. But in Wagner's work the F acts as Phrygian dominant (upper leading-tone) of E^7 (see example 7-26).

Fig. 33

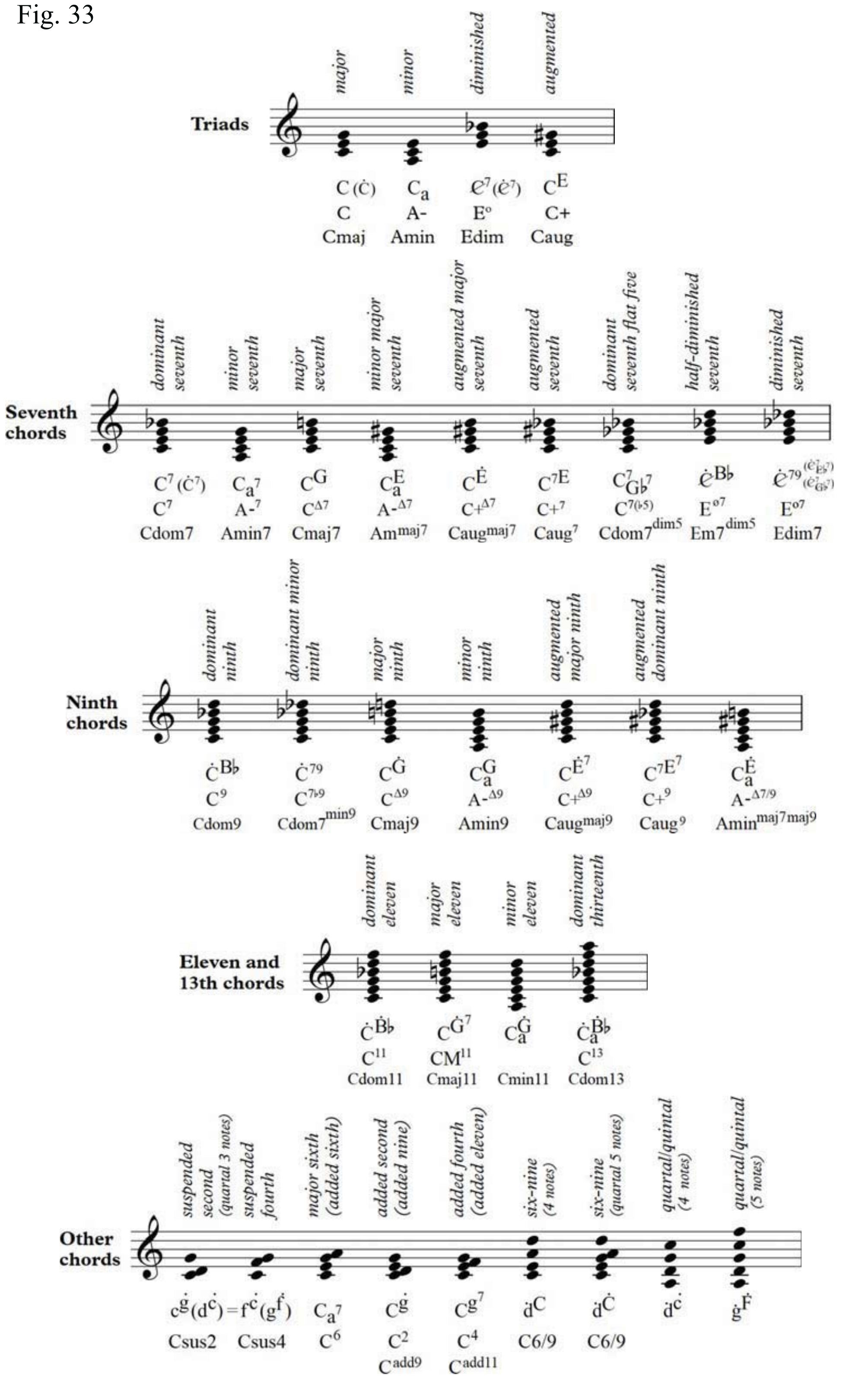
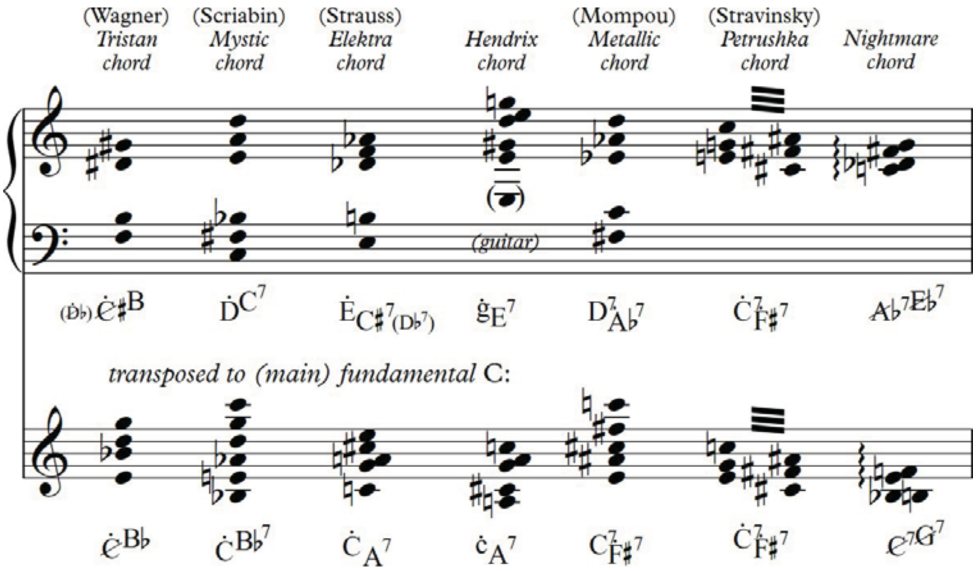

Triads
major
minor
diminished
augmented
C
A-
E°
C+
Cmaj
Amin
Edim
Caug
Seventh chords
dominant seventh
minor seventh
major seventh
minor major seventh
augmented major seventh
augmented seventh
dominant seventh flat five
half-diminished seventh
diminished seventh
Cdom7
Amin7
Cmaj7
Caug7
Cdom7
Em7
Edim7
Ninth chords
dominant ninth
dominant minor ninth
major ninth
minor ninth
augmented major ninth
augmented dominant ninth
Cdom9
Cmaj9
Amin9
Eleven and 13th chords
dominant eleven
major eleven
minor eleven
dominant thirteenth
Cdom11
Cmaj11
Cmin11
Cdom13
Other chords
suspended second
(quartal 3 notes)
suspended fourth
major sixth (added sixth)
added second (added nine)
added fourth (added eleven)
six-nine (4 notes)
six-nine (quartal 5 notes)
quartal/quintal (4 notes)
quartal/quintal (5 notes)
Csus2
Csus4
C6/9
C6/9

Fig. 34



3.4 Chord inversions and their optional symbology

We have already seen in 2.3 the strength of the octaves and deduced the functional equality between inversions. Whatever the inversion, the notes of the chord, as harmonics, are repeated in their upper octaves. This causes that, regardless of the bass of the chord, the ratios between intervals remain the same and by the virtual fundamental effect the ear functionally matches the chords and assigns them the same fundamental(s). In figure 16 we set as example the inversions of the major triad, but could be any other chord.

In general, the harmonic function of a chord changes little if you change the order of notes, including the bass.

Due to the significant power of harmonic *3f* (fifth) there are cases where (changing bass) we are close to the boundary of the functional change (as we have seen in 2.3), as would be the case of the cadential $^6_4$ chord where, in a tonal diatonic context, the upper notes can also be heard as appoggiaturas notes with a tendency to «resolve» in the harmonics *3f* and *5f* of the bass. In fact, it is not a question of discussing whether it is one thing or another but, as so often happens in harmonic analysis, be aware that both facts are perceived simultaneously.

Minor chords can be separated in two fundamentals, this explains the different functional consideration that has been given to this chord —already since Rameu times— according to the note that is in the bass. As we have seen, the chord ACE can be considered an A minor chord or a C (major) chord with the *sixte ajoutée* if C is on the bass. In jazz, in order to symbolize the chord CEGA the C^6 symbol is used when C is on the bass and is functionally used as a C major chord, even as final chord. Our symbol C_a reflects this duality of the A minor chord with that of C major sixth.

Therefore, having a note or another in the bass may have its importance, especially with respect to *sonance*, but in some cases may also vary the function of the chord.

This is why in the analyzes of works where the chords are not very complex a special symbology might be used to differentiate the inversions, but for many other analyzes, in which one only needs to get an idea of the harmonic functional progressions, it will not be necessary to differentiate between inversions. In fact, in most examples of chapter 7 this symbology is not used.

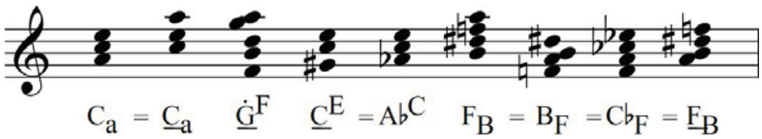
We will next do a proposal of symbology that differentiates inversions in case one wants to incorporate it to the analysis or to do a harmonic scheme of a score without the notes.

Our fundamental symbology already incorporates all the information of the notes and is independent of the musical scales, so it is not necessary to add numbers (other than 7 and 9) on the bass to specify the rest of the notes, as is done with the baroque figured bass. In jazz theory inversions are usually specified by placing a slash after the chord symbol and adding the bass note if it does not match with the chord root. For example, the 3rd inversion of the dominant seventh chord with root C is symbolized C^7/B♭ (B♭ in the bass).

With our intention of seeking simplicity in symbols that at first sight may seem complex and because, if we are doing the analysis with the score, the bass note is easily localizable, we will only indicate if the chord is an inversion or not. To do this we will simply put a line under the symbol in case the chord is not in the fundamental state (or does not have the main fundamental on the bass). Looking at the bass in the score we will know quickly which inversion is involved. We have already used this method in figure 6, and in figure 35 we give some more examples. We will always consider that a chord is in a fundamental state when the bass of the chord coincides with the fundamental that is in the lower position in the symbols, even it is in lowercase.

If we have a virtual fundamental, the fundamental does not exist in the chord, in these cases we can consider fundamental state the chord with the M3 of the virtual fundamental in the bass, which is the most stable position since we have m3 and tritone of the bass above (the structure of the harmonics of a virtual fundamental). The symmetrical chords should be coded according to a correct tonal context.

Fig. 35



As we have said, if music or chords have some complexity, it is not necessary to indicate the inversions if we want only to get an idea of the harmonic tensions that are at stake. For very paradigmatic cadences in musical theory, such as the cadential $^6_4$ (the tonic chord with the dominant in the bass), we will use the symbol D$^6_4$ in the tonal analysis (D of dominant).

If our symbology is used to harmonize melodies (without drawing complete chord notes), then we can specify, as in jazz/modern music, the bass note below the line (when the chord is not in the fundamental state).

As for pedals points, we will represent them by placing the pedal note with a line that will cover its entire duration, below the rest of the symbols representing the rest of the notes. See examples 7-7, 7-9, 7-16, 7-21 or 7-25 of chapter 7.

4. Secondary relaxions and other successions of fundamentals

We have seen that the two local (homotonic) main resolutions or relaxions of the functional fundamentals are (in the next chord) the lower fifth (or upper fourth) fundamentals (htonal resolution) or the lower minor second (or upper major seventh) fundamentals (Phrygian resolution).

This was the result of the tendency of the auditory system to «adjust» the «false-fifths» or «quasi-fifths» which are formed with the M3 and tritone intervals.

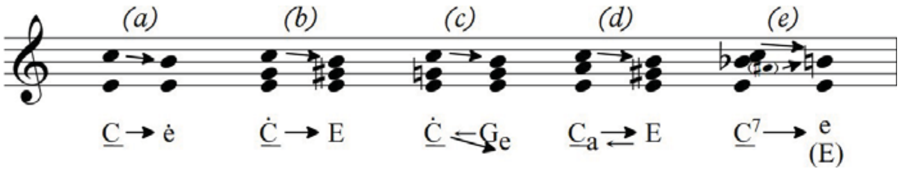
4.1 "Locrian" relaxion

There is another succession of fundamentals that produces significant relaxion, although is little used in tonal music because it does not fit within the diatonic scales, except in few cases, as in some half cadences in the minor mode.

Consists of the passage from a functional fundamental of a chord to another fundamental at M3 distance. For example, Ċ-Ė (CEG-EG♯B chords). I call it «Locrian» distension.

The acoustic demonstration of this relaxion would also be related to the resolution of the "quasi-fifth" of the M3, but this time only considering the fundamentals of the chords or intervals (figure 36).

Fig. 36



In this relaxion it is important that in the «resting» (second) chord the fundamental has its fifth since it is precisely the note of resolution. If the E is a functional fundamental (has its M3 [figure 36b]), the relaxion that occurs is clear despite the chromatism involved (G-G♯). If the fundamental of arrival is not functional, for example, the fundamental (lowercase) of a minor chord, then the relaxion is not so clear because there is a sequence of tension between fundamentals in the opposite direction (C-G) (figure 36c). But we must distinguish between successions of tension and successions in which there is relaxion in the two directions.

There is also relaxion in both directions if the first chord is from the minor family (C_a) because we also have tension in the opposite direction (a-E) (figure 36d). These two chords (C_a-E) could be continuously linked as a kind of *perpetuum* *mobile*, and either could be the final chord because there is homotonic relaxion in both directions (Locrian and htonal) (see also example 6-32 and figure 39). Therefore this Locrian relaxion or resolution establishes a relaxation between chords with certain conditions. The Locrian relaxion «wins» to the htonal one in the other direction if C_a chord is in second inversion, because C and A sound like appoggiaturas of the chord with fundamental E. Scholastically speaking we would be in the case of a cadential $^{6}_{4}$ in the minor mode (for example, see chapter 7 example 7-15 bars 78-79).

Melodically, without chords, the two notes that form an M3 interval can be «resting» notes; in this case the tonal field is determinant to establish the direction of relaxion (if we have strongly established, for example, a tonic C, then we have melodic relaxion in the two cases: E→C o A♭→C).

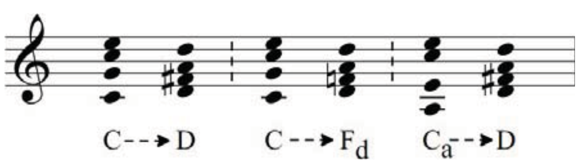
This resolution becomes much clearer if the fundamental of the first chord has the minor seventh since then (for example, C^7→E/e) there is a double resolution of «quasi-fifths» EC and EB♭ that resolve in the perfect fifth EB. B♭ is usually enharmonized to A♯.

In fact, Locrian relaxion is a variation of the Phrygian one since what causes relaxion is the Phrygian melodic resolution of the interval of M3 by means of a descending step of m2 of one of the notes.

4.2 “Dorian” relaxion

There is another succession of fundamentals that also produces some local harmonic “relaxation”, but in this case I would no longer use the name «resolution» since it is very weak. It is the ascending major second succession of fundamentals (figure 37). This harmonic progression is used continuously in tonal music. Much more than its inverse sequence, the succession of fundamentals by a descending major second. As examples we could include the typical tonic-supertonic (I-ii /i-ii), subdominant-dominant (IV-V), deceptive cadence (V-vi) in the major mode, ii-V, an so on. I call it «Dorian» relaxion. We cite it only to take it into account, but it has little weight compared to the other three homotonic relaxions. The most powerful Dorian succession is that of genre $C_a^{(7)}$-D if in the bass there is a jump of P5 (a→D).

Fig. 37



4.3 Successions of fundamentals without homotonic tension

Of the twelve intervals (six interval classes) that can be formed with the twelve notes palette of the equal-tempered chromatic scale we have established so far — once these two secondary relaxions have been added —eight intervals (four interval classes) between fundamentals, which determine successions of tension or relaxion. They are the fundamental (lower/upper) jumps of P5/P4 (htonal), m2/M7 (Phrygian), m6/M3 (Locrian) and m7/M2 (Dorian). We would only have pending to study the intervals of unison, m3 and tritone. These intervals form a tonal axis (according to Bart?k/Lendvai's theory of tonal axes).

The sequence of chords whose fundamentals form one of these intervals (they are in the same tonal axis) do not establish, locally, nor succession of tension or homotonic relaxion (would have tonal tension if they were established in a tonal field). They are unusual chord successions in tonal music because they do not fit within the palette of notes of the major and minor scales. In figure 38 we have some examples, they are sequences of chords that seem to be floating, not knowing exactly where they are going.

Fig. 38

4.4 Summary of homotonic tensions and relaxions

In Table 1 we have all local (homotonic) tensions and relaxions between the functional fundamentals of two chords, that is to say, independently of a possible tonal memory (tonal field) in which they could be immersed (remember that a capital letter represents an M3, for Example C = CE, having its fifth or not).

In case these chord progressions occur in a musical neighbourhood where the tonic is very established, the tonal tension/relaxion should be added (not to be confused with the homotonic htonal relaxion, which is simply a certain local jump between fundamentals). For example, the plagal cadence: the tonal relaxion (resting in the tonic chord) has more force than the homotonic local tension between fundamentals (the fundamental jumps an upper fifth and therefore is a local succession of tension). In general, to know the overall tension between two chords in a piece of music, we have to take into account three factors: tonal tension (if we have an established tonic), homotonic local tension and sonance tension. The sonance tension simply means that the passage from a dissonant chord to a consonant one produces relaxion, tension in the opposite case.

Tabla 1

$\longrightarrow$ / $\dashrightarrow$ } *resolution or relaxation between functional fundamentals*		*no tension*
htonal	$C \longrightarrow F$ (or f)	C(c) - - - - E♭(e♭)
phrigian	$C \longrightarrow B$ (or b)	C(c) - - - - F♯(f♯)
locrian	$C \dashrightarrow E$ (or e)	
dorian	$C \dashrightarrow D$ (or d)	

When the chords have more than one fundamental (functional or not) the issue is somewhat complicated, but as a general rule we can say that if the fundamentals form homotonic distension in both directions, the sequence will not have as much tension or will have relaxion in the two directions. This will depend on each specific case.

For example, Table 2 is the result of applying Table 1 to all link possibilities between major and minor chords. An arrow in both directions means that there is homotonic relaxion in both directions, dashes without arrows mean that there is neither tension nor relaxion. To these local harmonic tensions the slight tension of sonance, also local, should be added when passing from a major chord to a minor one (since the minor chord is less consonant than the major chord) or relaxion in the opposite sense. In the case of double arrow, the notes in the bass (and also in the soprano) can be decisive.

Tabla 2

$C_a \longleftrightarrow F$	$C_a \longleftrightarrow F_d$	$C \dashrightarrow F_d$
	$C_a \dashrightarrow A\flat_f$	$C \longleftrightarrow A\flat_f$
$C_a \longrightarrow B$	$C_a \dashrightarrow B_{g\sharp}$	$C \dashrightarrow B_{g\sharp}$
$C_a \dashrightarrow D$	$C_a \dashrightarrow D_b$	$C \longrightarrow D_b$
$C_a \longleftrightarrow E$	$C_a \longleftrightarrow E_{c\sharp}$	$C \dashrightarrow E_{c\sharp}$
	$C_a \longleftrightarrow G_e$	$C \longleftrightarrow G_e$
C_a - - - - E♭	C_a - - - - $E\flat_c$	C - - - - $E\flat_c$
C_a - - - - F♯	C_a - - - - $F\sharp_{d\sharp}$	C - - - - $F\sharp_{d\sharp}$
C_a - - - - A	C_a - - - - $A_{f\sharp}$	C - - - - $A_{f\sharp}$

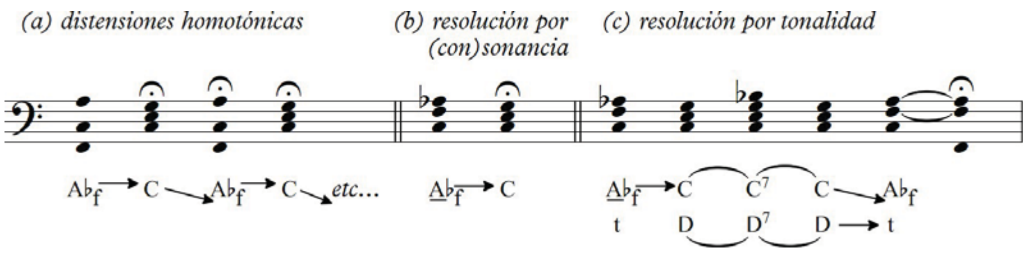
It is not necessary to pay much attention to the apparent complexity of Table 2, as we say is simply the result of applying Table 1, which is the one really important; and, within it, the two main relaxions stand out: the htonal and the Phrygian ones.

As we have been saying, the tonal memory also influences (and much). For example, the authentic cadence in the minor mode actually has homotonic relaxions in the two directions; in F minor (C-$A\flat_f$) we have htonal and tonal resolution (C→f), but in the opposite direction we would obtain the Locrian resolution (A♭→C); if the tonic is F, we have more rest in the F minor chord, but if F is not a clear tonic, if we only play these two chords —detached from a tonal field— we can rest in either.

When we have this equilibrium, a variation of sonance can decant the balance. For example, by placing the F minor chord in second inversion (figure 39).

We will insist in saying that the global tension between two chords is always the sum of the homotonic, sonance and tonal tensions. Now we are dealing only with the homotonics, that acquire more force in weak or indeterminate tonal fields. The tonal tensions will be studied in the next chapter.

Fig. 39



4.5 Homotonic relaxions and tonal axis theory

If the Locrian homotonic resolution was in the opposite direction, we would have obtained a perfect demonstration of the tonal axis theory proposed by Ern Lendvai (1955 and 1993), which, according to this author, was often used by Béla Bartók in his compositions.

But we shall see that, even so, the formulation of these homotonic relaxions gives sufficient consistency and reason to this theory, or, if one means otherwise, the two agree on many points.

Lendvai's theory, in short, comes to say that we can divide the 12 notes of the equal-tempered chromatic scale into three groups (axes) of notes (which can also be understood as chords roots or tonal regions) and that between them we found similar relations to those of tonic, subdominant and dominant.

The notes of each group are separated by a m3 or a tritone or, in other words, each axis would form a diminished seventh chord (although the theory goes beyond the diminished seventh chord).

For example (that the group is tonic, subdominant or dominant will depend on the compositional discourse):

- tonic axis or group: C, A, F♯, E♭

- subdominant axis or group: F, D, B, A♭

- dominant axis or group: G, E, D♭, B♭

If we join these notes with a line in the circle of fifths, these groups will form 3 perfect squares.

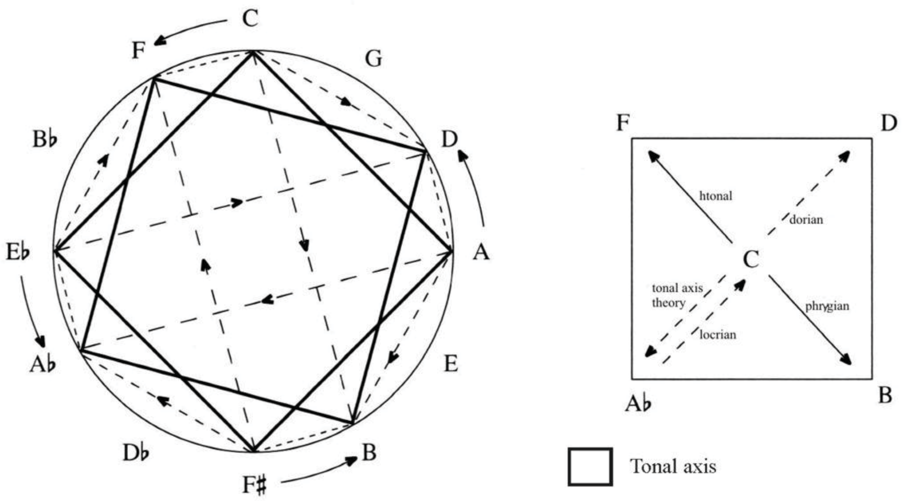
If we understand a «dominant» as a note or a chord resolving in a «tonic», according to tonal axis theory, the notes or chords of the «dominant» axis «resolve» in notes or chords of the «tonic» axis (and the same with the tonic-subdominant and subdominant-dominant axes). Well, our homotonic theory largely coincides with the tonal axes one.

That is, according to tonal axes theory, we have the following resolutions or relaxions: G→C, G→A, G→F♯, G→E♭, E→C, E→A, E→F♯, E→E♭, etc. If we look at Table 1, we will see that they coincide with the htonal, Phrygian, Dorian resolutions and fail with Locrian. However if we look at Table 2, we can see that if we take minor chords, the «Locrian sequences» are in equilibrium (due to the htonal relaxion that occurs in the opposite direction). In addition, Lendvai's theory divides each axis into two tritones, if we only take the resolutions between the «primary» tritone of each axis, we get our main htonal and Phrygian resolutions.

Theories also coincide in considering that between notes or chords of each axis there is neither (local) tension nor relaxion.

The coincidence between relaxions of the two theories is reflected in figure 40. If we apply the htonal, Phrygian and Dorian relaxions to points of a square, we obtain points of the following square. If we only take into account the htonal resolution, the squares (tonal axes) tend to rotate and resolve counterclockwise — towards the square (axis)— a lower fifth. As we have said, it only fails with the Locrian resolution since, if it is applied to the points of the same square above, it gives points of another square, which is not drawn in the figure (the GED♭B♭ axis).

Fig. 40



The similarity between these two theories will help us, when we study tonality, to identify tonal chordal functions in complex tonalities or in remote but transient modulations.

Going back to our theory, if chords have several functional fundamentals, as a rule, the homotonic local tension-relaxion logic between chords will follow the logic of tensions-relaxion between fundamentals, always keeping in mind the tonal field that is being created, which can quickly change the overall relaxion between chords. The utility of local homotonic relaxions is to find a fluid sequence of chords (rather than to look for cadences), especially in chromatic fragments where the tonal vectors change rapidly; in these cases the tonal field weakens and the homotonic and sonance tensions gain weight. Of course, if that is what we want in our composition, since we might be interested in the opposite, namely, to obtain sequences of chords always in tension, both tonal and homotonic or sonance; in this case we will continue applying the same principles, but in the opposite sense.

5. Tonality

5.1 The tonal field and its vectors

According to Carl Dalhaus (1967) and the New Grove encyclopedia, the term tonal or tonality was introduced by Castil-Blaze in 1821 —although this term had already appeared before, at least in the dictionary of Alexander Choron (1810)— to give relevance to three notes or three intervalic relations: the tonic, its 4th and its 5th (calling them *cordes tonales* in contraposition to the *cordes mélodiques*), that is, what would later be generalized as subdominant (4th) and dominant (5th) of the tonic.

Now instead the term tonality is closely intertwined with hierarchical relationships of notes within musical scales and more specifically of two scales or modes: the major mode and the minor mode.

Thus, for example, according to how the notes of a musical piece are fitted with the structures of certain major or minor scales, we say that that piece or fragment is in the «key» of D major, or is in the key of E minor, etc.; in practice the term key disappears and we simply say that the fragment is in C major and then modulates to G major, A minor, etc. But a representative major or minor scale is usually in mind.

Our concept of tonality goes some way back to its origins, we consider the major and minor modes of the same tone as two modes that have a different color, but which are subject to the same laws of tension and resolution. The two have the same note in which the chords tend to rest and have four identical significant notes: tonic, subdominant, dominant and leading-tone (although these notes may occasionally be altered in some musical fragment without losing «the key of the work»); the other notes simply add color to the musical discourse. Therefore, it would also be correct to say that we are in the «key of», without specifying whether it is major or minor; actually tonality is independent of the use of a scale, which does not necessarily have to consist of 7 notes.

Continually, in the course of any musical piece, a hierarchy is established between the 12 notes of the musical palette so that there are some that the auditory system gives them the power to be more resolutive or more stable than others; to say it in a simple way: there is a note (or several) that are the best candidates to finish a melody or to be root of an ending chord since they provide the highest degree of «resolution» or rest. This note would be, according to our way of seeing the tonality, the tonic, regardless of the scale or mode that can be associated. Sometimes I like to use the term *tonal vector* when, in a certain melodic or harmonic transition or in a transient modulation, some note is tonicized (I usually employ the term: tonal vector «towards note x»). There may be several vectors at a time, and with different lengths (their strength). Our graphic vision of the «tonality» of a piece of music consists of this living organism formed of arrows (tonal vectors) which, in the course of a work, are lengthened and shortened. This 'tonal field' can be very static for pieces in which the tonic is always the same or very dynamic for chromatic pieces that are continually changing their tonic. In pure twelve-tone works this tonal field would once again be static and would be formed by an organism of 12 almost equal vectors without one of them predominating over the others. This is our vision of tonality.

Therefore, our work in analyzing tonally complex works will not be to find, at any moment, a scale of reference, but rather the tonic (or tonics) that appear in the course of the work. In more diatonic pieces, we will simply be in a major mode if the third of this tonic is usually major and we will be in a minor mode if the third one is usually minor, although they may eventually vary, for example, using the Picardy third (major 3rd) in the minor mode.

5.2 Tonality and the 7M3 structure

We have seen in the previous chapters that there is a very powerful structure that creates a strong tonal vector: the structure 7M3. This structure creates a tonic (transient or not, even if the tonic note, to which it tends, is not present in the musical fragment) and is formed by the fifth, the fourth and the leading-tone of this tonic (as notes, not as chords). That is, in conventional theory, they would be the dominant, the subdominant and the leading-tone of the key. It varies slightly from the term introduced by Castil-Blaze in 1821, since he quoted the structure formed by the tonic, the fourth and the fifth, although if we consider this structure formed of functions or fundamentals of chords instead of notes, it comes to be a similar concept.

Finding the 7M3 structures in a musical work will therefore be very important in order to establish its tonal vectors.

We will see below that from the 7M3 structure we deduce the major and minor scales (with leading-tone). They are the only two scales of seven distinct notes that contain a single 7M3 structure and therefore have no conflict of tonics.

Let us take, for instance, the key of C:

The fixed notes of the 7M3 structure in this key are G, B, and F. Apparently, in order to maintain C as resting tone, the other notes may be any, but we will rule out certain combinations.

D♭: This note creates another 7M3 structure with the fixed notes: D♭-F-B (C♭), that is, a tonal vector towards G♭. If there is also E♭, another is formed: E♭-G-D♭ with a vector towards A♭. If, in addition, A is natural, we find a hidden one: A-D♭(C♯)-G with vector towards D. This note (D♭) is discarded if we want a single tonal vector in the scale.

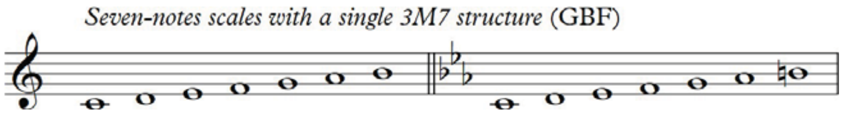
E♭: If A is natural, it also creates another 7M3 structure: F-A-E♭ and a tonal vector towards B♭. In addition there would be a second 7M3 structure with the note B: B-E♭(D♯)-A and a tonal vector towards E. Therefore, E♭ can only be accompanied with A♭ if we only want a single clear tonic.

A♭: If E is natural, it creates another 7M3 structure with D: E-A♭(G♯)-D and a tonal vector towards A.

Therefore, **the only scales with a single 7M3 structure are the major and harmonic minor (with leading-tone) scales** (figure 41).

In practice, these note combinations that we have discarded are used continuously in tonal music, even in the simplest, but we have tried to show that by applying only the tonal force of the 7M3 structure the two most common forms of major and minor scales are deduced.

Fig. 41

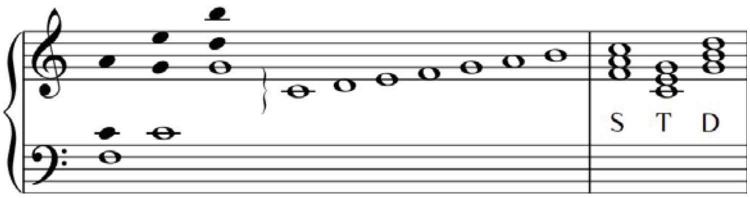


As we have seen in 2.7, in the minor mode, in order to avoid the exotic interval A♭-B♭, the notes A♮-B♮ are sometimes used locally in the ascending tunes or B♭-A♭ in the descent ones. In the latter case, what is lost in tonal vectorial power is gained in internal scalar coherence (fifths F-C-G and fifths A♭-E♭-B♭) and we get an ascending melodic tonal resolution A♮-B♮-C and a descending melodic Phrygian resolution to the dominant (B♭-A♭-G), known as *basso di lamento.*

5.3 Tonal functions and their symbols (functional symbology)

Before continuing with the 7M3 structure we will make a parenthesis and apply what we have learned in the previous chapters about the tensions that appear in the chords of the diatonic scales and will see the coherence of the true chord fundamentals with the reduction of the tonal functions to three: the tonic, dominant and subdominant functions (theory officially consolidated by Hugo Riemann).

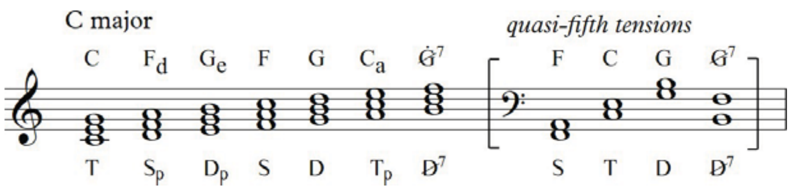
The compendium of tonality in three «chords» or main functions was already present in J. P. Rameau and especially in J. F. Daube. Rameau, in his *Démonstration du principe d'harmonie* (1750), deduces the major scale as a result of considering the fundamentals C, F and G and their first five harmonics. Daube, four years later, in his work *General-Bass in drey accorden* (1754), argued that any accompaniment can be made with three basic chords: the tonic chord (CEG), the subdominant chord with an added sixth (FACD) and the dominant seventh chord (GBDF). De facto, matching inversions, with Daube we find a succession of degrees I, II, V, which is also the foundation of the functional theory of jazz. Also William Jones in his book *A Treatise on the Art of Music* (1784, p. 13) says: "Therefore these three chords [C, G, F] comprehend all the native harmony of the octave; and the three notes C, G, F, are the fundamental notes, because they carry all the degrees of the octave in their accompaniments". Recall here figure 18 of Chapter 2:



We will introduce our functional symbology (not to be confused with the fundamental symbology of the chords), making some changes with respect to the Riemann school's symbology, applying everything we have seen in the previous chapters.

In Figure 42 we have the triadic chords of the major diatonic scale. Above the pentagram we have our separation of chords into fundamentals (fundamental symbology, seen in 3.1) and below the functional symbology used by Riemann and his followers. These symbols show the importance of the M3 and tritone intervals to establish the tonal tensions and therefore their functions. In the major diatonic scale, C, F, and G are the only notes that have an upper major third. And the only tritone found is B-F that determines its virtual fundamental G. Functionally these are the main fundamentals of the scale. Note that the uppercase (functional) fundamentals are only three (C, F, G) and match the T, S, and D of the Riemann school. In the minor mode, tonic and subdominant may be minor chords, but the dominant G is always in major (otherwise, we would be in eolian mode). That is, as dominant, always has the note B, the leading-tone. Having a subdominant chord (either major or minor) and a dominant chord automatically implies having the 7M3 structure, since, as we say, the leading-tone is always included in the dominant chord.

Fig. 42



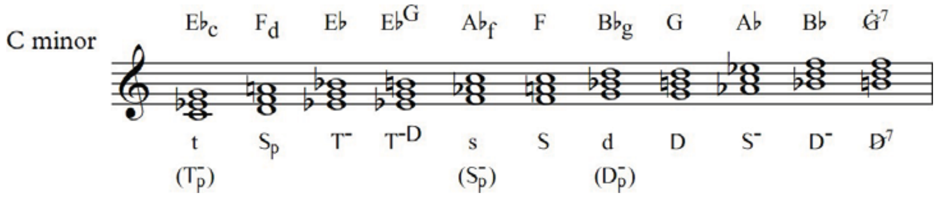
The minor mode is much more complicated. Riemann was dualist, that is, he considered the minor chord as an inverse reflection of the major chord. As we have seen that the diatonic major scale can be deduced in various ways from harmonics (see 2.6), the dualist theory believes that the minor chord and the minor mode (although there is no single minor scale) can be deduced from the symmetrical inversion of the major chord and the major scale, that is, as if the musical notes also had a kind of subharmonics (fact that is not an acoustic reality).

Following this theory Riemann designed symbols for the minor mode that complicated, and in our opinion distorted, his great success, which was precisely to simplify the tonal functions to three. Some followers of their symbols, such as Wilhem Maler and Diether de la Motte, were also aware of this and changed their symbols, among others, that of the minor tonic chord, that of the minor subdominant and that of the «minor dominant», putting simply the symbols ***t***, ***s*** and ***d***. This is what we have also done, though for other more complex chords our functional symbols are different in order to be consistent with the fundamental symbology we have seen in Chapter 3.

As can be deduced from what we have been explaining about our concept of tonality, our theory is not dualist, we consider the major and minor modes of the same tonic, as we have said, two modes with different character and color, but ultimately submitted to the same tonal laws towards the same tonic.

In figure 43 we have triadic chords of notes within the orbit of the different scales of C minor (we could still place more). Above the pentagram we have again the chord symbols according to its fundamentals and below our tonal functional symbology of the chords with respect to tonic C.

Fig. 43

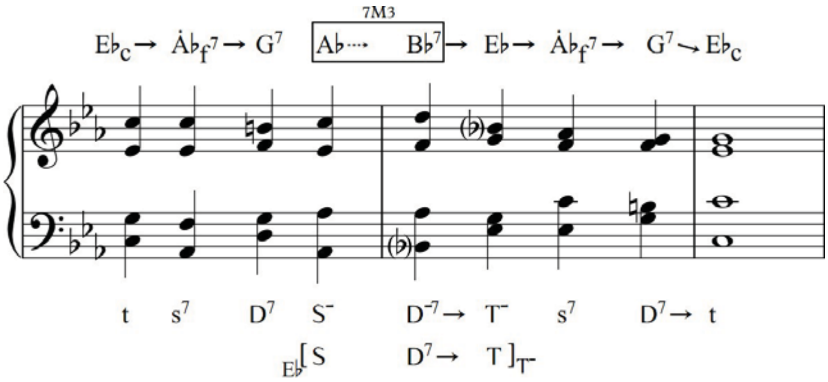


What is the meaning of T^-, S^-, D^-?

T^-, S^- and D^- are the tonic, the subdominant and the dominant of the relative major (key) of the minor, key to which always tends the minor mode when the 7th degree is not the leading-tone (they have the same key signature). If we have a musical fragment in minor mode with the functions T^-, S^-, D^-, these symbols will give us information that in that part we can have a transient modulation to the relative major. For example, in figure 44. We could use either of the two symbols below, using T^-, S^-, D^- or the brackets: when we have a small transient modulation, we can put the new functions in the new key inside brackets (depending on the length of the modulation).

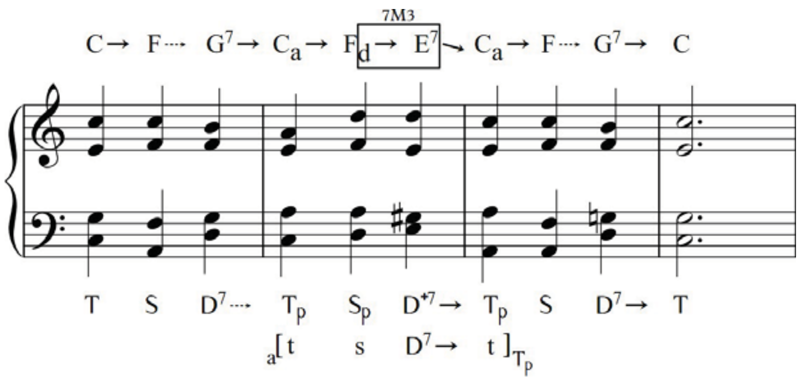
The inversions of the chords are not specified in the functional tonal symbols, except in a few cases (see 5.6).

Fig. 44



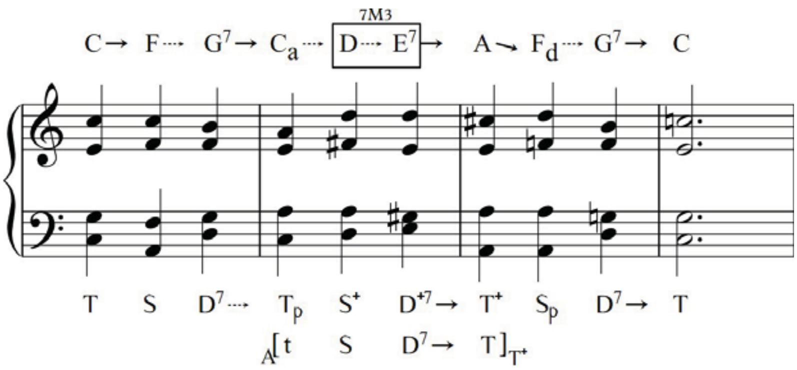
We could do something similar to the major and its relative minor, as, for example, the fragment of Figure 45 (C major / A minor / C major)

Fig. 45



or with its "relative major" (C major / A major / C major) (figure 46).

Fig. 46



In the latter two examples, the new symbol D^+ appears as the dominant function of the relative minor, and in figure 46 the symbols S^+, D^+, T^+ as subdominant, dominant and tonic functions of A major (within a context of C major). When we see the symbol D^+, it means that we have a tonal vector in the direction of the relative minor of the major mode (or the relative in major if T^+ appears).

I prefer to leave aside if we are in a major or minor mode and I like to talk about the upper relative or the lower relative of a *tone* (they belong to the same tonal axis). The symbols T^-, S^- y D^- give us information on a trend to the upper relative and the symbols S^+, D^+ y T^+ information of a tendency to the lower relative of the tonality in which we are established, characterized by the symbols S, D, T (or s, D, t).

Symbols are different if viewed from the perspective of one tone (key) or another. For example, in figure 47 we have the functional symbology of the triads in A minor from the perspective of A (in a tonal context of A minor) or from the perspective of C (in a tonal context of C major). See also 1.8 with figures 9 and 10.

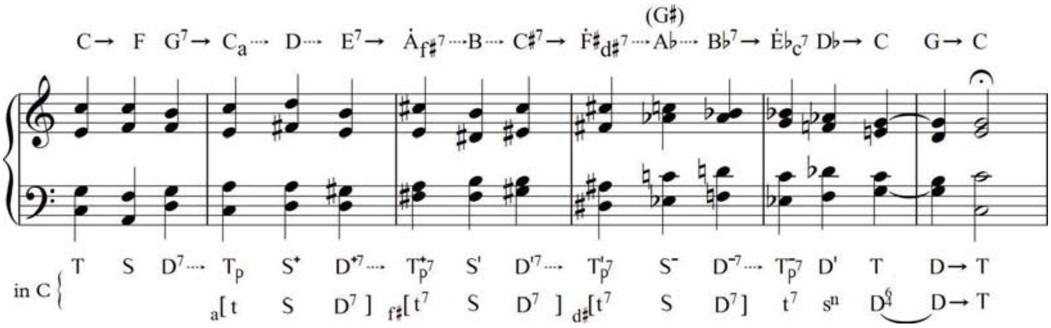
As we have said, the symbols T_p^-, S_p^- y D_p^- will be simplified by **t**, **s** y **d** (T_p^- = t, S_p^- = s, D_p^- = d) most of the time. When a transient modulation occurs towards C from A (C is the tonic of the upper relative [T^-] of A) these symbols can be used since A ceases to be momentarily the tonic.

Fig. 47

Note that the chord's fundamentals of the functions T^-, T y T^+ form minor third intervals (also the fundamentals of S^-, S, S^+ and those of D^-, D, D^+). That is, **they are on the same tonal axis**, as seen in 4.5.

Only three tonally functional fundamentals would be missing to complete the circle of fifths with all 12 possible fundamentals. These three fundamental would form a tonal vector towards the tonality which, in conventional theory, is considered the most distant one: that which is at a distance of tritone (a fact debatable in our theory and that of Lendvai). We will put the symbols S', D' and T' to the functions that represent these fundamentals. The function D', in fact, is not a strange function with respect to T, since it is its «Phrygian dominant» and is used continuously in tonal music. For example, in C, the function D' would be a chord with fundamental D♭, which resolves, with Phrygian relaxion, in C. The D' function can share the subdominant function with respect to the original tone if the subdominant is in the bass (would form the well-known Neapolitan sixth chord) and is followed by the classical tonal dominant. Into the Andalusian mode used in *flamenco* music, this chord (transposed to F) is also called dominant, in its classical version resolving in E (major) (C_a-G-F-E). This function (D') is also usually used as the «(Phrygian) secondary dominant» of the dominant, often in the form of a augmented sixth chord (see 1.5).

Fig. 48

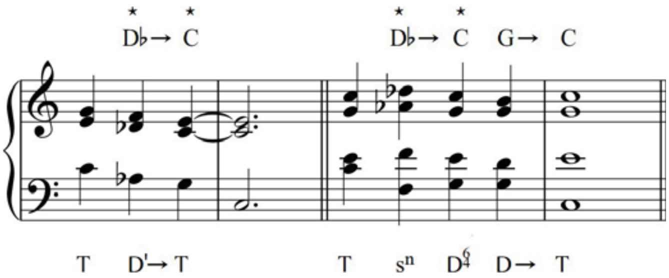


In figure 48 we have an example, within a tonal context of C, where we see transient modulations following the tonal axes (C, a, f♯, d♯), and the respective dominants are D, D^+, D' y D^-. The symbols appearing in the second lower line are alternative notations (to which we will give preference in the analyzes to provide greater simplicity and clarity).

s^n (in bar 5) is the Neapolitan sixth chord (acting as subdominant, such as the postriemannian school symbolizes it) and D^6_4 the cadential $^6_4$ chord (as appoggiatura of the real dominant). They are the only two chords that —depending on its inversion and the function of the subsequent chords— can have a double tonal functionality. And this because of the strength of the note that is in the bass which, in these cases, acquires a greater functional protagonism. In figure 49 we have examples of D♭

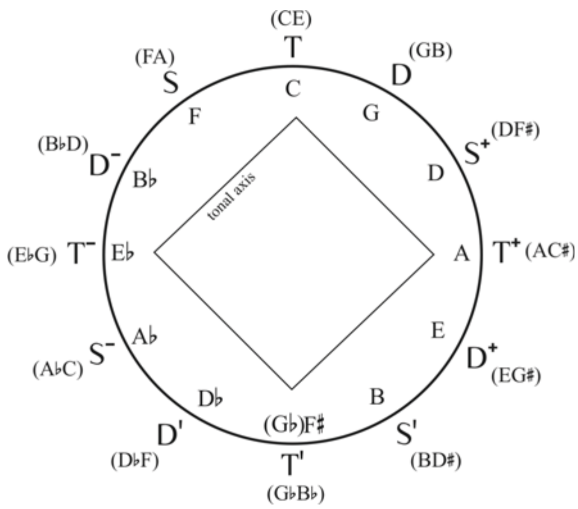
major chord acting as Phrygian dominant (D’) or as subdominant (s^n) and the second inversion of C major chord acting as tonic (T) or as dominant (cadential $^6_4$) (D^6_4).

Fig. 49



Taking a tonic as reference, for example C, any fundamental (or fundamentals) of a chord, however strange it is to the tonality of C, is functionally classified using these symbols, which are summarized in the circle of fifths of figure 50. The symbols are representative of the tonal tensions of M3 in a tonal context of C.

Fig. 50



It should be made clear that the only symbols that determine the classical concepts of tonic, subdominant and dominant functionality, as they are known since Rameau and Riemann, are the symbols T, S and D (and their variants in the secondary chords T_p, S_p, D_p, Đ, but without the $^-$, $^+$, or ’ additions).

Most harmony books speak of tonic, subdominant and dominant chords, but, apart from saying that they correspond to the chords of degrees I, IV and V (with leading-tone) of the scale and to give rules of progressions between them, their meaning with respect to musical perception is somewhat ambiguous.

There are coincidences in saying that the tonic is the function of rest, of end, of resolution, of «returning home». The dominant is defined as the function of tension, which is resolved satisfactorily if it is followed by the tonic function. The function of subdominant is darker, it is spoken of «expansion», passage chord towards the dominant… It is also said that the tonic function is in the center, that of dominant towards one direction (an upper 5th) and the one of subdominant towards the opposite direction (a lower 5th) and that then the subdominant and the dominant define the key and the tonic is in the middle and is its center of gravity. Riemann said: «Thesis is tonic, antithesis is subdominant, and synthesis is dominant».[1]

Objectively, from a harmonic (and from the harmonics) point of view, we can only say that T is a htonal and tonal resolution of D, S a htonal relaxion of T, D a doric relaxion of S in the major mode and a Phrygian relaxion of S in the minor mode and that, certainly, S and D define a key or tonality (a tone of rest, a tonic) since (together) they contain the 7M3 structure (when I speak of T, S, D, I also include —in a key— the chords T_p, S_p, D_p, Đ, D^7 but note, not T^7 and S^7, which, by themselves, already include 7M3 structures strange to the original tone). These homotonic relaxions have been the cause that they are the most used progressions in classic tonal music: $T_{(p)}$-$S_{(p)}$ more than $S_{(p)}$-$T_{(p)}$, $S_{(p)}$-$D_{(p)}$ more than $D_{(p)}$-$S_{(p)}$ and D-T as paradigmatic final resolution.

As we say, T, S and D are the basic tonal functions and the symbols T^-, S^-, D^-, S^+, D^+, T^+, T', S', D' basically indicate tonal vectors towards the tonic relatives. However, if we extend the concept of dominant as any chord that discharges its tension in the next chord or we extend the concept of subdominant as a chord that links well with the dominant and produces local (homotonic) relaxion when it sounds after the tonic, then some of these symbols can also be considered somehow tonic, subdominant or dominant with respect to the original tone. Specially, as we shall see, T', S' and D' since the D'-T sequence gives us a Phrygian resolution (D', Phrygian dominant), just like the sequence T-S', which is also a Phrygian homotonic resolution. In fact D'^7 has two of the three notes of the 7M3 structure included within the dominant seventh chord D^7. They have the same tritone (F-B) and, therefore, share their tension and resolve it similarly towards T.

In a much less pronounced way we also find homotonic resolutions in the sequences D^--T (especially D_p^{-7}-T, which also contains two notes of 7M3: G and F) and T-S^+, which form two weak Doric homotonic resolutions. There are other combinations with Phrygian resolutions like D^+-T^-, D^--T^+, D-T', T^+-S^-, T'-S, T^--S^+ and we obviously have the htonal relaxions D^+-T^+, D'-T', D^--T^-, which, as we have already seen, indicate a small transient modulation to the relative tones.

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1 Riemann "Musikalsche Logik" (extracted from Harrison [1994], p. 267).

Due to this large coincidence of behaviors (functionalities) similar to the classic functions of T, S and D we consider that can be convenient to use these new symbols eventually and occasionally in strange (and isolated) chords within a tonality. But when there is a clear transient modulation, we will prefer to use the symbols T, S, D of the new key (between brackets). We will also prefer to put the secondary dominants (htonal or phrygian) in parentheses before the tonicized chord. For example, (D)→D (dominant of the dominant) is the same as S^+→D; and (D')→D (Phrygian dominant of the dominant) is the same as S^-→D; they are dominant chords with respect to the dominant, but with respect to the tonic are subdominants that link with the dominant. In Chapter 7 we can see examples.

5.4 Tonality and tonal axes

In figure 50 we have the tonal axes (variations of T, S and D) in a similar drawing to Ernö Lendvai's theory of tonal axes. As we have seen in 4.5, this theory roughly says that one could substitute chords of the same axis (tonic, subdominant or dominant axes) with one another and this is what, according Lendvai, Béla Bartók did in many of his works. For example, replace S with S^+ or S^-, or D with D' or D^+, etc. I do NOT say that this can be done, except for a few exceptions, if we want to keep the attraction of a SINGLE tonic. Indiscriminately substituting families of T, S or D of the different axes with each other soon loses the attraction of a single tonic and we enter in a multitonal field, which can be perfectly a system of composition (as can be the twelve-tone of Schoenberg), but moves away from the classic tonality concept with a main tonic, which is what we are studying in this chapter.

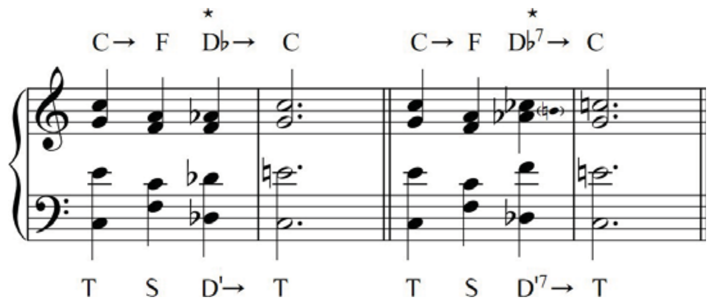
We have presented all these symbols now because, sporadically, it is possible to put strange chords to a tonality without being considered modulation and this symbology helps us to locate the function of any chord in reference to the main tonality.

Classifying functionally (within a key) a foreign chord has been a problem in all schools when you want to take a scale (major or minor) as reference. In the Riemannian school many strange chords can not be functionally symbolized. Schoenberg himself, in his book Structural functions of harmony (1948), simply cross out the degree of the root when it is a chord that does not fit within the scale.

That said, however, as we have advanced before, **some chords can be substituted, without losing the main key tonal vector, when in the passage from subdominant to dominant or dominant to tonic or tonic to subdominant we use homotonic (htonal or Phrygian) relaxions.**

For example, in figure 51 we have substitutions of D for D'. In the case of D'^7 we find the familiar tritone substitution used in jazz (two notes of the chord are changed but the tritone [F-B] is conserved and therefore also its tension). It converts an authentic cadence (tonal dominant D) into a Phrygian one (Phrygian dominant D'). I have put the tonic chord in major, but it could be in minor.

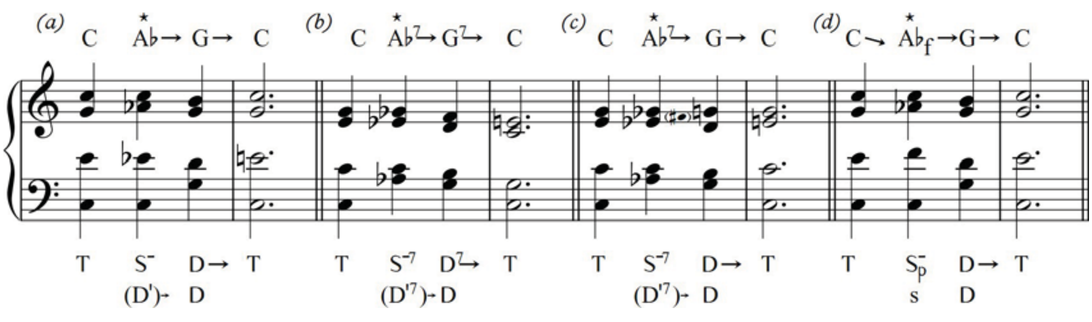
Fig. 51



In figure 52 we have substitutions of S for S^-or S^-_p. The passage from subdominant to dominant then becomes a Phrygian resolution or, if it is to say otherwise, the subdominant makes the role of phrygian dominant of the dominant. If we want to see this way, we may put D' in parentheses. When S^- is a seventh chord, if G♭ (melodically) functions as F♯ (resolves in G) and in the bass there is the fundamental, we are in the case of the scholastic use of the Italian and German augmented sixths (case [c], in the case of the German one the «Mozart fifths» appear). As before, the tonic chord may be major or minor. In case of major mode, these substitutions imply giving it a color of minor.

If the tonic is well established, S can also be replaced by S^+ (then S^+ makes the role of dominant of the dominant, i.e., the htonal resolution to the dominant), but it would take more than four chords if we do not want the tonal vector towards the dominant exceeds that of the tonic.

Fig. 52



Of course we can also make these substitutions in a consecutive way, as in figure 53, or even putting two fundamentals of the same tonal axis within a same chord

(Figure 54); in this case, if the fundamentals are separated by a tritone, we have (two) seventh dominant chords with a lowered fifth (in the same chord: symmetrical chord). Case (b) is the known succession of the French augmented sixth.

As before, the tonic chords of figures 52, 53 and 54 could also be minor chords.

Fig. 53

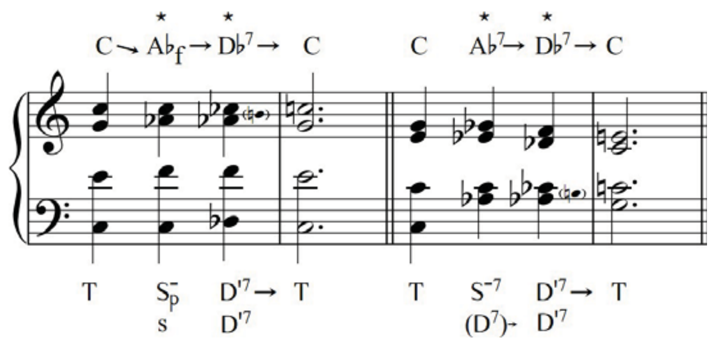
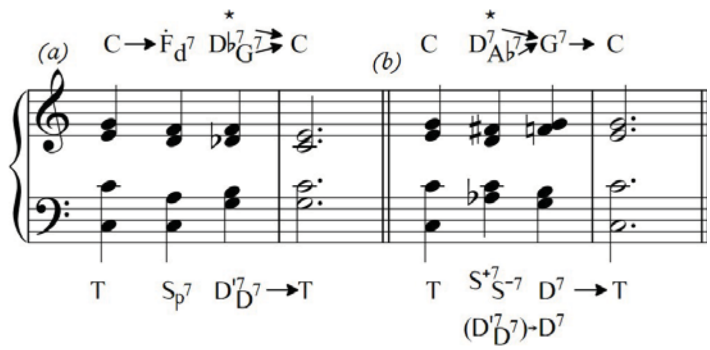


Fig. 54



5.5 Cadences

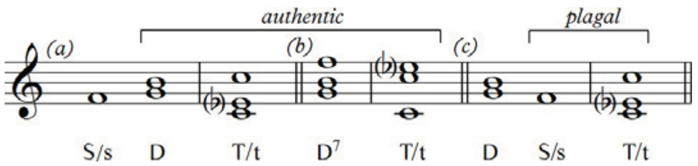
We could divide the cadences into conclusive ones —which serve to finalize a work or a musical fragment (such as the authentic cadence or the plagal cadence)— and «suspended» cadences which do not fully discharge the tension but allow a certain rest or local relaxation (such as the half cadence or the deceptive cadence).

In the concluding ones, and in view of the tonal music literature from the end of the 19th century until today, we would extend the cadential progressions simply by saying that we found a cadence when there is a rest in the tonic, that is to say, mainly a resolution in a chord of the family of the major or minor chords (may include major sevenths and other intervals, see definition in 3.1.1 and 3.1.2) normally with the tonic as fundamental when it (the tonic) has been well established. See for example the previous figures 49, 51, 53 or 54 although there are many more possibilities.

If the tonic has been well established, any chord (degree) can go before the final tonic chord, without losing conclusive meaning. Hovewer, if in the previous chords we hear the 7M3 structure (the one that defines the tonic), this conclusive cadential sense will be more intense and definitive.

The tonal cadential skeleton par excellence would be that shown in figure 55, being much more used progressions (a) and (b) than (c).

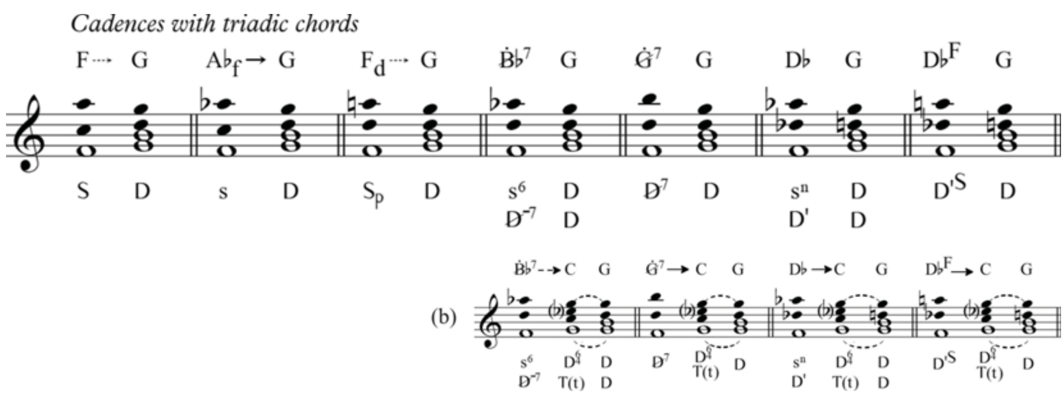
Fig. 55



In the skeleton of figure 55 other notes could be added and if these notes do not form new 7M3 structures they do not lose the conclusive cadential sense. But even if new 7M3 structures appear, the resulting new progression can give us a satisfactory cadence, as shown in the examples in figure 5 of chapter 1 (1.4). In fact we could include again all part 1.4 of Chapter 1 here, so I suggest to the reader his rereading.

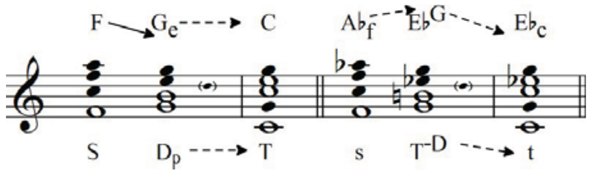
By making a more systematic and traditional arrangement of the different possibilities of filling the conclusive cadential skeleton we would obtain the examples of figure 56 (we only give two chords previous to the not shown tonic chord). In these examples the cadential 6/4 may be interleaved, especially when the sub-dominant has also dominant character (the last four) (Figure 56b).

Fig. 56



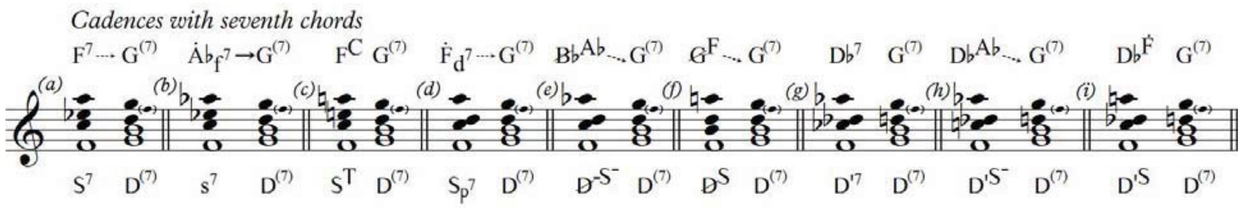
In the dominant part one could also add E or E♭ forming the chords EGB or E♭GB; then the dominant chord is somewhat «tonicized» and at the same time E or E♭ can be heard as appoggiaturas of D (figure 57), although D does not have to sound in the cadence. Riemann considered the E minor chord to have a dominant function (*Dominant-Parallele*), but many harmony treatises consider it with function of tonic. Indeed, the E minor chord (EGB) has two notes of the tonic triad chord (CEG) and two notes of the dominant triad chord (GBD), but I believe that the dominant part prevails over the tonic part due to the tension of the M3 (GB) (dominant + leading-tone) and if we separate the chord into fundamentals according to the harmonic structure (see 1.7, 2.7 and 3.1), the chord has the functional fundamental G, which is the dominant. But one has to take into account the tonic component of this chord, especially if we place E in the bass. As is often the case in analysis, there is no need to opt for one thing or another, but at certain times the two functions can be shared.

Fig. 57



In these examples (figures 56-59) we are placing the subdominant and the dominant in the bass, but it would also work by putting the chords in any inversion, although the resolution effect, with tension release, of the cadence, can diminish according to the note that is on the bass. Recall also that the figures are cadential skeletons, they are not examples of voices conduction.

Fig. 58



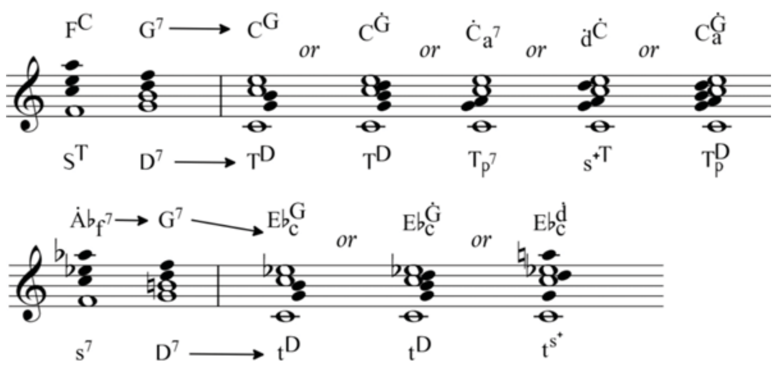
The subdominant may also be included in a seventh chord and also, obviously, the dominant and the leading-tone (figure 58). In example (a) we should have the tonic previously defined because the chord F^7 (contains the 7M3 structure of another tone) defines another tonic, although after listening to GB (with the previous F) we have C tonicized. Something similar happens in (g), but here, even if the first chord defines the tonic G♭, this chord is at the same time Phrygian dominant (D'^7) of the final tonic C.

In all the examples of figures 56 and 58, resolution in the tonic chord (major or minor) is not shown. This resting tonic chord of the cadence can also have the major seventh (B) and even the ninth (D), which gives a slight dominant feeling to the tonic chord and a half cadence air to the cadence, but it is fully conclusive, widely used in jazz. The ear accepts this dissonant chord as final because it has the main harmonics of the main harmonic of C, which is G (figure 59). You can also use the sixth (A) as this note does not create a new functional fundamental. If the sixth goes along with the ninth (D), it (the 6th) loses meaning as root of a possible chord based on A minor since the auditory system accepts A as harmonic (fifth) of D. Located within the funcional harmonic limits one could even add F♯ to complete the D chord, forming a global chord C^{GD} by fifths (of fundamentals). Even all these added notes (BDF♯A) may be used together in the final chord.

With the minor chord you can also use the major seventh, but we have an even more dissonant ending (since B does not have now the support of E). We could also put the sixth (A) and the ninth (D) together (because they support each other forming a fifth), but putting them appart would be more conflicting due to the tension with the E♭, although neither can be ruled out as final chord.

In all these cases, we have *sonance* tension in the final chord but at the same time a tonal resolution (figure 59).

Fig. 59



The two classic non-conclusive cadences are half cadence and deceptive (or interrupted) cadence.

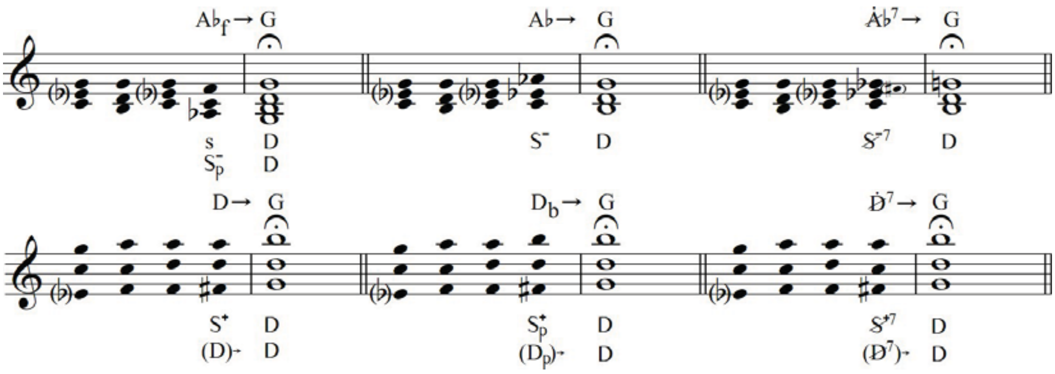
The half cadence, as its own name indicates is an «incomplete» cadence. Creates a suspension in the dominant chord and therefore does not resolve the tonal tension. We could say that the ear is so accustomed to authentic cadences that when the dominant chord sounds, it already has the tonic chord in mind before it actually sounds, creating a sort of suspensive expectation if this familiar resolution does not occur.

However we can found chords, previous to the dominant chord, that help half cadence to acquire a more «cadential» character. They are the chords resoving homotonically towards the dominant using a Phrygian or htonal relaxion; and, in fact, are the most used progressions in classic half cadences.

In C, for the Phrygian resolution to the dominant, we would need A♭ (S^-) as functional fundamental and for the htonal resolution to the dominant we would need D (S^+) as functional fundamental. The htonal resolution is more conclusive because in fact it consists of a small local modulation (see figure 60: the (♭) in parentheses means that E can be natural or flat).

Other half cadences come from the subdominant F (S) as functional fundamental, producing also a weak Doric homotonic resolution (see 4.2).

Fig. 60

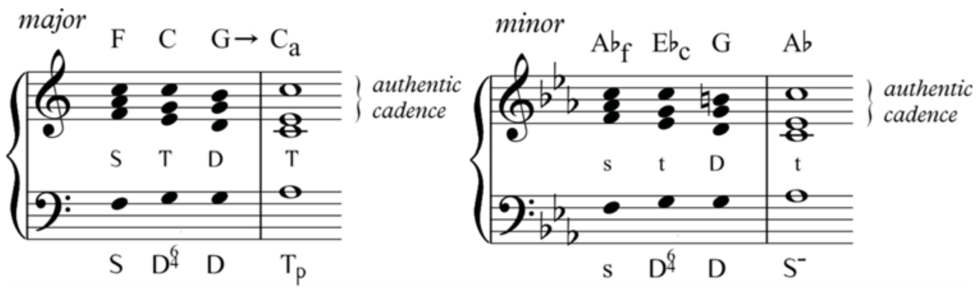


The deceptive cadence, like the half cadence, does not completely resolve tonal tension. It also leads us to a sort of temporary suspension which calls for clarification.

The 6th degree chord to which the dominant chord is resolved is not a tonic chord but is familiar to it because it has two of the three notes of the tonic triad. In the major mode the functional fundamental of the 6th degree chord is precisely the tonic and taking the functional fundamentals of the 5th and 6th degrees we obtain a homotonic htonal distension.

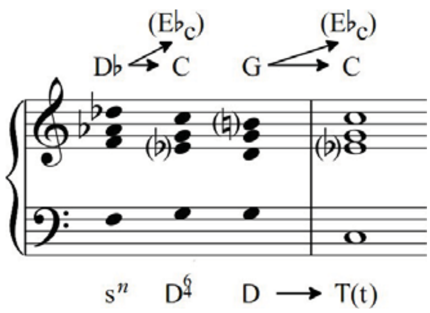
In figure 61 we have a deceptive cadence in major and minor modes. In both cases, if we only considered the notes of the upper staff, that is, all but the bass, we would have an authentic cadence. The bass, in the final chord, converts the authentic cadence into an unexpected progression, but at the same time the ear hears the familiar authentic cadence of the other notes, producing this characteristic and sweet pseudo-cadence.

Fig. 61



In section 5.3 we already mentioned the Neapolitan sixth chord when using the cadential skeleton. But the most usual progression of the Neapolitan cadence is placing —before the dominant chord— the $^6_4$ cadential chord. We already know that the $^6_4$ cadential chord is the second inversion of the tonic chord. Considering it as a tonic chord this sequence gives us a more fluid progression since between the Neapolitan sixth chord and the tonic chord we have a Phrygian homotonic resolution (figure 62, see also the example of figure 6d).

Fig. 62

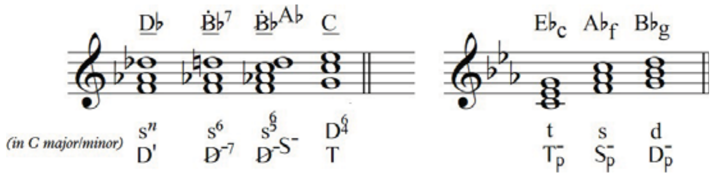


5.6 Functional symbology in inversions

In general, with the functional symbols we have introduced (the functional symbology), we do not distinguish between a chord and its inversions since in most cases the inversion does not significantly vary the tonal function of the chord. But, as we have seen, there are exceptions. Depending on the note in the bass and depending on the tonal progression that follows, a chord can acquire different tonal functions or, in fact, share the two at the same time. Therefore, for some inversions and progressions the functional symbology used may be different for the same chord. They are relatively few cases. In figure 63 we have the only chords with possible dual tonal functionality and therefore with two different functional symbols depending on the tonal situation in which they are immersed.

We also add in this figure, as a summary of the different symbologies for the same chord, the two possible functional symbology of some of the minor mode chords that we have already seen in 5.3 (t, s and d).

Fig. 63



5.7 Modulation

We have already explained in 5.1 our vision of the tonal field as a living set of dynamic tonal vectors appearing during a musical work and that signalize new tonics or tonicization processes.

When this process of tonicization takes on a certain consistency, it is referred to as "transient" modulation and when the tonic is firmly established, then we speak of true modulation. With the algebraic simile we use we would draw the case of one of these new vectors clearly highlighting the others. Also, for example, the use of secondary dominants would create very small vectors as a tonicization of the fundamental to which it resolves.

Let us remember that these vectors signal tones, that is to say, concrete notes and secondarily modes, therefore the changes in a musical fragment based on a major scale that lowers its third degree (and optionally the sixth degree) and becomes minor (or vice versa) for us it is not exactly a modulation. A modulation implies a change of tone.

Although the term modulation is often used when passing from major to minor (or vice versa) of the same tone, theorists like Piston and Schoenberg also seem to suggest something similar: «At first glance it might appear that the tonalities of C major and C minor are quite distant, since there is a difference of three flats in the key signature. But because of the similarity of their harmonic functions, these two tonalities can be considered the same in many aspects, since they have the same tonal degrees and actually differ only in the third degree» (Walter Piston, Harmony, p. 223 in the Spanish version) and «In classical music the minor and major modes are frequently exchanged without so much formality, in the sense that a passage in major mode can be followed by a passage in minor mode without a harmonic help and vice versa» (Arnold Schoenberg, Structural functions of harmony, p. 90 in the Italian version).

Picardy third is well-known to us: when a work in minor mode ends with the major triad chord. Although it has its origin in modal times, its use has remained relatively alive in the baroque and in classicism and also later composers like Chopin have made use of it (see for example 7-22).

We could also quote as examples Beethoven's Symphony No. 5 in C minor that ends in C major, not only because of the last chord, but because it uses this mode in most of the last moviment; or, for exemple, the Rondo of his Sonata No. 21, Waldstein, where, from bar 153, major and minor chords alternate each other; or also the start of *Also sprach Zarathustra* by Richard Strauss. We would find numerous musical examples of the direct alteration of the third of the tonic converting passages from major to minor and vice versa, thus producing a change in color but not a change in tone, that is to say, without changing the note that does tonic function, the note that our auditory system assigns as the best note of rest or resolution.

For there to be a change of tone (key), transient or not, again the 7M3 structure is fundamental. This structure creates a powerful tonal vector (a well-established tonality has a unique 7M3 structure associated to it). If in a musical passage (except modal ones) the 7M3 structure is unique, we can say that «we are» in the tone (key) associated with this 7M3 structure and the (major/minor) key will depend on whether the third of this tone is, in this passage, mostly major or minor.

Hence a modulation will simply consist of moving from one 7M3 structure to another. Let us keep in mind that a musical fragment can share one or more 7M3 structures at a time, in this case the power of the tonal vectors will depend on the harmonic progressions that are used.

The most effective way of modifying a 7M3 structure (and to avoid duplication of 7M3 structures) is precisely by varying the notes that shape the 7M3 structure. Doing this way will get the tonalities that are considered more «close» to the original or starting tonality.

For example, let's stand in the tonality of C (C major and C minor scales). The 7M3 structure consists of notes G, B and F (GBF).

If we change G for G♯[2] , we obtain, in C major, the new 7M3 structure EG♯D creating a tonal vector towards A (minor mode, since we have C natural).

If we change B for B♭, we obtain, in C major, the new 7M3 structure CEB♭ creating a tonal vector towards F (major) and, in C minor, the new 7M3 structure B♭DA♭ creating a tonal vector towards E♭ (major).

If we change F for F♯[3] we obtain, in major and minor,[4] the new 7M3 structure DF♯C creating a tonal vector towards G (major).

---

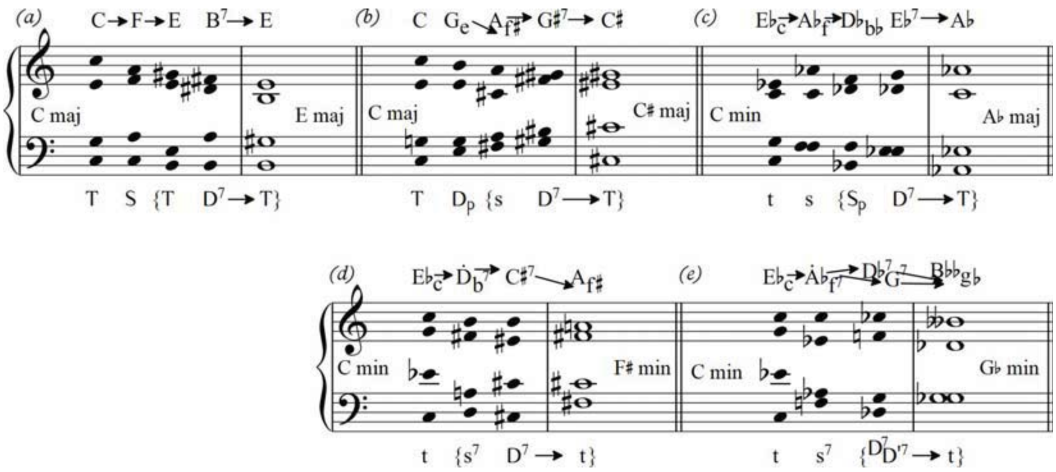
[2] If we change G for G♭, the ear actually hears G♭ as F♯ since F♯ is the P5 (harmonic 3) of the 7th degree B and the M3 (harmonic 5) of the 2nd degree D, so we will consider it as a variation of F, case we will see next.

[3] If we change F for F♭, the ear hears it as E since E is the P5 (3rd harmonic) of the 7th degree B and the M3 (5th harmonic) of the 1st degree C.

That is, by changing the notes of the 7M3 structure of C we obtain the tonal vectors towards A (relative of C major), Mi♭ (relative of C minor), F (lower fifth of C) and G (upper fifth of C), which are the tones (keys) considered the closest to C.

But we can work out modulations to other more «distant» tones by simply showing the 7M3 structure of the new tone we want to go to, and if we want them more consistent, we can apply one or more cadential processes, discharging the tensions of the 7M3 structure in a tonicized chord, as we have seen in 5.5.

Fig 64



One procedure that can help us to modulate in a fluid way would be to apply htonal and Phrygian homotonic relaxions between the chord progressions responsible for the modulation. In figure 64 we have some examples. The arrows indicate htonal or Phrygian homotonic relaxions. They are fast modulations with few chords, illustrative purposes. Surely modulations too fast from a classical and romantic point of view, where key changes occur in a more gradual way and in closer tones. In example (d) we modulate very fast from C minor to F♯ minor using sharps; in example (e) we modulate to the same tone (enharmonic)(G♭ minor), also with few chords, but using flats and a symmetrical chord with dual function of htonal dominant and Phrygian dominant introduced by a double homotonic resolution from the previous chord $\dot{A}\flat_{f7}$. In the two examples we start from the same C minor chord but advance in opposite directions, in (d) we apply a Phrygian resolution and in (e) a htonal one, resolutions which, as we know, are at tritone distance (their resolution fundamentals).

---

4 In C minor we would also get enharmonically the structure A♭CG♭, but as we have said in note 2, the ear hears *G♭* as *F♯*, creating a conflicting 7M3 structure, but we have to keep in mind the fact that this tendency towards D (major) would also exist.

5.8 Recapitulation

During these chapters we have seen that the phenomenon of harmonics shapes the mechanisms of musical apprehensions. In particular, the third harmonic (the fifth), the perfect consonance, is responsible for creating the «laws» that govern our perception of harmonic tensions.

Harmonics create the palette of available sounds and intervals, and the second harmonic (the octave) is the cause that all intervals between notes can be reduced (apart from unison) to six: m2 (M7), M2 (m7), m3 (M6), M3 (m6), P4 (P5) and tritone. Of these intervals there are two very close to the perfect fifth consonance (half-tone difference), are the «quasi-fifth» intervals of M3 and tritone, which discharge their tension if one of their two notes is directed toward the note (or fundamental) that would «resolve the tuning of the fifth». These are the resolutions we have called htonal and Phrygian, very important interrelationship between chords for the establishment of harmonic and tonal tensions and relaxions.

If we take these two intervals together, so that we match the resolution notes (which are separated by a tritone), we obtain the 7M3 structure, formed by an M3 and a tritone (for example CEB♭), structure that reinforces and increases the resolutive tendency towards these two notes (F and B♮ for CEB♭).

This structure (7M3 = M3 + tritone) together with the tendency of tones to resolve a lower fifth (due to the harmonics structure) is what generates tonality (the notes that form the 7M3 structure are known as dominant, leading-tone and subdominant of the tone in which they want to discharge their tension).

These principles (we might also call them musical forces) are therefore very simple and elementary (in fact everything could be reduced simply to the tendency to tune the interval of fifth and to rest in the lower fifth), but, as in nature, combinations of elemental principles (or forces) can give very complex perceptual structures.

In the next two chapters we will give examples of how we can apply these principles in composition or musical analysis.

6. Examples of harmonic progressions with homotonic relaxions

In this chapter we will see examples of (local) homotonic relaxions between fundamentals in weak tonal fields, that is, sequences of chords that create a continuously fluctuating tonality so tonal tensions do not influence much because they are weak. It is in this situation that homotonic relaxions acquire greater meaning, in spite of the chromatism of the chord links.

An arrow will indicate a htonal or Phrygian relaxion between fundamentals.

The examples can be heard in a playlist on youtube (https://www.youtube.com/user/llbape/playlists).

6.1 Relaxions between chords with a single functional fundamental

In this section we will give examples of htonal relaxions using major, minor and dominant chords with a single functional fundamental, that is, only containing an interval of M3 or tritone (or both).

6.1.1 Htonal relaxions following the circle of fifths

In Example 6-1 we can see htonal relaxions between major triads following the circle of fifths.

Ex. 6-1

n Example 6-2 we have htonal relaxions between minor triads following the circle of fifths. The chords are in first inversion but with the functional fundamental in the bass. If other inversions were used, the effect of homotonic relaxion between chords would continue to occur in the same way, perhaps adding possible *sonance* tensions or relaxions if the inversions of the chords were different between them.

Ej. 6-2

Example 6-3 shows htonal relaxions between dominant seventh chords following the circle of fifths.

Ej. 6-3

In Example 6-4 we have htonal relaxions between minor seventh chords (or, in other words, major triads with the sixth) following the circle of fifths. They are also in first inversion with the functional fundamental in the bass.

Ej. 6-4

In these four previous examples we have seen sequences of relaxion between chords with the same structure and therefore do not intervene other tensions like those of *sonance* (the tonal tensions act very slightly, since their vectors are changing continuously). But any combination of intervalic structures in these chords, using any inversion, would be possible without losing the relaxed sense produced by the homotonic progression. As, for example, the progressions in Example 6-5.

Ej. 6-5

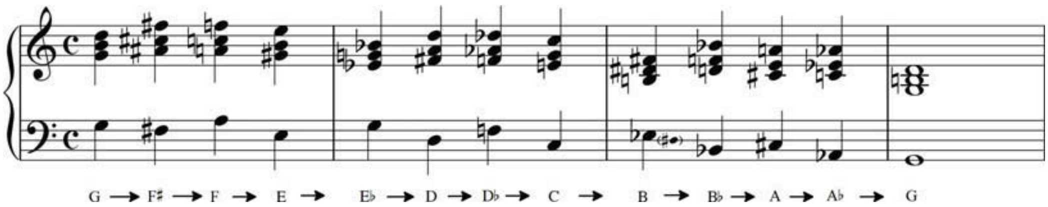
The consecutive htonal successions are very abundant in the musical literature. In the following chapter we will see many fragments, like Examples 7-1, 7-2, 7-3 and 7-5 of Beethoven and Brahms.

6.1.2 Phrygian relaxions following the 12 tones of the chromatic scale

In this section we will make examples of Phrygian relaxions using major, minor and dominant chords with a single functional fundamental.

In Example 6-6 we can see a progression of major triads (some in first inversion to avoid parallel fifths or octaves, although in Phrygian successions the parallel fifths do not sound bad, because they are chromatic) whose fundamentals are traversing the 12 tones Phrygianly.

Ej. 6-6



In Example 6-7 we hace done Phrygian successions using minor triads (in first inversion).

Ej. 6-7

Between the chords of Example 6-7 a small local tonal tension is continuously produced —or a (tonal) relaxion if the sequence were in the opposite direction— since any two contiguous chords contain notes corresponding to a 7M3 structure (we have a M2 interval between fundamentals, see 1.7.1). For example, between the first two chords we find AC♯G and therefore the two chords create a tonal vector toward D, which is the dominant tone of the G minor chord (the first one), so there would be a micro-tonal rest in this chord if we were playing these first two chords repeatedly.

In Example 6-8 we can see Phrygian successions between dominant chords, the second one with a virtual fundamental.

Ej. 6-8

In Example 6-9 I use minor seventh chords, alternating root positions with first inversions. We also have slight tonal tensions (due to the tonal relaxions that are observed in the opposite direction).

Ej. 6-9

As before, any combination in the chords structure (and in any inversion) is possible without varying the sensation of Phrygian relaxion, as long as the step between fundamental is Phrygian; as in Example 6-10. To the relaxions appearing in it a *sonance* relaxion should be added at the beginning of each bar, since we have a more consonant chord.

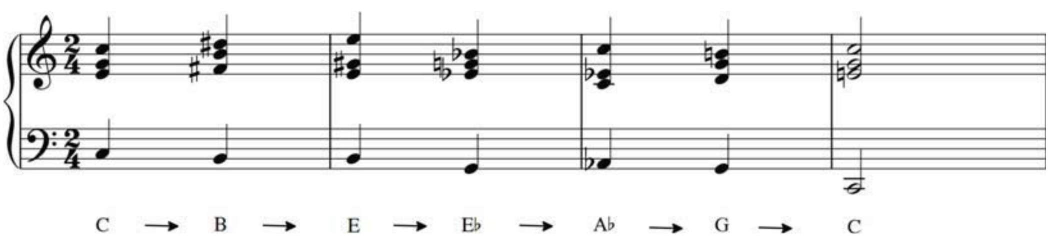
Ej. 6-10

These Phrygian relaxions are relatively abundant in music literature. Similar sequences can be seen in Brahms symphonies (Examples 7-6 and 7-18) or in Chopin (Example 7-19).

6.1.3 Combinations of htonal and Phrygian relaxions

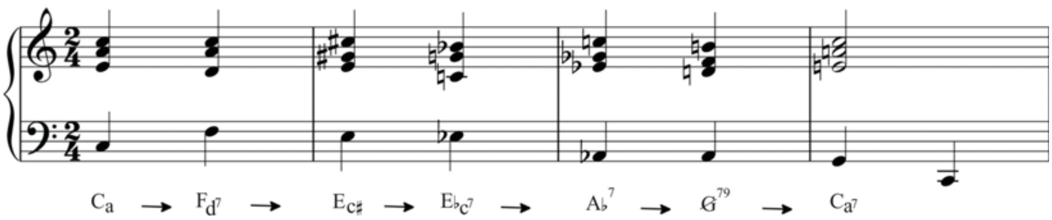
Fluid chromatic chord sequences can also be obtained using combinations of htonal and Phrygian relaxions, as in Example 6-11, in this case using only major triads (with inversions in the 2nd bar).

Ej. 6-11



In Example 6-12 we have a progression similar to the previous one with a mixture of more coloured chords, but the functional fundamentals continue forming htonal and Phrygian relaxions.

Ej. 6-12



Or as in Example 6-13 that harmonizes an ascending whole tone scale in the soprano voice using htonal and Phrygian sequences between functional fundamentals.

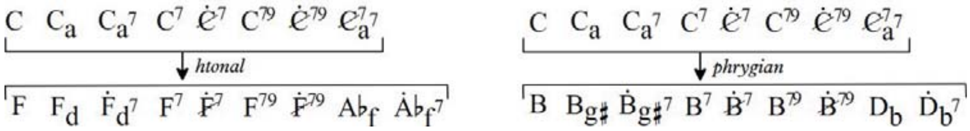
Ej. 6-13

6.1.4 Using the full palette of chords

In the previous sections we have seen only a small part of the possible sequences of chords with htonal and Phrygian homotonic relaxions. In Table 3 we have a summary of htonal and Phrygian relaxions between the main chords with a single functional fundamental (still more chords, more exotic and dissonant, could be shown).

This table expands Tables 1 and 2, which incorporated only major and minor triads, although they also contained secondary relaxions.

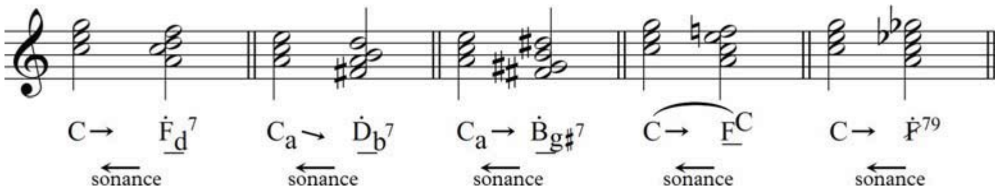
Table 3



Any chord with C as functional fundamental can link to any chord with fundamentals F or B (functional fundamental or not, uppercase or lowercase) producing homotonic relaxion although in some cases tonal and *sonance* tensions can be produced (all applied to all 12 notes and their transpositions —C, of course, is an example).

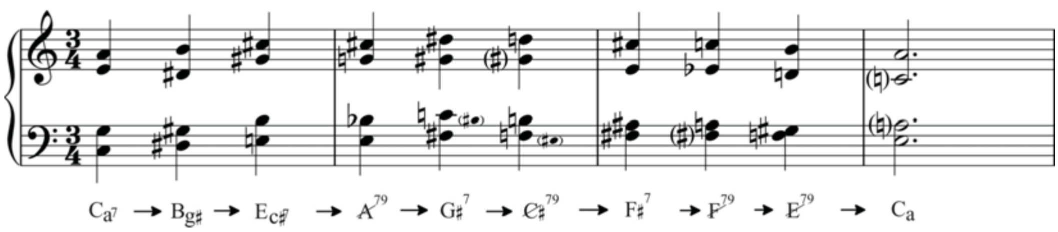
Sonance (consonance/dissonance) of chords is another kind of harmonic tension. A chord can be resolved homotonically towards another more dissonant chord, as in Figure 65. We have homotonic resolution against *sonance* tension (the harmonic resolution chord is more dissonant). Many times resolution by consonance can "win" in relaxion to an homotonic resolution, but using homotonic resolution against *sonance* tension can be a way of sweetening dissonant chords.

Fig. 65



In example 6-14 we can see a new sample of the use of htonal and Phrygian homotonic relaxions, using this time somewhat more dissonant chords: a chromatic sequence of chords within a weak tonal field (indeterminate before the final cadence).

Ej. 6-14



In Figure 66 we have represented some of the possible combinations of Table 3 where htonal and Phrygian homotonic relaxions take place. In Figures 65 and 66 we have indicated the inversions by simply placing a line below the symbol (see 3.4) (in the other examples we do not distinguish between inversions in the symbology).

In the next chapter we can see many examples of works by composers using htonal and Phrygian combinations, which are the most significant local homotonic relaxions.

Fig. 66

6.2 Relaxions with one or more functional fundamentals

6.2.1 Two functional fundamentals separated by a tritone (symmetrical dominant chords)

The chord formed by two M3 at a distance of tritone (see 3.1.5) is very interesting because it resolves homotonically towards other fundamentals in a htonal and phrygian way at the same time. If one fundamental resolves htonaly, the other resolves Phrygianly and vice versa. Also two tonal vectors are created (separated by a tritone) since the major third of one fundamental is the minor seventh of the other one and vice versa, that is to say, we have in this chord two 7M3 structures at tritone distance, therefore is a useful chord for modulating to "distant" keys, as we have seen in Figure 64(e). From a classical point of view would be a chord consisting of two seventh dominant chords with a lowered fifth. The French augmented sixth chord has the structure of this chord, enharmonizing one of the minor sevenths as an augmented sixth.

A 7 over the fundamentals can be placed or not, since the seventh is implicit. To make these examples cleaner we have preferred not to draw it now although, if this chord is found isolated, it is preferable to do so.

In Example 6-15 we have a progression with intermediate resolutions towards major triads. A major chord also resolves homotonically to a symmetrical chord in a htonal and Phrygian manner at the same time. In the latter case we have harmonic relaxion, but tension of *sonance*.

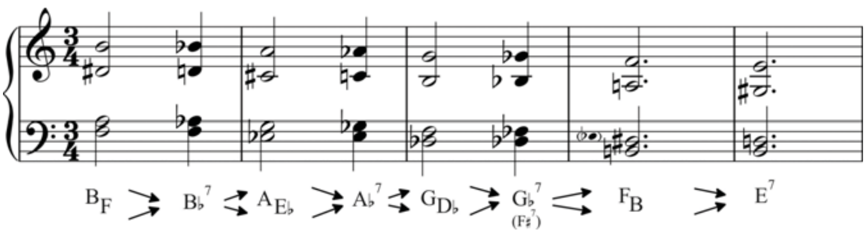
Ej. 6-15

As this chord is symmetrical the progression can be made by changing the direction of resolution of the fundamentals. What was htonal is now Phrygian and vice versa:

Ej. 6-16

In order for the *sonance* of chords to be more similar, seventh chords could be used instead of triads, with the same homotonic result:

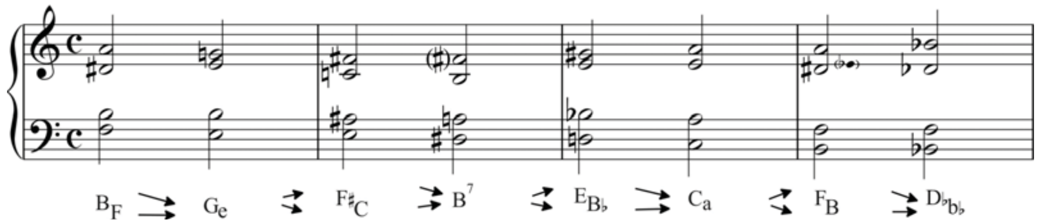
Ej. 6-17



Resolutions of this type can be seen in the next chapter, among others, Examples 7-16 (bar 137), 7-21 (bar 13) and 7-45 (bar 8) (Brahms, Chopin and Mompou).

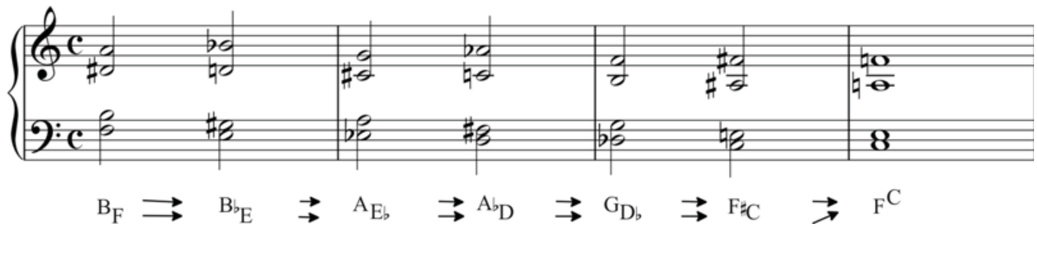
We could also resolve in minor chords (example 6-18), that is, towards non-functional fundamentals (in lower case). Let us remember from other chapters that a fundamental in lower case can serve as a «resolutive» fundamental but not as fundamental «to be resolved» since it does not represent tensions of M3 or tritone; the one that has these tensions is the (upper case) functional fundamental of the minor chord (the fundamental that have its upper M3).

Ej. 6-18



But this chord can also resolve in another chord with the same structure. In this case four homotonic relaxions are produced at the same time (!). It is the link that occurs at the famous beginning of *Tristán e Isolda* by Wagner between the chords formed in the last eighth note of the third measure and the first eighth note of the fourth (see Example 7-26). Either of the two fundamentals is (htonal or Phrygian) relaxion of any of the two previous fundamentals. In Example 6-19 we have only put two arrows, but in reality they should be four.

Ej. 6-19



We could also resolve this family of chords using virtual fundamentals in the resolution chords and in general utilizing all the types of chords that appear in Table 3. In Figure 67 we have some examples. In this figure we only put an arrow although the resolution is double, and we also mark the inversions.

Fig. 67

6.2.2 Two functional fundamentals separated by a M2 (dominant chords family)

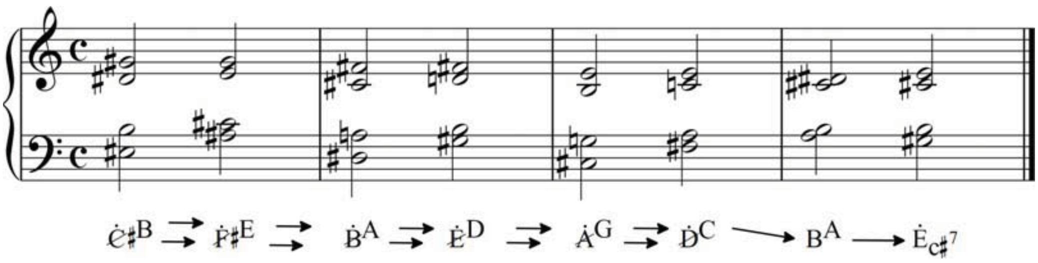
The unitonal dominant chords contain a single 7M3 structure and their natural resolution is towards chords that have the fundamental pointed out by the tonal vector they create (the two fundamentals are the local dominant and subdominant of the fundamental to be resolved). They can also relax Phrygianly. Thus, a chord type $C^{B\flat}$ can resolve or discharge the tension in chords having the fundamental F/f or B/b. Secondarily, taking into account B♭, they would also provide a certain relaxation the fundamentals E♭ and A, as htonal and Phrygian resolutions of the second fundamental B♭.

As happens with all chords that have two functional fundamentals, they can resolve in another chord of the same family producing a double htonal or Phrygian relaxion, as in Example 6-20, where we have double htonal relaxions and we take advantage of a double Phrygian relaxion in the third bar to return to the chord of the beginning and resolve at the end in the "tonic" functional fundamental E♭ while A♭ resolves secondarily in C (lower case) according to a Locrian relaxion.

Ej. 6-20

The Tristan chord also belongs to this family of unitonal dominant chords (F♮ enharmonized to E♯), with the characteristic that its main fundamental is virtual.[1] In Example 6-21, in the first bars, we have a sequence of chords with the same «Tristan structure” (with double htonal homotonic relaxion). In the link between mesasures 3 and 4 it is the secondary fundamental that resolves Phrygianly (C→B) in another dominant chord, which resolves to “its tonic” as in the previous example (but with another inversion of the chord taking advantage of the bass melodic Phrygian relaxion: A-G♯). The process of tonicization of E is also facilitated by the melody of the soprano voice.

Ej. 6-21



These dominant chords with two fundamentals also link well with the dissonant augmented chords (type C^E) since it is the resulting chord if we make a htonal leap from one fundamental and a Phrygian leap from the other fundamental, as can be seen in Example 6-22. Here we have also placed the fifth of the main fundamental (dotted on the symbol) and in the case of the last chord we have placed both fifths of the two fundamentals. This last chord is quite dissonant, but at the same time, harmonically, it is a very resolution chord (in spite of its tonal instability) because of the htonal and Phrygian homotonic relaxions between fundamentals that are formed with the previous chord. We do not need to repeat here the difference between *sonance* resolution and harmonic or tonal resolution.

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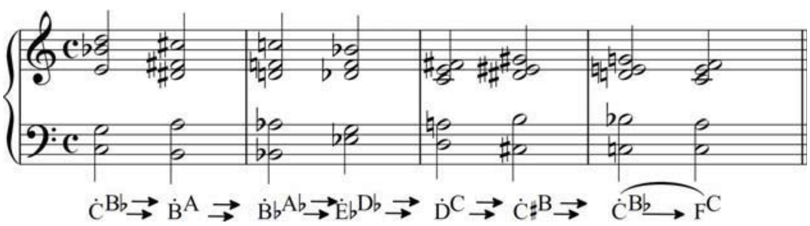
[1] In my book *La convergència harmònica*, 1994, I devote a whole section to show examples of the use of the Tristan chord in 14 musical works, with their different resolutive combinations.

Ej. 6-22

In Example 6-23 we have a sample of relaxed Phrygian progressions between chords of the same family, setting in the middle (measures 2-3) a htonal relaxion to break up the monotony of the descending Phrygian movements; ending the fragment with a chord of two fundamentals of the family of major chords, which we will see in the next section.

Ej. 6-23



The relaxed combinations of this chord with other chords following the htonal and Phrygian resolutions between fundamentals can also be very varied. In Figure 68 we can see some examples.

Fig. 68

In the following chapter we can see resolutions of this type, among others, in the Symphony No. 4 of Brahms (Example 7-17, bars 229, 233, 241, 243-246), Wagner's

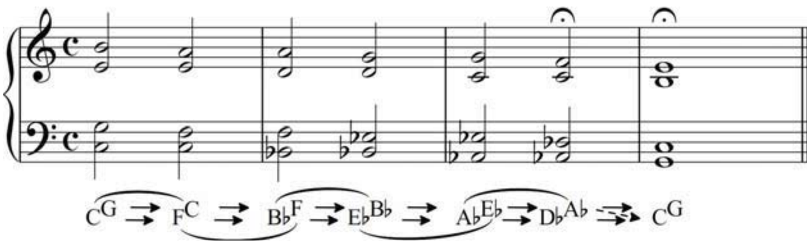
Wesendonk lieder (Examples 7-30 [bar 78] and 7-31 [bars 5-7]) or Rachmaninov's *Prelude* (Example 7-37).

6.2.3 Two functional fundamentals separated by a P5 (major chords family)

These chords have no tonal tension because they do not contain the 7M3 structure and in addition the second fundamental is the main harmonic (the fifth) of the principal functional fundamental. They have the relative tension of the two M3 they contain, especially that of the main fundamental and the *sonance* tension of the m2/M7 interval being formed.

As usual, tensions can be solved htonally or Phrygianly (secondary resolutions apart). Using htonal relaxions between chords of this family we obtain a curious and sweet progression of fourths and fifths, as shown in example 6-24, broken at the end by a Phrygian relaxion ($D\flat^{A\flat} \rightarrow C^{G}$) that gives it a cadential air, surely reinforced by the Locrian secondary relaxion that is formed (A♭→C).

Ej. 6-24



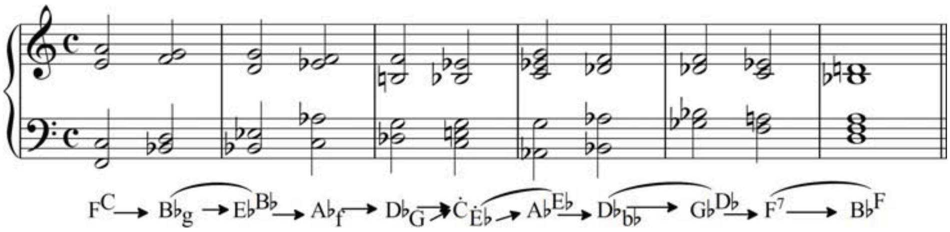
A similar but more chromatic progression can be seen in Example 6-25, this time resolving in a symmetrical dominant chord by means of the htonal and Phrygian relaxion (at the same time) of the second fundamental without changing the main fundamental ($C^{G} \rightarrow F\sharp_{C}$) which resolves Phrygianly (C→B) (and the other htonally [F♯→B]) by returning to a chord of the same family we are studying ($F\sharp_{C} \rightarrow B^{F\sharp}$).

We take advantage of the tonal tension of the symmetrical dominant chord adding at the end one of its two possible resolution tones in the soprano voice (obtaining the local leading tone - tonic sequence), which gives this semi-conclusive air despite the dissonance of m2 sounding in the upper voices.

Ej. 6-25

In Example 6-26 we resolve this chord several times into minor chords by means of a htonal relaxion between functional fundamentals. In measure 3 we use a symmetrical intermediate chord ($D\flat_G$) and a major-minor type chord ($C_{E\flat}$) (blue chord). In order to make a somewhat more conclusive ending —in this short fragment without a clear key— we end with a dominant chord resolving again in a chord of the family we are studying in this section, that is, major with two fundamentals separated a fifth, in this case —to give color— in second inversion.

Ej. 6-26



Harmonically relaxed htonal and phrygian combinations of this family of chords, like the previous ones, are very varied, as shown by some examples in figure 69.

Fig. 69

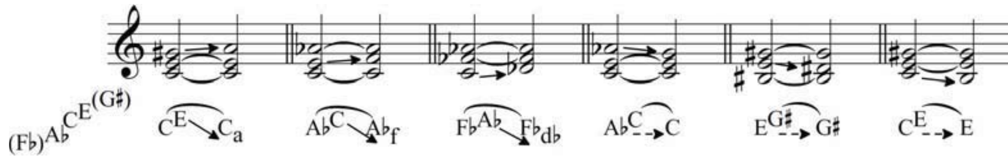
6.2.4 Two functional fundamentals separated by a M3 (augmented chords)

These chords are symbolized with two capital letters, but all chords with two fundamental at a distance of M3 actually have another hidden functional fundamental, also separated by a M3. Therefore, in fact, these chords would be symbolized with three capital letters representing the three fundamentals. For practical reasons, we place the two that best fit the tonal or harmonic context in which they are immersed, but this third fundamental must always be taken into account.

This chord, in its simplest version, that is, with three fundamentals without fifths, is a chord of three notes that divides the octave into three equal parts. Of the three fundamentals there is no principal one, unless one has his fifth. From a scholastic point of view it is a triadic chord with an augmented fifth, but, without a tonal context of reference, any of the three fundamentals could be the augmented fifth/ minor sixth.

We could consider this chord as a chord that shares simultaneously six appoggiaturas (or suspensions if they come from another chord) to six major or minor triads and therefore one of its possible relaxions would be the resolution of the appoggiatura to one of these six chords, as shown in Figure 70. Actually, using these links there are only two different types of relaxions, raising or lowering by semitone some of the three fundamentals, which result in a htonal relaxion or in a Locrian one (resolved melodically Phrygianly).

Fig. 70



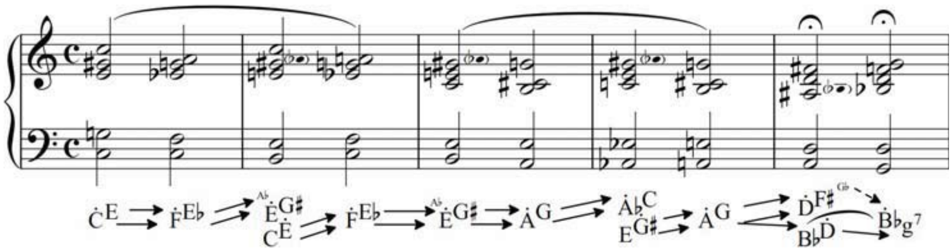
If we resolve htonally the two (three) fundamentals of this chord, we obtain another chord of the same family (as it happens in all other types of chords since the intervalic relation between fundamentals remains the same); if the two resulting fundamentals are then Phrygianly resolved, we get the chord of the beginning, as shown in Example 6-27 (the beginning of each measure have the same chord). In this example we have, therefore, the harmonization of a melody using only two chords, except for the final chord where we have added a lower minor third; this ending chord remains in the same family since no new functional fundamentals are created, but acquiring genes of F minor.

Ej. 6-27

If we resolve htonally one of the fundamentals of the augmented chord, and the other resolves Phrygianly, we obtain a chord of the dominant family. If we do the same with this resulting dominant chord, that is, one fundamental resolves Phrygianly and the other htonally, we again get a chord of the augmented chord family, as we have already seen in Example 6-22. Depending on the fundamental resolving Phrygianly and the one resolving htonally we can also obtain again the original chord, as shown in Example 6-28.

In this example we have added to the augmented chords the fifth of one of its fundamentals, which gives us a more dissonant chord since there is a minor 2nd collision between two notes of the chord. The example is finished, in last bar, with one of the relaxions (appoggiaturas) that appear in Figure 70, in this case resolving in a minor triad with (minor) 7th or, if it is to say in other words, resolving in a major triad (Ḃ♭) with an added lower minor 3rd (of the fundamental) (in this case in the bass).

Ej. 6-28



In Debussy's *Proses lyriques* Example 7-39 we can see, between bars 14 and 17, a Phrygian succession of augmented chords (the two —three— fundamentals resolve Phrygianly).

As with the other chords, the possible relaxed combinations of this chord, by means of htonal and Phrygian resolutions between fundamentals, are very varied. In Figure 71 we have some examples.

Fig. 71

Also we can see, among others, resolutions of a chord belonging to this family in Examples 7-46 (bar 19) and 7-47 (final) (Mompou and Schoenberg).

6.2.5 Other chords

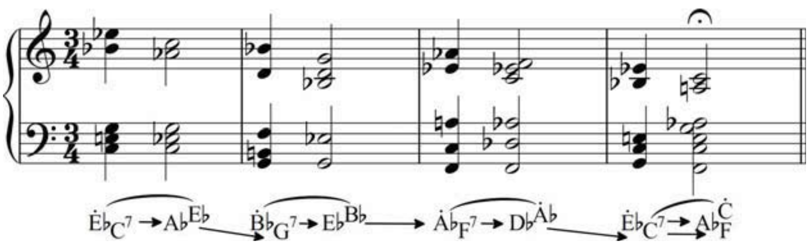
Of the chord families with functional fundamentals, only the family with two functional fundamentals separated by a m3 (major-minor chords) and the family with two functional fundamentals separated by a semitone (cluster chords) would remain to be studied. They are dissonant chords, but we can "soften" their *sonance* if we make the auditory system clear what is the "identity" of each of their notes.

The major-minor chords have two functional fundamentals at a distance of minor third or, what is the same, are formed (at least) by a major triad and a minor triad, both starting from the same scholastic root (e.g. CE♭E♮G, which is symbolized with $E♭_C$), which gives two functional fundamental separated by a m3 (CE/E♭G). One is the fundamental of the two major-minor triads (C) and the other is the functional fundamental of the minor triad ($E♭_c$), that is, a minor third above the first fundamental C.

The most common use of this chord is incorporating the fifth of the upper fundamental as this gives support to it and sweetens the *sonance*, although the chord has more notes (e.g. CE♭E♮GB♭). However, in this layout of five different notes (three complete triads with fifth) a 7M3 structure is created (the main fundamental obtains the minor seventh), which causes the chord to acquire "genes" of dominant chord. In this way this chord is usually used in jazz, particularly in blues; although always with the main fundamental in the bass.

In Example 6-29 we have this chord at the beginning of each bar and we do htonally resolve its second fundamental towards the lower fundamental of a chord of the major chords family (as we saw in 6.2.3). The last chord is the sum of a major-minor chord and an augmented chord, with three functional fundamentals (four if we consider the third hidden fundamental of the augmented chords). In isolation it is a very dissonant chord (we have the cluster G-A♭-A♮), but here, as it proceeds from a double htonal homotonic relaxion, the ear somehow "justifies" its dissonance because it perceives internal harmonic resolutions from the previous chord. This last chord of 6 different notes has genes from three families of chords: that of the major (F^C, which gives stability and sustains the chord), that of the augmented ($A♭^C$) and that of the major-minor ($A♭_F$).

Ej. 6-29



The members of the cluster chords family are very dissonant because they have two functional fundamentals separated by a m2 and, having only one semitone of difference between them, the ear does not perceive any harmonic relationship, except a very remote relation between a fundamental and its fifteenth harmonic or, rather, the fifth harmonic of his third harmonic (the major third of his fifth). In addition we have, at least, two semitone shocks: between its fundamentals and its thirds (as long as its fundamentals are not virtual). If any of its fundamentals are virtual, the dissonance is significantly reduced (see, for example, the resolution of the cluster chord in bar 355 of the first movement of Brahms Symphony No. 2, Example 7-12).

Of the two htonal homotonic resolutions of the two fundamentals, the relaxion of the second fundamental (the fundamental corresponding to the lower semitone) produces more relaxion than the other, because the fundamental to which it resolves is closer harmonically. For example, if we take C^B, these two fundamental resolves htonally in F and in E. C accepts the resolution B→E because E is one of its main harmonics, whereas B «does not like» the resolution C→F because F is at tritone distance with respect to B and forms with it a «false-fifth». Therefore, the resolution C^B→E is more relaxed than C^B→F (apart from having the Locrian relaxion C→E, see Brahms' previous example). In the symbology we put the B above because it is a harmonic of C, but functionally it would also be good to put B^C. The symbols of the cluster chords can be put either way, that is, by placing the upper seventh major or the upper second minor above.

In Example 6-30 we have a succession of cluster chords with Phrygian and htonal relaxions. We take advantage of the strongest htonal relaxion of the second fundamental to resolve the penultimate chord to a fundamental based on F♯ (C♯→F♯) resolving the other fundamental Phrygianly (D→C♯), which, in addition, gives us a chord of the family of major chords ($F\sharp^{C\sharp}$), suitable for an ending. But this would give us a too consonant and resolutive ending that would contrast with the dissonances of the previous chords, so we have added to the final chord a third fundamental (another upper fifth) (G♯: —F♯/C♯/G♯—), which gives us a chord with genes from the family of the major chords (if we think from F♯) and of the dominant family (if we think from G♯), in this example, the first configuration predominates, due to the fifth F♯C♯ in the bass.

Ej. 6-30

We have shown only two examples with major-minor chords and cluster chords, but, as in the other sections, the harmoniously relaxed combinations between these chords and other chords of other families remain very large. In addition, due to the dissonance in these two families, we can easily obtain an added relaxion by *sonance* resolution.

We would only have one family of chords to study, that of suspended chords. These chords are defined precisely because they are the only ones that do not have functional fundamentals, therefore they do not have homotonic relaxions between them, since the homotonic relaxions only resolve the tensions of the functional fundamentals (representatives of the "quasi-fifths" of the chord, M3 and tritone intervals). But they can be chords of resolution if they come after a chord belonging to other families.

Suspended chords are basically chords by fourths or fifths (up to three if there are four different notes)[2] also known as quartal and quintal chords. In the case of three consecutive fifths or fourths a very weak tonal vector is formed since in the chord we have included two fundamentals (two fifths) at a distance of M2. For example, let's take the chord FCGD. FC could be considered a subdominant part and GD a dominant part (without the leading tone) towards the key C. This chord could «resolve», then, with some relaxion, towards a tonicized fundamental C. As we have said, it is not properly a htonal homotonic distension, but it could be taken into account, as shown by the sequences of suspended chords of Example 6-31, where the chords «resolve» locally in this weak way that we have commented towards the first fundamental of the next chord. All chords are chords with three consecutive fifths (or fourths) (in different inversions), except the one in the end formed only by two fifths and therefore is a more stable and consonant chord. It would have been possible to put a simpler example —probably with more relaxion between chords— by putting the sequence of fifths in the bass (as in Examples 6-1 to 6-4 of the beginning). We leave it as an exercise for the active reader.

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[2] If we put a fourth fifth, we would get a functional fundamental.

Ej. 6-31

Samples of this small dominant character that quartal/quintal chords have can be seen in Examples 7-33 (bar 29) and 7-38 (bar 18-19) (Bruckner and Debussy).

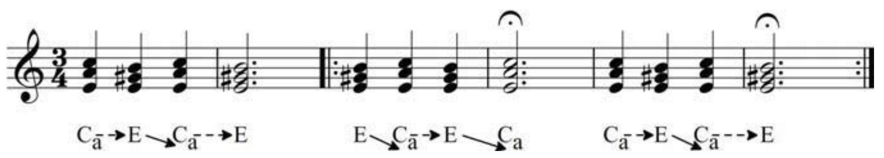
6.3 Locrian homotonic relaxion

We have already seen in Chapter 4 an introduction to secondary homotonic relaxions. In our opinion, the one that implies a greater relaxion is what I denominate Locrian relaxion, that is the fundamental jump of an (upper) M3. The first fundamental must be functional, the fundamental of resolution can be functional (upper case) or not (lower case), but the resolution fundamental should have its fifth since, as we have seen in 4.2, it was an important note when we did the demonstration by means of the resolution of the "quasi-fifths".

It is not a sequence too used in tonal music because it does not fit with the chords that are formed in the diatonic scales except in an important succession: the passage from tonic to dominant in the minor mode, taking into account the functional fundamental of the minor triad. In A minor we would be talking about the succession C_a→E. This sequence has a left-to-right (Locrian C→E) relaxion but a right-to-left (htonal E→a) resolution. The htonal relaxion is more powerful, but if in the tonicized chord we add a little tension of *sonance*, for example, putting it in second inversion, then the resolutive effect of the two relaxions may be similar, as shown in Example 6-32, where the piece rests cadentially in any of the two chords. In the last five bars of *Vexilla regis* of Bruckner (Example 7-35) we can see a variant of this sequence (E without the third), which is also introduced by the Locrian relaxion A♭→C in bars 30- 31.

Secondary relaxions are indicated by a dashed arrow.

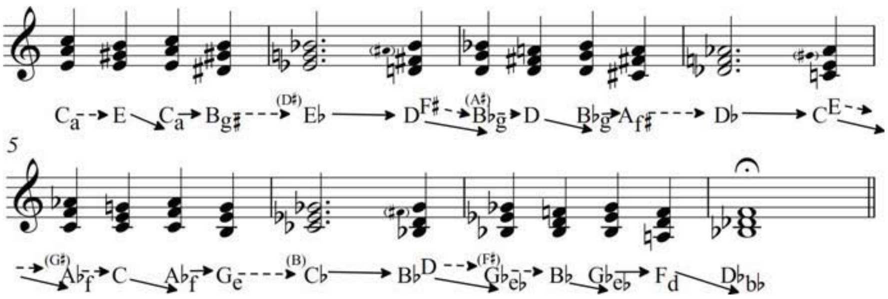
Ej. 6-32



Therefore, in the minor mode, the authentic cadence using cadential $^6_4$, from the point of view of the fundamentals is a Locrian + htonal sequence and in the same minor mode the half cadence from the tonic is a Locrian relaxion.

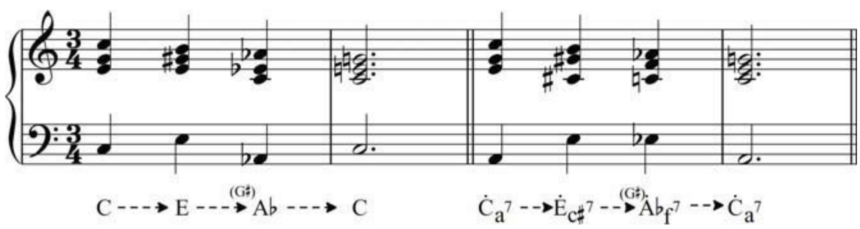
In more chromatic passages we can link Locrian sequences with other chords in many ways. As the fragment of Example 6-33. All voices are moved by semitone and the descending progression may continue indefinitely, but in bar 8, instead of resolving Locrianly, we resolve the chord F_d htonally (F→b♭).

Ej. 6-33



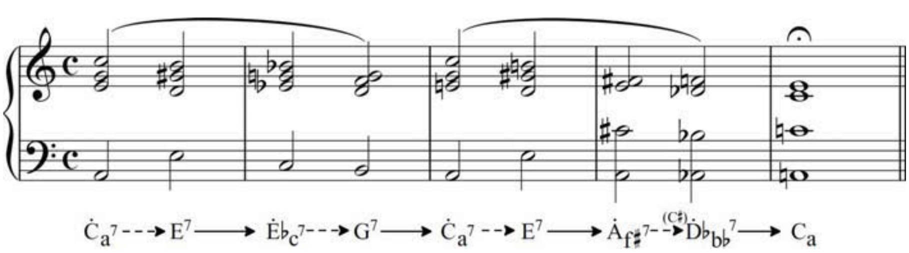
Three successive Locrian homotonic sequences take us to the same starting chord (or to the same functional fundamental), since the interval M3 divides the octave into three equal parts. We have two samples in Example 6-34.

Ej. 6-34



In Example 6-35 we have Locrian relaxion between the two seventh chords of each bar combined with other Phrygian and htonal relaxions.

Ej. 6-35



Like the other homotonic relaxions, the possible Locrian sequences between chords are very varied. In Figure 72 we give some examples between chords with a single functional fundamental.

Fig. 72

Some of these sequences would also produce relaxion in the opposite direction, that is, if the two chords were played in the reverse order, since «false-fifths» or «quasi-fifths» are resolved in both directions.

In chromatic music, Locrian successions occur constantly, but often it can simply be something inevitable, the result of chance. Samples where it seems to take advantage of its local relaxing property (including the half cadence ‘i-V’ in the minor mode) can be found, inter alia, in Examples 7-11 (bars 251-252), 7-12 (bars 355-357), 7-18 (bars 276 and 278), 7-28 (bars 18, 20 and 22), 7-35 (bars 25-27) and 7-48 (bars 41 y 45) (Brahms, Wagner, Bruckner and Schoenberg).

7. Examples of homotonic and tonal analyses

In this chapter we will give examples of works by different composers that will show the utility of separating the chords according to the fundamentals that define them, as we have seen in chapter 3, since they provide us with information about their internal harmonic tensions, mainly the "quasi-fifth" tensions of M3 and tritone intervals. In turn, thanks to the structure of the significant harmonics for the auditory system (which, in relation to the fundamental, also form intervals of M3 and tritone), are fundamental tones representing the chord.

The local homotonic relaxions between chords (see summary in 4.4) are represented by an arrow. When they are secondary relaxions (Locrian or Dorian) or when the «quasi-fifth» tensions resolve in complex chords or in secondary fundamentals, we draw the arrow with a dashed line, although we do not always indicate the secondary homotonic relaxions, especially the Doric one, since they are very numerous.

We have looked for examples with great amount of homotonic relaxions in few measures. Searching in the musical literature is not at all difficult to find them, but normally these relaxed progressions between chords are not so concentrated.

To the right or below the title of the musical work we have placed a diagram of the homotonic relaxions of the analyzed fragment and below the last staff (in some examples on the first staff) we can see the detailed homotonic analysis. Sometimes, we have united sequences of chords with fundamentals in the same tonal axis with a "vertical square bracket line".

In most examples we have also added a line (or two) with the analysis of the tonal functions according to the functional symbology explained in 5.3. Sometimes we place a second line or another symbol below to show an alternative functional symbology. We also draw an arrow when there is tonal relaxion, which may or may not coincide with the homotonic one. We feel tonal relaxion basically when we hear the tonic chord (which may be local) or when music rests on the dominant (which can also be local). If the tonal relaxion is weak (because its tonal field is also weak), we also indicate this with a dashed line. Transient modulations are indicated with brackets and inside them we show the function of the chords from the point of view of the new tone, which is specified in the lower part of the first bracket. At the bottom of the second bracket, which closes the transient modulation, we put the new tone function with reference to the main tone/key. Sometimes we put the functional symbology from the point of view of the two tonalities, in two lines, one with brackets and the other without them.

Something similar happens with secondary dominants. The two main homotonic relaxions, honal and Phrygian, since they discharge their tension in the chord in which they resolve, are considered with dominant function (secondary or not) and when they do not resolve into the tonic, we put their function (D) in parentheses with a small arrow: (D)→ in case of honal resolution, and (D')→ in case of Phrygian resolution. Seen from the tonic, when they resolve into the dominant, they have a subdominant function and alternatively we can also put the symbols S^+ (dominant of the dominant) and S^-(phrygian dominant of the dominant), respectively (without the parenthesis since this is the function from the point of view of the main key).

In most examples, in order not to complicate the symbols, we do not distinguish between inversions of chords, since they do not normally vary their harmonic function (except in specific cases, such as cadential $^6_4$). In fact, we have only indicated the inversions, as explained in 3.4, in Examples 7-5, 7-33, 7-37 and 7-40 (apart from Examples 7-3 and 7-6 where all chords are in root position).

Nonchord tones like appoggiaturas, suspensions, passing tones, etc., have been indicated by changing the note-head with a white diamond figure.

If somewhere appears the equal sign (=) between two fundamentals, it indicates that they are on the same tonal axis (and there is no tension or relaxion between them).

As in the previous chapter, examples can be heard in a playlist on youtube (https:/ /www.youtube.com/user/llbape/playlists).

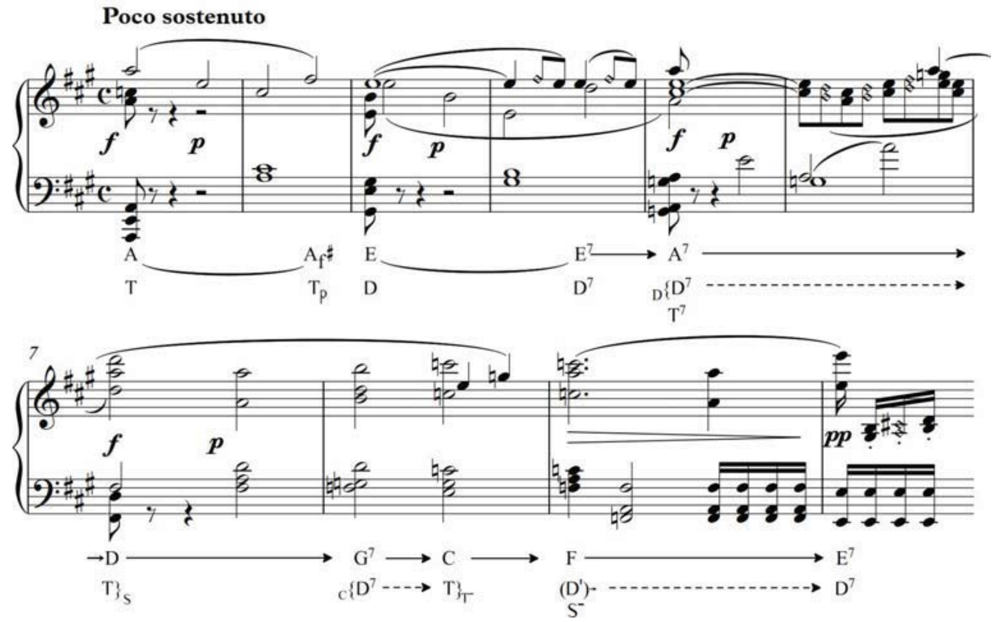
* * *

At the beginning of the Symphony No. 7 of Beethoven (Example 7-1) we find five consecutive htonal relaxions and then a Phrygian one resolving into the dominant.

In bar 5 we have homotonic relaxion but tonal distancing (actually in bar 6, since G♮ of bar 5 only lasts an eighth note) since a tonal vector appears towards D (major). Something similar occurs in bar 8 towards C. In bar 10 we have homotonic and tonal relaxion, because it returns to the original key of A by the Phrygian sequence (D')→D^7 or, which is the same from the point of view of A: S^-→D^7. (D') is the Phrygian dominant of the dominant, but has a subdominant function (S^-, Dorian sixth) from the point of view of the tonic or the main key.

Ej. 7-1

Beethoven. Symphony No.7-I ($E^7 \rightarrow A^7 \rightarrow D \rightarrow G^7 \rightarrow C \rightarrow F \rightarrow E$)



In the following examples we will focus on the symphonies of Brahms where there are plenty of homotonic relaxions.

Ej. 7-2

Brahms. Symphony No. 1-II ($G\sharp^7 \rightarrow C\sharp^7 \rightarrow F\sharp^7 \rightarrow B^7 \rightarrow \dot{E}^7 \rightarrow \dot{A}^7$ $D \rightarrow C\sharp^7 \rightarrow F\sharp^7 \rightarrow B$)

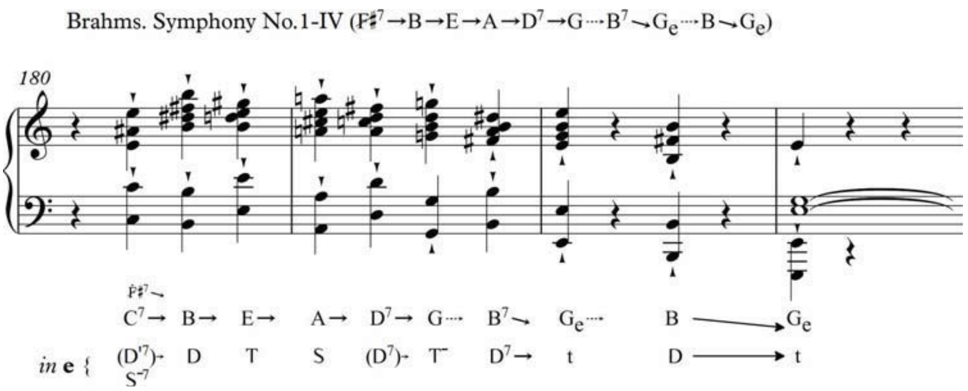
In the second movement of his first symphony (Example 7-2) we find six consecutive htonal relaxions between bars 85-87 —in bars 86-87 between chords with virtual fundamentals (diminished seventh chords).

Then follow Phrygian and htonal relaxions until reaching the tonic of this fragment (which is the dominant of the initial key in E). We have tonal relaxion with functional progressions similar to the previous ones in bars 89 and 90 (apart from bar 86), using the dominant of the dominant: $(D^7) \rightarrow D^7 \rightarrow T$ or, alternatively (taking as a reference the tonic B): $S^{+} \rightarrow D^7 \rightarrow T$.

A fragment with 10 chords communicated by homotonic relaxions, including four consecutive htonals (five if we consider the htonal side that every augmented sixth chord resolution has), is found in the fourth movement of Symphony No. 1 (Example 7- 3a).

In bars 181 and 182 we also find two Locrian sequences (G→B) towards the dominant of this passage, in E. An identical sequence is heard beyond in bar 363 (Example 7-3b), this time in C. In both cases the succession begins with the Phrygian dominant of the dominant in its form of an Italian augmented sixth chord. As we have seen in previous chapters, augmented sixth chords also take advantage of the other tritone's tensing direction (all tritone has two possible htonal and Phrygian resolutions), so the minor seventh is enharmonized as an augmented sixth.

Ej. 7-3 (a)



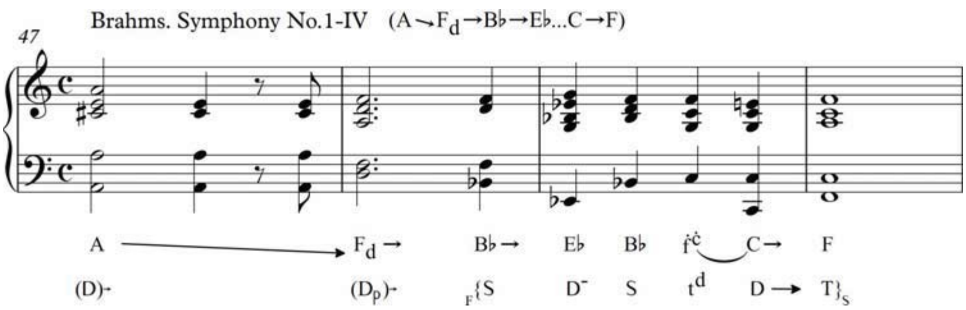
Ej. 7-3 (b)

In this same movement we can also see three consecutive htonal sequences (Example 7-4).

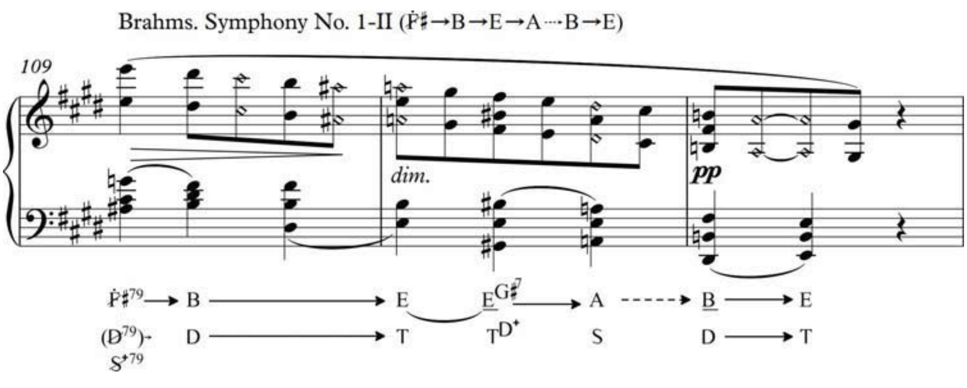
Like all minor triad chords, D minor chord has two fundamentals. The non-functional fundamental D resolves the tension of the "quasi-fifth" C♯-A of the previous chord. The functional fundamental of this D minor chord (F) creates the "quasi-fifth" tension A-F, which is resolved by the fundamental B♭ (fifth B♭-F) of the following chord, which in turn creates the "quasi-fifth" D-B♭, resolved by the fundamental E♭ (E♭-B♭) of the next chord. This is, as we have seen, how homotonic htonal relaxions work.

In bar 50, tonal relaxion is added to the htonal relaxion, in this typical local modulation towards F (S-...-D-T), with the intermediate suspended chord $f^{\dot{c}}$ as appoggiatura of the dominant chord.

Ej. 7-4



Ej. 7-5



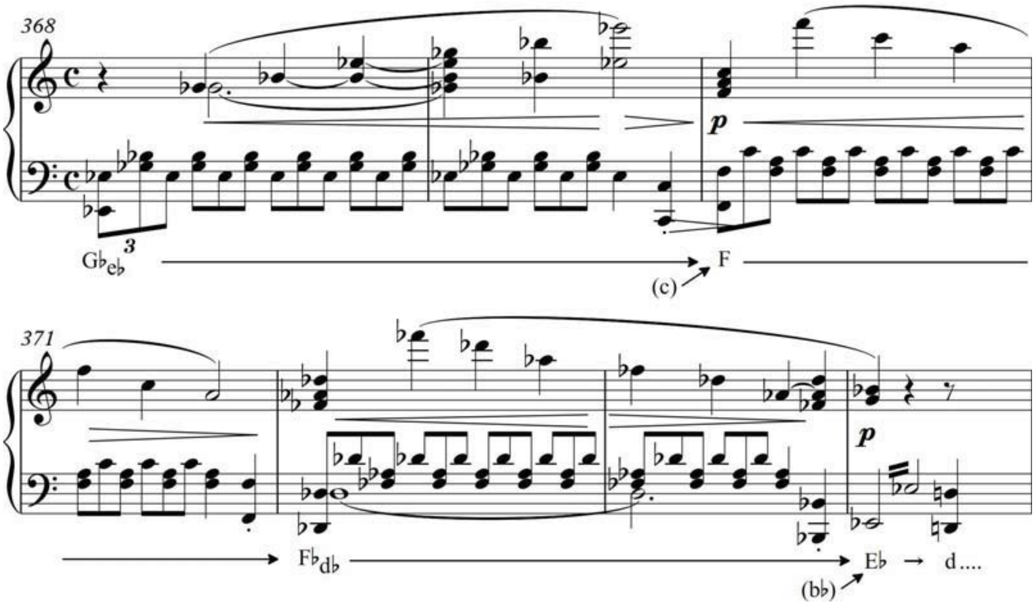
In the second movement of his first symphony we find similar successions. We have in bar 109 (Example 7-5) the «dominant of the dominant» in the form of a diminished seventh chord (S^{+79}) reaching the subdominant by three htonal relaxions (F♯-B-E-A), with some passing tones and appoggiaturas and the passing chord ($E^{G\sharp}$), ending the fragment with the tonal cadence S-D-T. Every S-D sequence implies a

Dorian secondary homotonic relaxion, indicated in this example (although this secondary relaxion is usually not shown in the examples). The sequence of bar 110 (E-G♯) could also be considered a Locrian homotonic succession, but here it is nor marked because G does not have its fifth (see 4.1).

Returning to the fourth movement of this symphony, in bar 368 we find four consecutive Phrygian homotonic relaxions, the first chords being in arpeggiated form occupying two bars each (Example 7-6).

Ej. 7-6

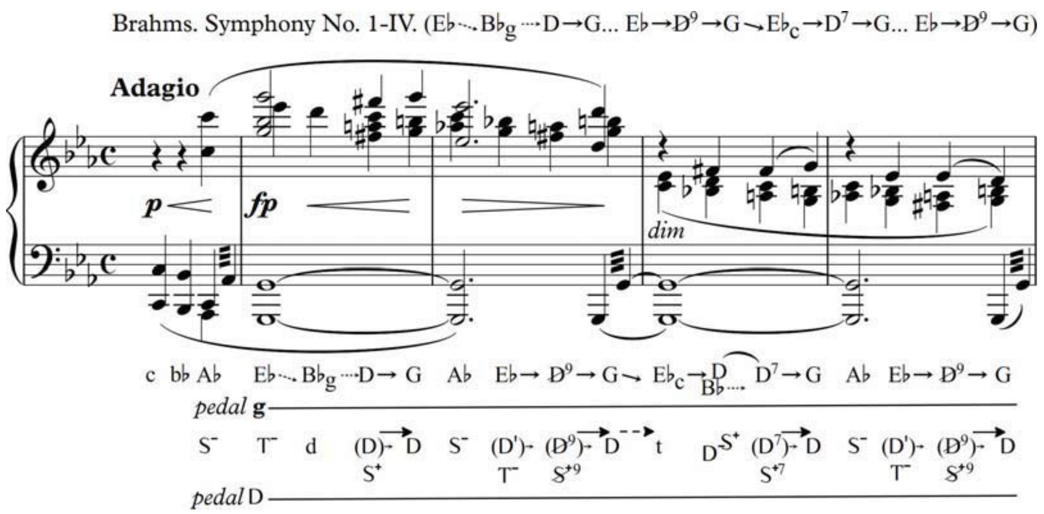
Brahms. Symphony No.1-IV (G♭$_{e\flat}$ →F→F♭$_{d\flat}$ →E♭→d)



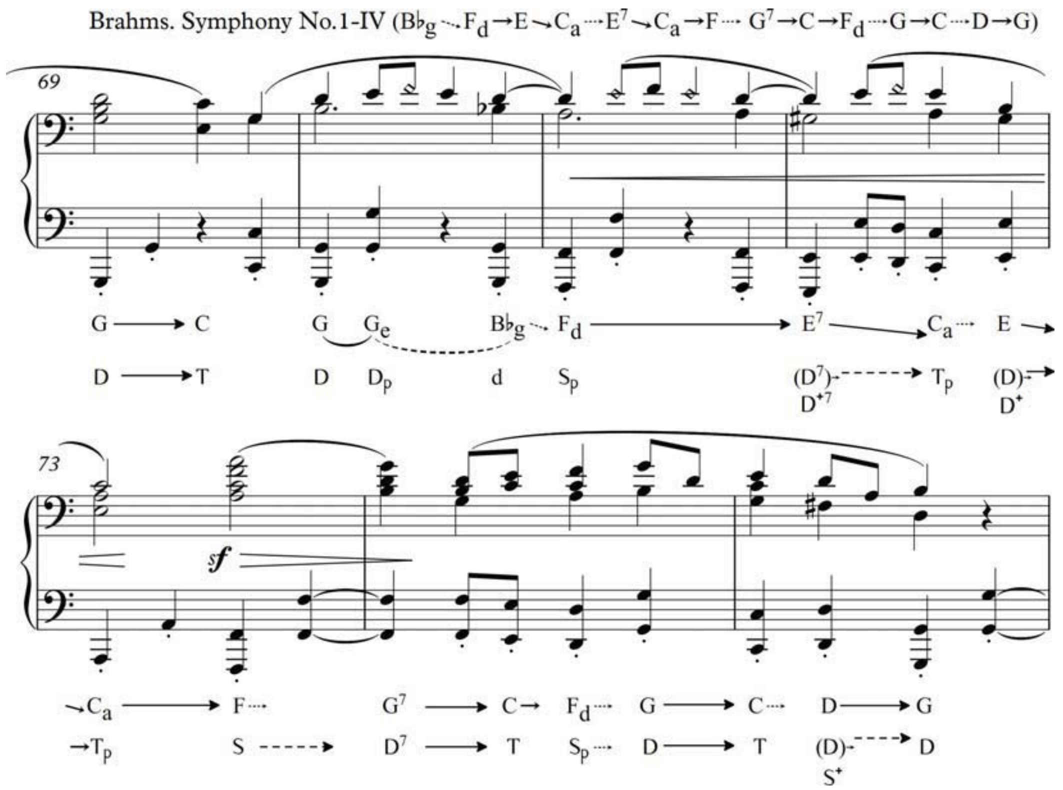
Brahms begins this movement (Example 7-7) with a dominant pedal point and above relaxed successions towards the dominant chord appearing at the end of each bar, forming a kind of deceptive cadence at the beginning of next bar (G-A♭). In all four bars the basic sequence of functional fundamentals is E♭→D→G. That is, using two secondary dominants, the first Phrygian and the second honal. E♭ resolves Phrygianly on the dominant of the dominant, which in turn resolves on the dominant. Seen from C minor, we have a functional sequence $T^{-}\to S^{\pm}\to D$. In the second bar, if we do not consider D as a passing tone, we have a passing chord (B♭$_g$) that forms two Locrian sequences with the anterior and posterior chords. Something similar occurs in bar 4, in this case forming an augmented passing chord with Locrian relaxion towards the next chord.

Apart from the homotonic relaxions we have tonal and *sonance* resolutions at the end of each measure, when we reach the dominant chord (on the dominant pedal).

Ej. 7-7



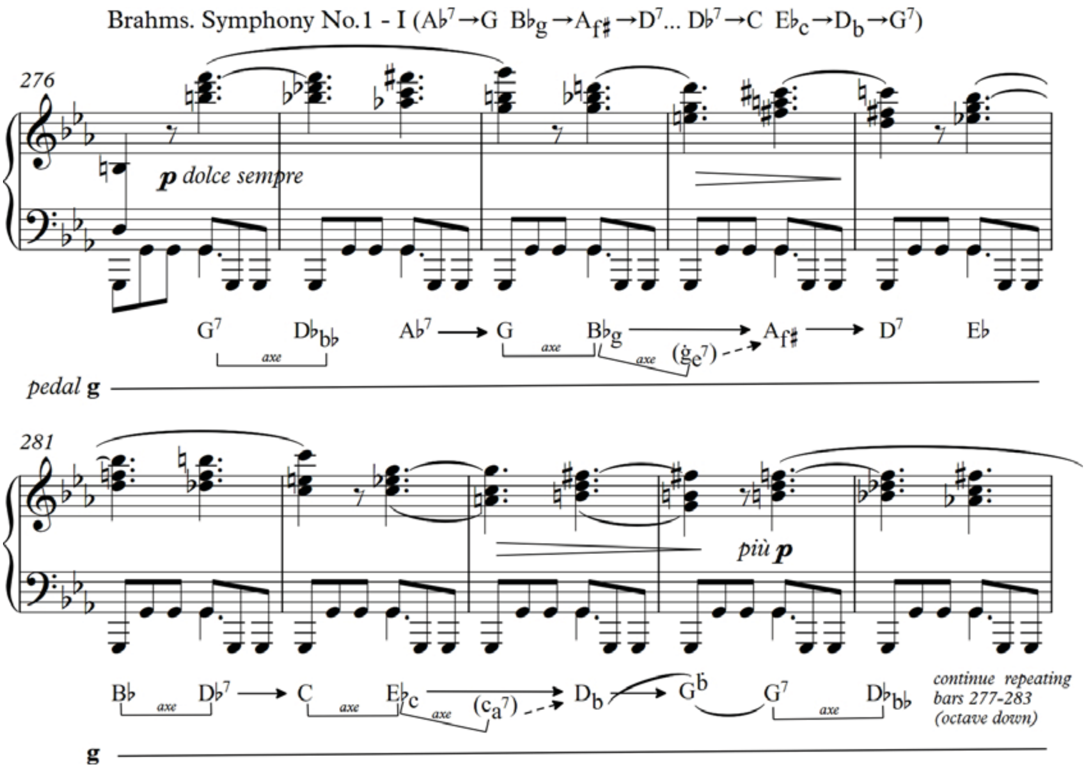
Ej. 7-8



In this same movement, from bar 69 we have a long series of htonal, Phrygian, Locrian and Dorian relaxions (Example 7-8). In bars 71-73, a tonal vector is formed towards A using the typical Phrygian-htonal sequence we have seen before $F \to E^7 \to a$ (bars 71-72), that is to say, Phrygian dominant of the dominant, dominant, and tonic, one of the more repeated successions used in romantic and post-romantic music. Being a short transient local modulation I put the tonal functional symbols from the point of view of the original tone/key (C), in addition, being relative tones between them (C-a) they have coincidences in tonal function (they are in the same tonal axis). But we could also have put this fragment, as in Example 7-1, between brackets: ${}_a[D^7 \to t\ D \to t]_{Tp}$ instead of $D^{+7} \to T_p\ D^{+} \to T_p$.

In the first movement of the same Symphony No. 1 we find again a dominant pedal and above some interesting progressions (Example 7-9). The sequences of functional fundamentals are A♭→G and B♭→A→D being repeated later a lower fifth from bar 281 (D♭→C and E♭→D→G). That is, the former go to the dominant (G) and the dominant of the dominant (D) and the second one to the (Picardy) tonic (C) and the dominant (G). From the second half of bar 284 the fragment is repeated an octave lower (only the upper voices). Brahms also uses sequences of chords with fundamentals within a same tonal axis, which, according to theory, produce neither tension nor homotonic relaxion.

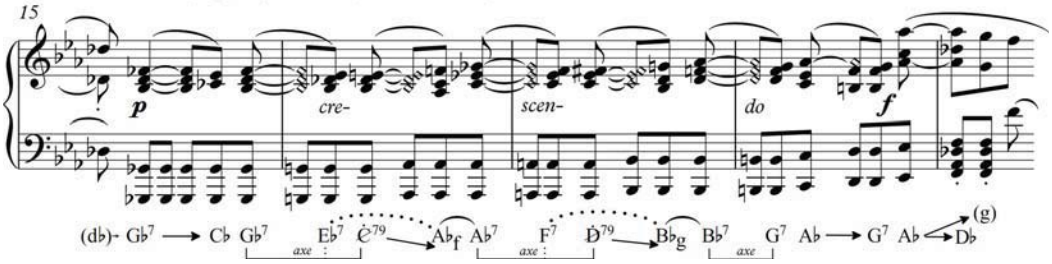
Ej. 7-9



In bar 15 of the same movement (Example 7-10) we can see a chromatic fragment with a complex homotonic relaxions interlacing, sequences within the same tonal axis and melodic and *sonance* resolutions in off-beat chords. In bar 18, the functionalities are clarified with a tonal relaxion to the dominant (G^7 A♭→G^7) followed, in bar 19, by a htonal relaxion towards the Phrygian dominant of the tonic (chord D♭), which lasts two bars, going afterwards to a dominant section in bar 21 (what could be scholastically understood as a dilated succession of Neapolitan sixth) (the example shows only up to measure 19).

Ej. 7-10

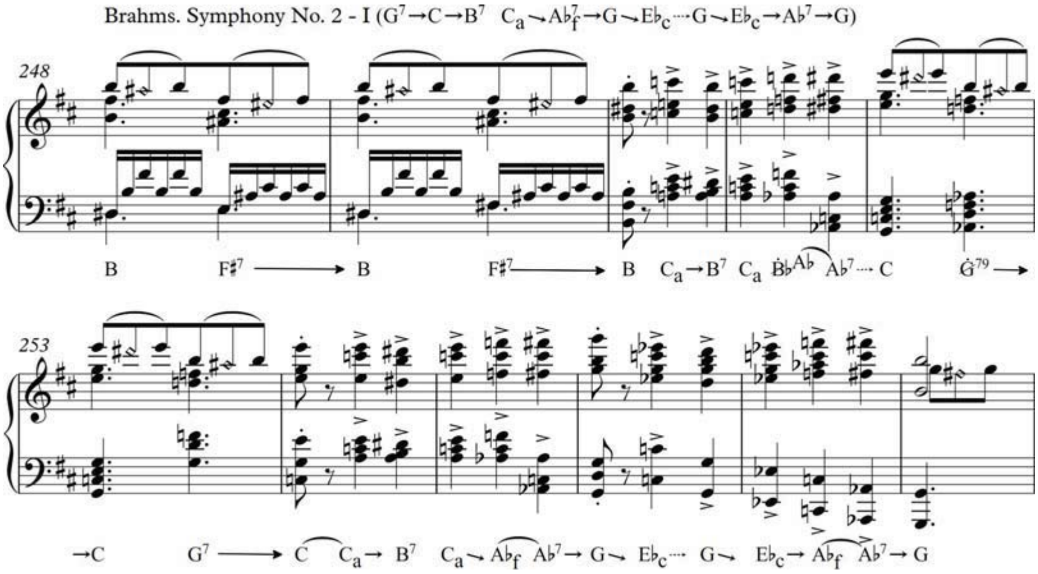
Brahms. Symphony No. 1-I (A♭→G A♭→D♭)



Let us now turn to the first movement of the second Brahms symphony (Example 7-11). From bar 248 we have three bars in B major and then a transition bar that leads to C (major and minor). The harmonic structure of bars 248-249 and 252-253 is the same, transposing the fundamentals an upper semitone. In both cases confirming its tonal vector (T D→T D→T).

Ej. 7-11

Brahms. Symphony No. 2 - I (G^7→C→B^7 C_a→A♭^7_f→G→E♭$_c$···G→E♭$_c$→A♭^{7}→G)

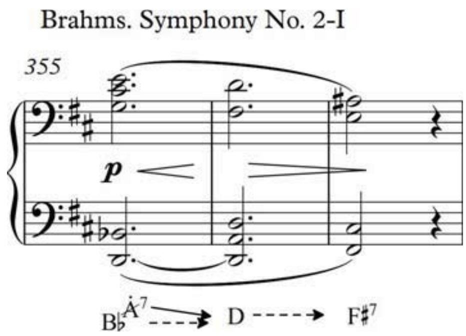


From bar 254 comes a long chain of homotonic relaxions to the dominant (G) of this transient tone.

In all chords with fundamental A♭, the minor seventh appears as augmented sixth (F♯), as leading-tone of the dominant, manifesting the two faces of the tritone contained in the chord. The main fundamental resolves Phrygianly (A♭→G) and the other (hidden virtual one) htonally ($\not{D}^7$→G) (remember that $\not{D}^7$ = F♯C). Exceptionally, at the end of bar 251, the augmented sixth chord resolves Locrianly in the chord whose fundamental (C) will be the next local tonic, but with the dominant in the bass.

In bar 355 of this movement (Example 7-12) we find a chord of the cluster chords family resolving both htonally and Locrianly towards the D minor chord, which resolves again Locrianly to chord F♯^{7}, preparing the entrance of the arpeggiated B minor chord in next bar, producing a slight local modulation to the relative tone of D.

Ej. 7-12



In Example 7-13 we have a long fragment of the first movement of Symphony No. 3. Here Brahms, htonal relaxions aside, uses chord sequences whose fundamentals are on the same tonal axes (i.e., they are separated by m3 or tritone intervals), which creates a great chromatism without this affecting too much harmonic tension, since in the theory of Lendvai (and partially in ours), fundamentals of the same tonal axis have the same tonal function and there is no homotonic tension or relaxion between them (but in this example there are differences of *sonance*, since we have chords with different intervalic structures). For example, at the end of the fragment, in bars 132-133, we have three different chords on the dominant tonal axis, resolving in the tonic in bar 134, the latter dominant chord being the most used and powerful one (D^7) since it alone contains the 7M3 structure, which determines the key. The next one most used before the tonic is $D'^{(7)}$ accompanied by $D^{(7)}$ as we have seen and will see in more examples. In addition, in this example, during this sequence of chords in the same axis, the diminished seventh chord appears many times, which is, by antonomasia, the chord with its four virtual fundamentals in the same tonal axis.

Ej. 7-13

Another long fragment analyzed is the beginning of the third movement of this Symphony No. 3 (Example 7-14). Again almost all chords are linked with homotonic relaxions. In this example we also mark the Dorian ones. In bar 3, note G in the melody, can be considered an appoggiatura of F minor or not (or, to put it better, both can be perceived together, that is why I have put the htonal relaxion E♭→A♭ with a dashed line). If it is not considered an appoggiatura, we have the added relaxion $g^7 \rightarrow c$ (which with the chord of the second beat of this measure forms the sequence: dominant→tonic). Will this double interpretation make it so pleasant to the ear?

Ej. 7-14

We can show more examples of almost consecutive homotonic sequences in the same movement, in bar 70 (example 7-15). Here Brahms uses a curious and remote succession of chords to finally reach the dominant of A♭.

Basically it consist of using (in bar 76) two htonal relaxions to reach B (C♯→F♯→B), which is the Phrygian dominant of the 'dominant of the dominant' (B→B♭), but then, once they have arrived to the 'dominant of the dominant' (subdominant S^- seen from A♭), instead of linking directly with the dominant (B♭→E♭), it connects with a chord of the same tonal axis at tritone distance (F♭), which is the Phrygian dominant of the dominant (subdominant S^{-}seen from A♭), a function we have already seen used in so many previous examples (i.e. the augmented sixth chord). But this time, to resolve in the dominant, it uses an intermediate chord (appoggiatura), which is the tonic minor chord in second inversion or, scholastically, a cadential $^{6}_{4}$ seen from A♭ (minor), although after hearing the dominant, music does not continue with the tonic A♭. This intermediate chord could also be seen as two Locrian relaxions to reach the dominant (F♭→a♭ and C♭→E♭).

Ej. 7-15

Let’s go to the fourth movement of this symphony (Example 7-16). We depart from the dominant to return to the dominant using a long homotonic chain.

The harmonic sequences of the first system (bars 134-137) and the second one (bars 138-141) are similar. In both cases there is a small transient modulation to C♭ (F♭-G♭→C♭), returning to F♭ as a Phrygian dominant of the dominant (helped by B♭, the dominant of the dominant) (bars 137 and 140). That is, the combination (D+D’)→D again. In the first system with a dominant pedal point.

If this fragment were considered a modulation to E♭ instead of being in A♭ (as always is a mixture of the two things), then the functional language would change the term ‘dominant of dominant’ to dominant. The Phrygian dominant and the htonal dominant would resolve in the tonic. Note that using a scholastic language we would be talking about converting the resolution of the French augmented sixth in a Neapolitan cadence (placing the correct notes in the bass, of course, we are thinking in fundamentals and chord-classes). That is to say, equaling inversions, the French augmented sixth is to the dominant what the Neapolitan sixth + dominant is to the tonic. In C would be: (A♭+D)→G, transposing a lower fifth we have: (D♭+G)→C. In both cases, we obtain the Phrygian and honal resolutions of the respective dominants (A♭ and D —Phrygian and htonal— dominants of G // D♭ and G —Phrygian and htonal— dominants of C).

Ej. 7-16

Example 7-17 of the first movement of Brahms Symphony No. 4 is very

Brahms. Symphony No.3 - IV ($E\flat^7 \to C\flat_{ab} \to B\flat_g = F\flat \to G\flat_{eb} \to C\flat \to B\flat^7{}_{F\flat^7} \to E\flat^7 \to C\flat_{ab} \to$ etc...)

interesting. Without a clear key at the beginning of the first measures of this fragment it is a sample of how the homotonic relaxions can be useful to give fluidity and sense to chromatic chord sequences.

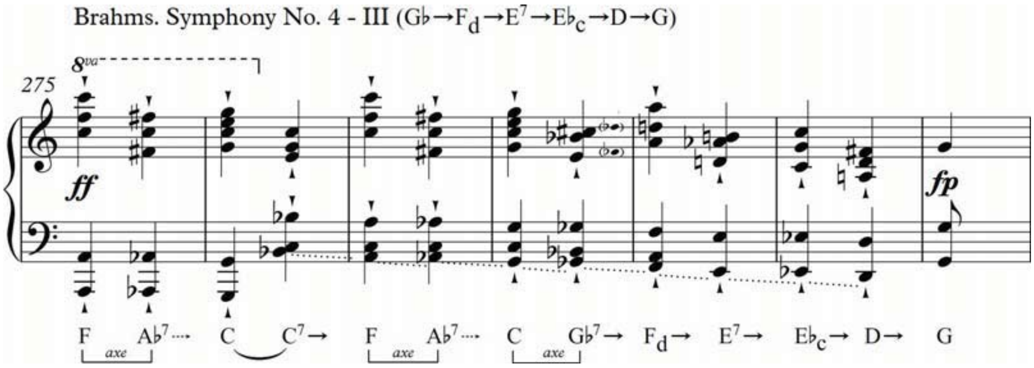
This fragment begins with the tonic chord (in major) and take advantage of the Phrygian relaxion to go to E♭^{7}, in the following measures use chords of the same tonal axis to go to other regions using htonal and Phrygian relaxions.

In bars 229, 233, 241 and 243 we see the Phrygian resolution of the second functional fundamental of the chord (Tristan chord subfamily from the dominant chords family); and from bar 243 to resolve on the dominant in bars 246-247. It is a variant of the typical Phrygian resolution to the dominant that we have seen in the other examples. In addition, bars 232-235 are a progression (an upper tone) of bars 228-231. The same thing happens between bars 236-237 and 238-239 (to htonally reach the dominant in bar 240). When, at bars 240-241, the same progression seems would be repeated, Brahms changes the chord in bar 241 in order to initiate again the cadence to the dominant.

Ej. 7-17

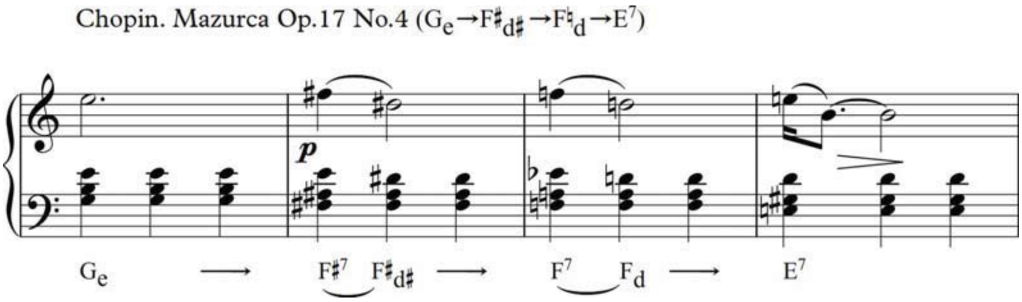
In the third movement of this same symphony we reach the dominant after five consecutive Phrygian relaxions (bars 278-281), after the tonic chord (in second inversion, in bar 278) has directly linked with another chord of its tonal axis (G♭). In bars 275 and 277 we have two clear Locrian relaxions (A♭^{7}→C). If we look at the bass line, from bar 276 we have eight melodic Phrygian movements until we reach the dominant of the dominant (D).

Ej. 7-18



Let's go to Chopin's music now. In his Mazurka Op.17 No.4 we can also see a Phrygian sequence of fundamentals until we reach the dominant (Example 7-19).

Ej. 7-19



Most of Prelude No. 20 is intertwined with homotonic relaxions (Example 7-20). The third system is a repetition of the second. In this example we place the symbology of the chords on the first staff and below the tonal analysis.

All bars have tonal distension in the last beat of the bar, either as a tonic or as a local half cadence (except perhaps in bar 5 [and 9] when the last beat rests in the minor dominant after a htonal homotonic relaxion). To these relaxions in last beats the very important *sonance* relaxion (in almost all bars) has to be added due to the resolution of the appoggiatura in the previous beat.

In bar 6 (and 10) we find the formula (D+D')→D that we have already seen (in the form of French augmented sixth) and at the end the formula (D'+D)→T, which does not become Neapolitan cadence because D' is not in first inversion (it does not have the subdominant in the bass).

Ej. 7-20

Chopin. Prelude No. 20

In Example 7-21 we have a homotonic analysis of its Nocturne Op. 27 No. 1. See again (in bar 13) the use of D+D’ to reach the tonic by means of its characteristic double homotonic resolution, sum of a Phrygian resolution and a htonal one.

Ej. 7-21

Chopin. Nocturne Op. 27 No. 1

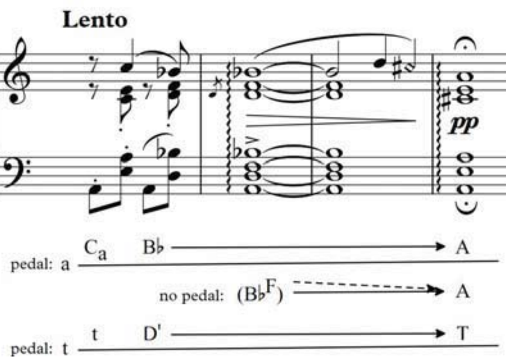
($C\sharp^7 \rightarrow A_{f\sharp} \rightarrow D \rightarrow E_{c\sharp}$ $B^7_{g\sharp} \rightarrow E$ $C\sharp \rightarrow A_{f\sharp} \rightarrow D \rightarrow E_c \cdots G\sharp^7 \rightarrow E_{c\sharp} \cdots G\sharp^7 \rightarrow E_{c\sharp} \cdots G\sharp^7_D \rightarrow E_{c\sharp}$)

In the last bars of Etude Op. 25 No. 4 by Chopin (Example 7-22) we have an ending with a Phrygian relaxion directly on the tonic (with Picardy third). A tonic pedal could be considered in this passage. If we do not consider A a pedal note, a

Locrian relaxion would be added to the Phrygian one, from the second fundamental of the previous chord (F→A).

Ej. 7-22

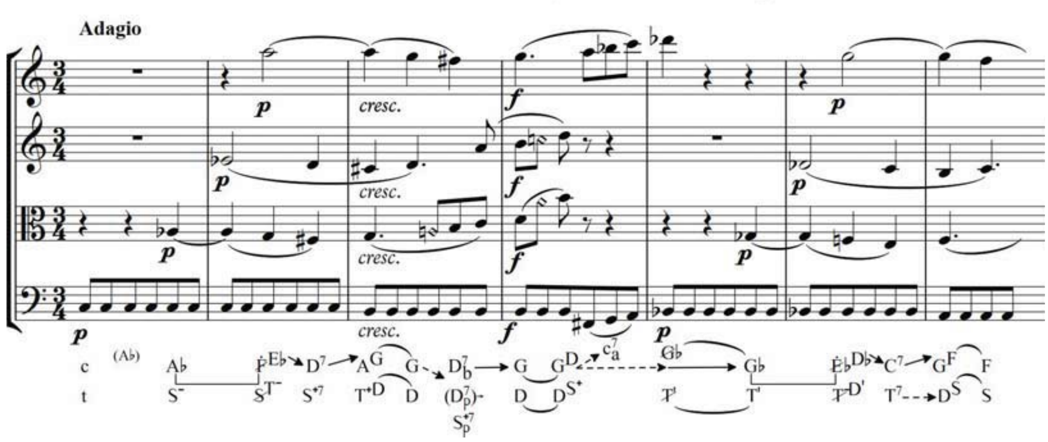
Chopin. Etude Op. 25 No. 4 ($B\flat^{F}\rightarrow A$)



Example 7-23 shows the beginning of Mozart's Quartet No. 19, nicknamed «Dissonance». To the chord A♭ of the beginning follows (with the entrance of the first violin) a «Tristan» chord-class ($F^{E\flat}$, being virtual fundamentals F and A♭ in the same tonal axis). The second fundamental Phrygianly resolves to the dominant of the dominant (S^{+7}) and then htonally towards the dominant ($D^7 \rightarrow G$ —A^G as an appoggiatura chord—). In bar 5 music apparently move away from key C by repeating the beginning a lower M2, forming a chord G♭ with the entrance of the viola (the link is Phrygian), but only apparently because, repeating exactly the same notes as the three first bars (a lower second), in bar 7 we come back to the key of C when we hear the fundamentals corresponding to the dominant and the subdominant, and therefore we hear again the 7M3 structure of C.

Ej. 7-23

Mozart. Quartet No.19 in C Major ($F^{E\flat} \rightarrow D^7 \rightarrow G \rightarrow D^7_b \rightarrow G \rightarrow G\flat = E\flat^{D\flat} \rightarrow C^7 \rightarrow {}_GF$...)



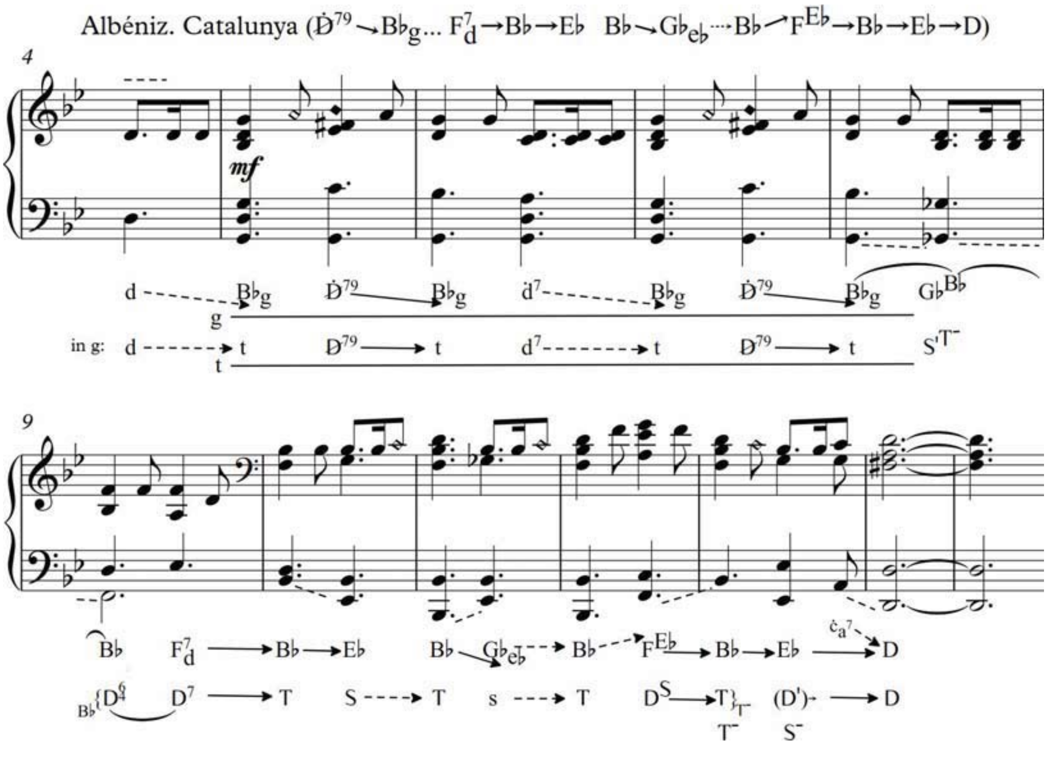
Ej. 7-24

Camil Saint-Saëns. Le Cygne (Carnival of the animals) (G→C_a G→$F\sharp^7$⇝D_b···$F\sharp^7$⇝D_b)

In the work The Swan by Camil Saint-Saëns (Example 7-24) notes in bars 1-2 and 5-6 are the same (apart from the tail of the cello theme in bar 5), basically within the tonic chord. In first instance the tonic chord resolves htonally towards the (S_p type) subdominant (G→C_a, in bars 2-3), but the second time resolves Phrygianly (G→F♯^{79}-F♯^{7}, in bars 6-7) to the chord that later will be dominant of B minor (F♯, Phrygian subdominant S' seen from G), making fit the second part of the cello theme within this new harmony (F♯→b) retaining the melodic drawing, although the intervals are not exactly the same.

In Example 7-25 we have the beginning of Catalonia by Albéniz. The first bars show (under a tonic pedal point) different types of htonal resolutions from the dominant to the tonic. From bar 9 there is a local modulation to the relative tone/ key B♭ (the relative tones are always on the same tonal axis), and in measure 12, it takes advantage of usual htonal sequences in the new key (F→B♭→E♭) to make a Phrygian relaxion towards the dominant D of the initial key (E♭→D). Typical cadence of the minor mode.

Ej. 7-25



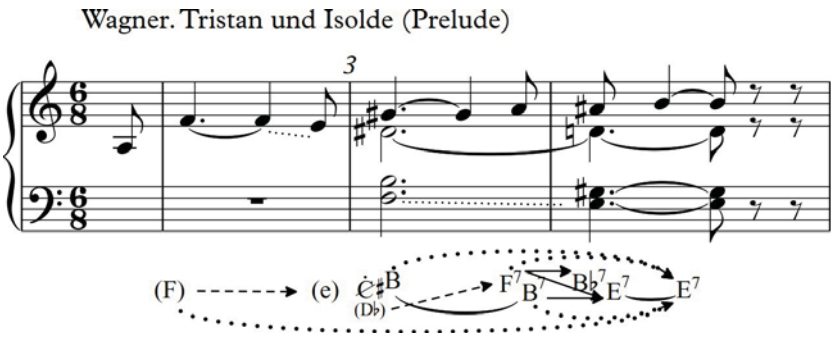
Now let's inquire into Wagner's music. As it could not be otherwise, we will begin with the beginning of the Prelude of Tristan and Isolde (Example 7-26), perhaps the musical fragment of the whole history of music that has been more often analyzed by theorists, with numerous divergences between them.[1] Many times they are not divergences but the consequence of the fact that the ear listens to several harmonic structures at the same time.

If we take a look at the fundamentals of the example, this beginning is an entanglement of htonal and Phrygian homotonic relaxions towards the local dominant of A minor (B→E and F→E).

We do a parenthesis to say that, harmonically, enharmonic notations could be used in several notes; Wagner uses the ones that best fit melodically, but both coexist at the same time. For example, in bar 3, there is no doubt that it is better to put F on bass, but the ear also listens this note with its function as E♯, forming part (as local leading tone) of the chord (C♯)E♯G♯B(D♯) and which would melodically link with note A♯ of the soprano in the downbeat of the next bar (E♯→A♯). But, at the same time, this A♯ could also have been written as part of the chord B♭D(F♭A♭) as the htonal resolution of the previous chord (the last eighth note of bar 3) (F^7→B♭).

Closing the parenthesis; in the last quaver of bar 3 and the first one of bar 4, two chords of the symmetrical dominant family (two 7M3 structures at tritone distance) are formed. This succession has the characteristic that four homotonic relaxions are produced at the same time (!), two Phrygian and two htonal (F→B♭, F→E, B→B♭, B→E), thus greatly softening the entry of bar 4 appoggiatura to the dominant chord. One might even consider the Tristan chord of bar 3 as a long appoggiatura to FBD♯A chord and summarize this beginning of the prelude as the only two-chord link (F^B→E^7), which is no more than the resolution of the French augmented sixth towards the dominant, i.e. the simultaneous htonal and Phrygian resolutions (F→E and B→E). In addition, secondarily, almost as an added anecdote, we have the Locrian relaxion of the virtual fundamental of the Tristan chord to one of the fundamentals of this augmented sixth chord (C♯/D♭→F).

Ej. 7-26



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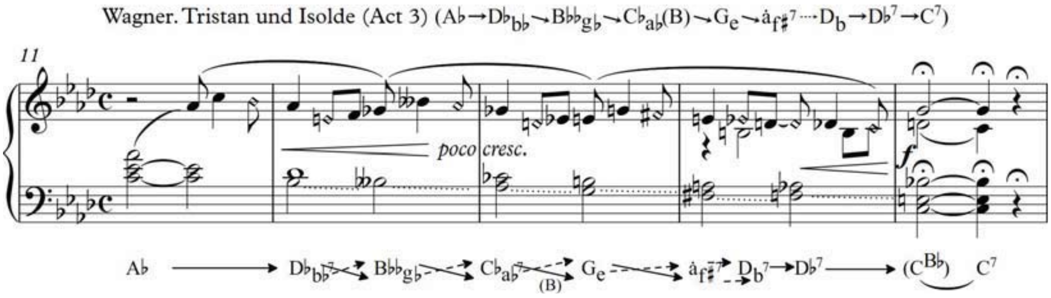
[1] *Der Tristan-Akkord und die Krise der Moderner Harmonielehre.* M. Vogel (1962).

In the introduction of the third act of the same opera we have a long homotonic chain to reach the dominant of the relative tone/key of A♭, passing through intermediate chords with htonal and Phrygian relaxions. In addition, the bass walks descending steps of m2, Phrygian melodic movement.

Ej. 7-27



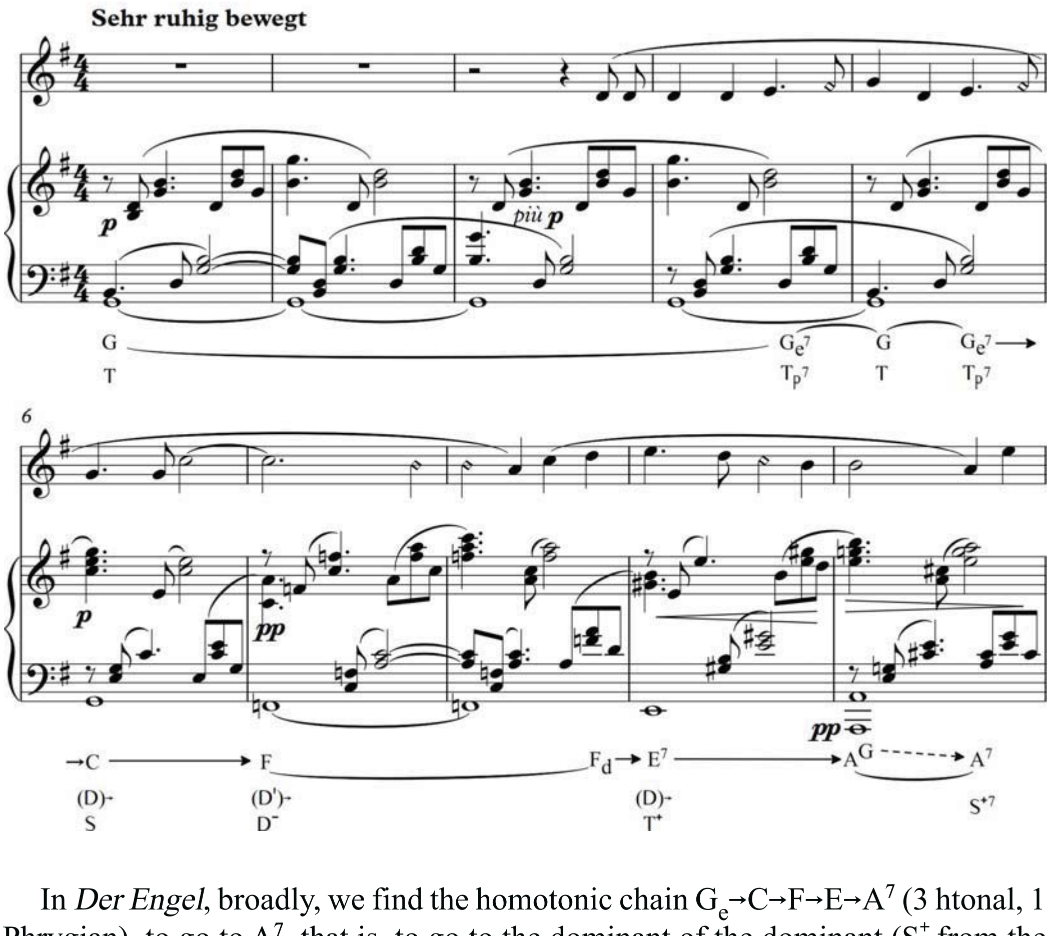
In the Pilgrim's chorus from Tannhäuser, from the end of bar 17, we have three progressions (of two bars, beginning in the last beat) that repeat exactly at a distance of m3, that is, they go through the tonal axes. The first two resolve htonally (F=A♭$_f$→D♭ and A♭=C♭$_{a♭}$(B)→E), but the third progression does so Phrygianly (B→B♭), to go to the dominant. Within each progression we can observe two Locrian relaxions. The return to the dominant (in bar 25) and to the initial key in order to repeat the progressions again consists of two new htonal and Phrygian relaxions from the same mentioned dominant (B♭^{7}→E♭↘C♭$_{a♭}$→B♭).

Note that in all the examples we have been showing in this chapter, Phrygian resolution to the dominant is more common than the htonal one (as dominant of the dominant). We have placed tonal functions, in this and in some other examples, in two lines, the first one is the symbology from the point of view of the key of the piece (E♭), and the second one an alternative notation that indicates, in this case, the functions within each transient modulation (in brackets, to D♭, F♭/E and G).

In the *Wesendonk lieder* (of Wagner), we also find numerous homotonic sequences.

Ej. 7-29

Wagner. Der Engel (Mathilde Wesendonk lieder) (G_e→C→F→E→A^7)



In *Der Engel*, broadly, we find the homotonic chain G_e→C→F→E→A^7 (3 htonal, 1 Phrygian), to go to A^7, that is, to go to the dominant of the dominant (S^+ from the point of view of G), though then does not immediately resolve to the dominant (we have to wait three minims, up to bar 12).

Note B in the melody in bar 10 could be considered appoggiatura of A; If we consider B to be part of the chord its function does not vary, because both the chord

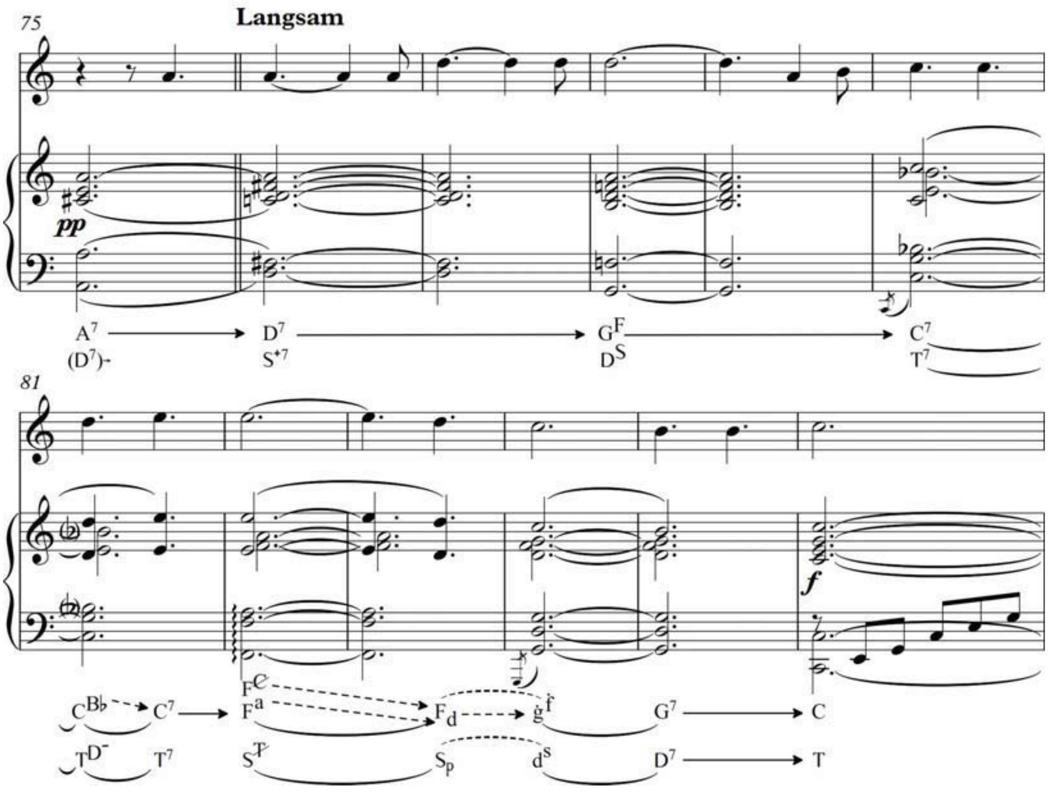
A^G and the chord A^7 contain the 7M3 structure (in this case towards D); as we have seen in 3.1.3, both belong to the unitonal dominant chords family. The resolution of A^G in A^7 is basically by *sonance* (the second is more consonant), but one could also speak of a weak Dorian relaxion between fundamentals (G→A).

In *Stehe still!*, from the same lieder collection, we find a homotonic chain consisting only of htonal relaxions ($A^7 \rightarrow D^7 \rightarrow G^F \rightarrow C^7 \rightarrow F_d$ ...$G^7 \rightarrow C$).

Using scholastic language we may say that it begins with the secondary dominant of the 'dominant of the dominant'. Using our symbology we write $(D^7) \rightarrow S^{+7}$. Continues, in bar 78, resolving towards the dominant (with the subdominant as second fundamental) ($S^{+7} \rightarrow D^S$), to resolve in the tonic chord with the minor seventh, which in turn resolves htonally in (scale 2nd degree) S_p ($D^S \rightarrow T^7 \rightarrow S_p$), considering notes E of bar 82 appoggiaturas of notes D of S_p. Finishing the fragment with the authentic cadence ($D^7 \rightarrow T$).

Ej. 7-30

Wagner. Stehe still! (Matilde Wesendonk lieder) ($A^7 \rightarrow D^7 \rightarrow G^F \rightarrow C^7 \rightarrow F_d$...$G^7 \rightarrow C$)



We find more homotonic sequences in *Matilde Wesendonk* lieder; for example in *Im Treibhaus* (Example 7-31), also to reach the dominant (harmonically the part of the piano is almost identical to the introduction of the third act of Tristan, although at 6x8 metre instead of 4x4).

The piece starts with three htonal resolutions to the tonic chord from a chord of the dominant chords family ($Ȼ^{B\flat}$, «Tristan» chord-class) (resolution to the functional fundamental of D minor chord: $Ȼ^{B\flat}\rightarrow F_d$).

As we have seen in 5.3 we have two alternatives to functionally symbolize the minor tonic chord. One is writing **t** and the other writing T_p^- (since its functional fundamental is the upper relative major tonic, in this example F=T^- of D minor). In this case perhaps it would be more convenient to put T_p^-, since we come from its subdominant (S^-) and from its relative dominant ($Đ^-$) and in fact, in these initial moments, the tonal vectors point more to F than to D.

In bar 9 we reach Phrygianly the dominant with the sequence $C^{B\flat}\rightarrow F_d\rightarrow B\flat\rightarrow A$. The homotonic string until the dominant (A^7) of bar 13 is even longer (and more complex and chromatic), it is not necessary to write it here, it is the one that appears in the example from chord F in the 2nd beat of bar 9 (in first inversion). Note also, from bar 10, that there are melodic Phrygian descents in the bass.

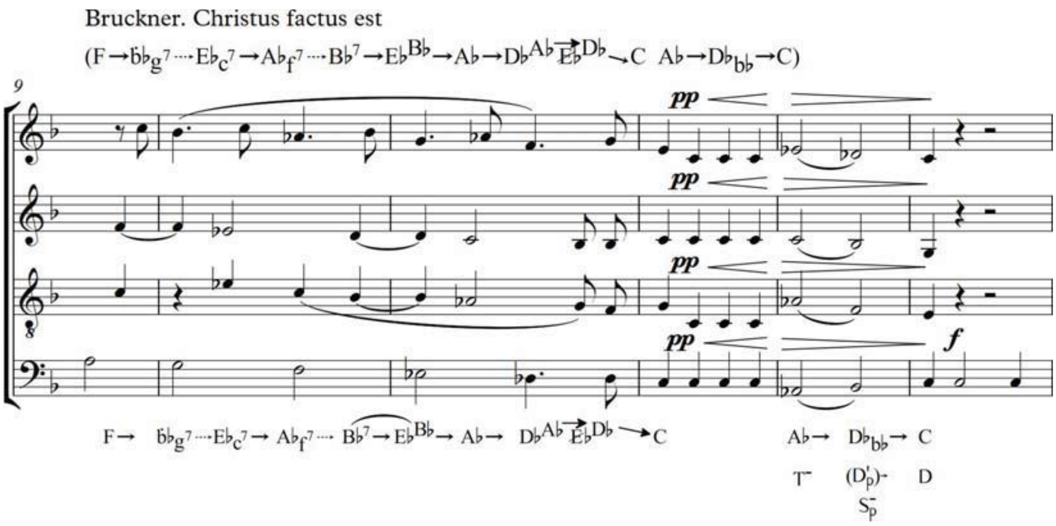
Ej. 7-31

The first chord of bar 10 may have several interpretations: the main one is as G minor seventh chord ($\dot{B}\flat_{g7}$) with C♯ as appoggiatura to D. But in the second quaver we could also consider C♯ as leading tone within the virtual dominant chord (A)C♯(E)GB♭, which gives it a dominant touch. But, by enharmonizing C♯ as D♭, we could also see the Tristan-class chord (E♭)GB♭D♭F with the M3 tension D♭F which resolves in the fundamental G♭ of the next chord. As always, it is not necessary to choose one of the three interpretations, the coexistence of the three can be considered. Something similar happens to the chord of the first beat of bar 12.

The following example goes to Bruckner, to his four-part motet *Christus factus est* (Example 7-32). Again we see a long htonal and Phrygian string between fundamentals (with a Dorian one in bar 10) to reach again the dominant in bars 12 and 14. The resolution to the dominant from the previous chord is also Phrygian in both cases, but using different chords.

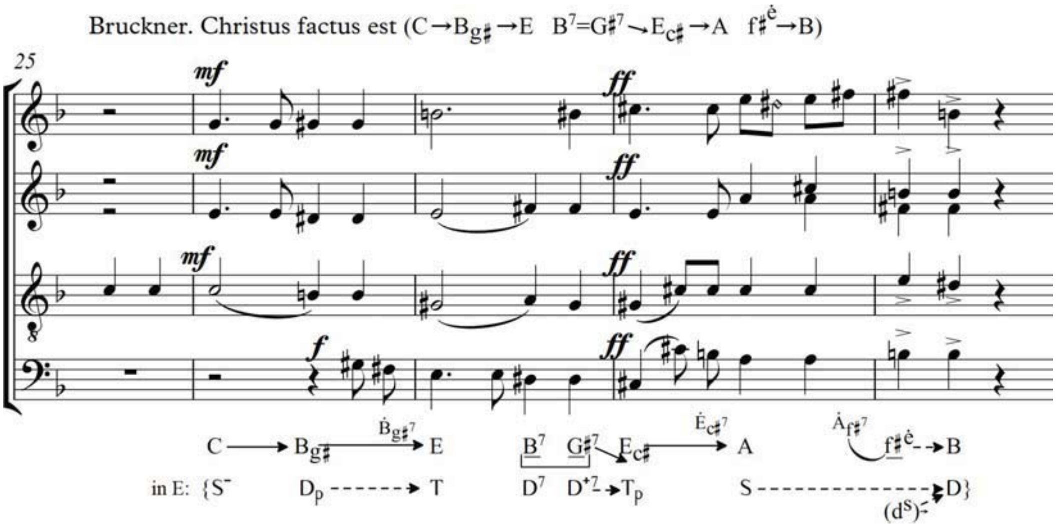
In the last beat of bar 11 we have a Tristan-class chord and is its second fundamental D♭ which resolves Phrygianly to the fundamental C. In bar 13 it is the functional fundamental (D♭) of B♭ menor minor chord, which resolves Phrygianly to C. Seen from tonic F, this B♭ minor chord is the subominant S^{-}_{p} (seen from the dominant C this chord has the function of Phrygian dominant type D'_{p}).

Ej. 7-32



In this same work of Bruckner we can see in bar 25 (Example 7-33) a modulation to E (major) using, from the dominant of F, a Phrygian and htonal successions to go directly to the tonic E chord (C→$B_{g\sharp}$→E). To reaffirm the modulation we have a homotonic string to reach the subdominant A (B^7=$G\sharp^7$↘$E_{c\sharp}$→A) and then a step to the dominant taking advantage of the small dominant character of the quintal chord (suspended chord $f\sharp^e$→B), aided by the soprano melodic leap of descending fifth to the dominant. This suspended chord (being the dominant on the bass) could also be viewed as an appoggiatura of the dominant chord (E→D♯ in the tenor voice).

Ej. 7-33



In Example 7-34 we can see the homotonic sequences of bars 21-30 in Bruckner's *Ave María* (the symbology of chords above the system and below the tonal functions). The scheme to reach the tonic chord in bars 21-25 is A↘F_d→B♭↘$\dot{C}_{a^7}$→F. Here the Phrygian succession to the 'dominant function' of C_{a^7} is not from D♭ but from the subdominant B♭ which resolves in A minor seventh, chord which has the dominant triad inserted (has functional fundamental C), in addition, in this case, with the dominant in the bass, therefore with a clear dominant function (functional sequence D^S-D_p-T).

In bars 26-28 there is a transient modulation to the subdominant B♭ colouring the chords of the typical sequence T-S-D-T (of B♭), and in bar 30 we return to the dominant C by means of the tonal tension of chord G^F (dominant + subdominant 'of C' as fundamentals). Note that in bar 26 we have a double htonal homotonic relaxion.

In *Vexilla regis*, also by Bruckner (Example 7-35), we see, in last bars, a modal ending with Locrian relaxions (B^7→D♯/E♭, A♭↘$E♭_c$, $A♭_f$→C, C_a→ė…), combined with the htonal relaxions E♭→A♭ and ė↘C_a (the latter very weak since E does not have its M3). The final Locrian relaxion is supported by *sonance* relaxion.

Ej. 7-34

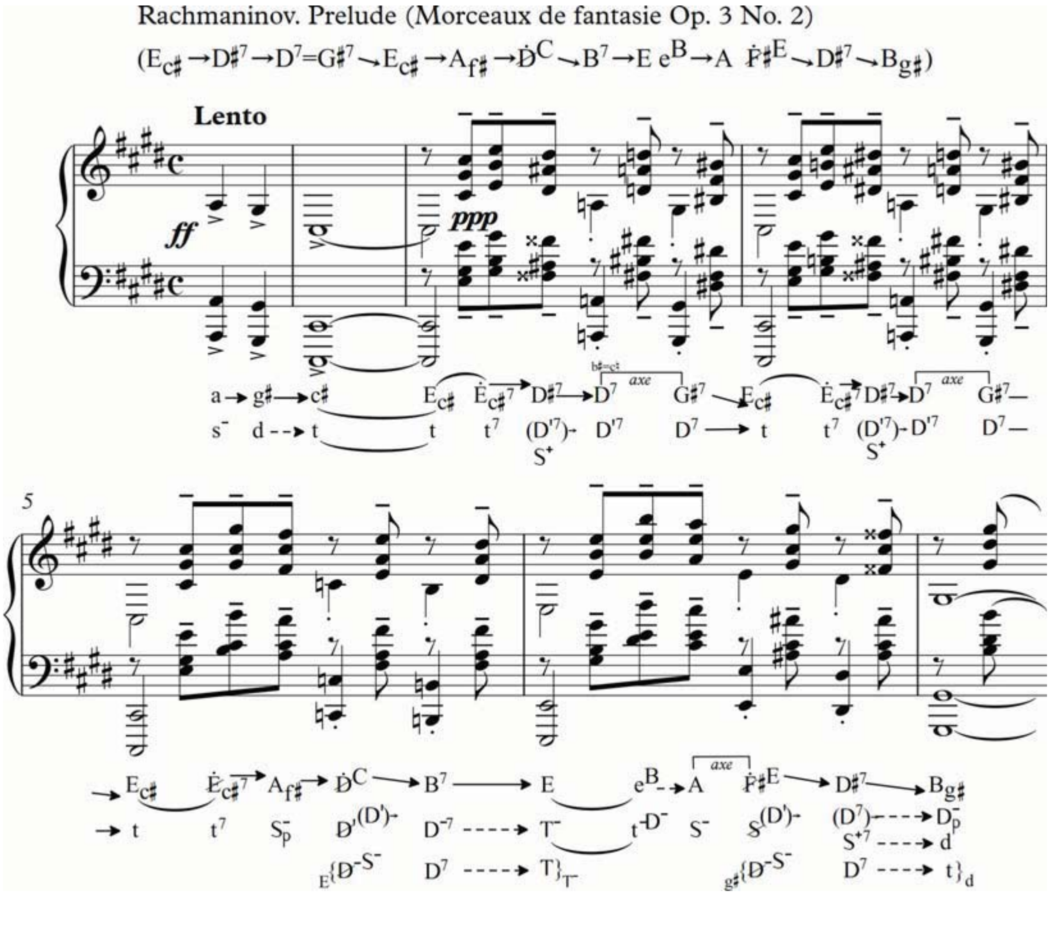
Bruckner. Ave Maria (A→F_d→B♭... Ċ$^{B♭}$→$Ċ_a{}^7$→F→B♭^{F}→E♭$^{B♭}$ F^7→B♭$_g$ Ġ^F→C)

Ej. 7-35

Bruckner. Vexilla regis (conclusion) (B^7···D♯...A♭···$E♭_c$→A♭$^{E♭}$→$A♭_f$···C···ė^7···C_a···ė)

The beginning of the prelude of *Morceaux de fantasie* by Rachmaninov (Example 7-36) seems to augur the transient cadences that will appear throughout the work. In fact we are talking about the cadential progression we have seen more often in the previous examples: Phrygian relaxion followed by a htonal relaxion towards a dominant or a tonic (transient or not). The prelude starts with this sequence (***ff***) but only with one (fundamental) note, repeated in three octaves (a→g♯→c♯). In bars 5-6 we found the same progression in the relative key/tone E (C→B^7→E) and in bars 6-7 towards the minor dominant (E→$D♯^7$↘$B_{g♯}$). In bars 3-4 there is a variation of this formula. It begins with a Phrygian relaxion ($D♯^7$→$D♮^7$) but instead of going to G♮, it links to the most similarly functional chord on its tonal axis (D^7=$G♯^7$, which shares the same tritone F♯-C/B♯) to, finally now, release the tension htonally to the tonic chord ($D♯^7$→D^7=$G♯^7$↘$E_{c♯}$) (the same thing occurs in bar 4).

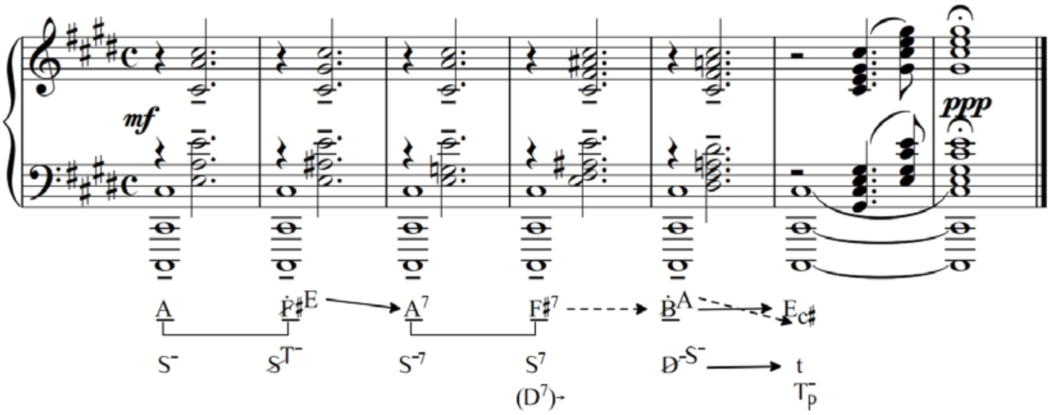
Ej. 7-36



As always, the second row of functional symbology (the third, counting all rows) is an alternative notation; both are correct. D in parentheses (D) refers to a secondary dominant (htonal (D) or Phrygian (D')), but from the point of view of the tonic is a subdominant (S^+or S^-). The fragments between brackets show a transient modulation and the functions within it are seen from the point of view of the new key/tone (indicated at the beginning of the bracket).

Ej. 7-37

Rachmaninov. Prelude (Morceaux de fantasie) (A^7=F♯^{7} ⋯ $\dot{B}^A \rightarrow E_{c\sharp}$)



At the end of this prelude (Example 7-37) we find a rare cadence to finalize a tonal composition (bearing in mind that the tonic is C♯) (Ɖ$^{-S^-}$→t), even though we have htonal homotonic relaxion between chords (F♯→Ƀ^A→$E_{c\sharp}$) (would be a more typical cadence of E major instead than C♯ minor). However, since we always have the pedal C♯ in the bass, we could also consider an ending with *sonance* resolution, since all the previous chords are more dissonant and none is in root position giving us a sensation of ending simply by de fact of hearing the tonic chord in its most consonant position, aided by the htonal relaxion towards the functional fundamental of the minor chord (Ƀ→E) and the Locrian one towards the root (A→c♯).

In Debussy's *La fille aux Cheveuz* (Example 7-38) we can see a long homotonic chain. In bars 12, 15, and 18 the same formula is used to create three tonal vectors: this passage take advantage of the slight property of quartal chords to tonicize next chord, which is a major triad in root position.

Quartal chords with four fifths (five notes, for example: FCGDA) functionally have the form $\mathbf{d}^S$, where **d** is the dominant with its fifth (GD) and S is the major triad of the subdominant (FAC) and form a small tonal vector towards C although the chord does not have the leading-tone. This is what happens in the above-mentioned bars, transposed to G♭, C♭ and E♭, in bar 18 incorporating the semiquaver C♮ at the end of the bar; if by its small duration we do not incorporate it, we have a chord with three fifths (four notes) that still conserves this small tonal vector ($\mathbf{d}^\mathbf{s}$); in this case we have dominant + fifth and subdominant + fifth. With this long chain

Debussy reaches the relative of G♭ mayor (E♭) but with the Picardy third.

Ej. 7-38

The following examples are from *Proses lyriques*, also by Debussy.

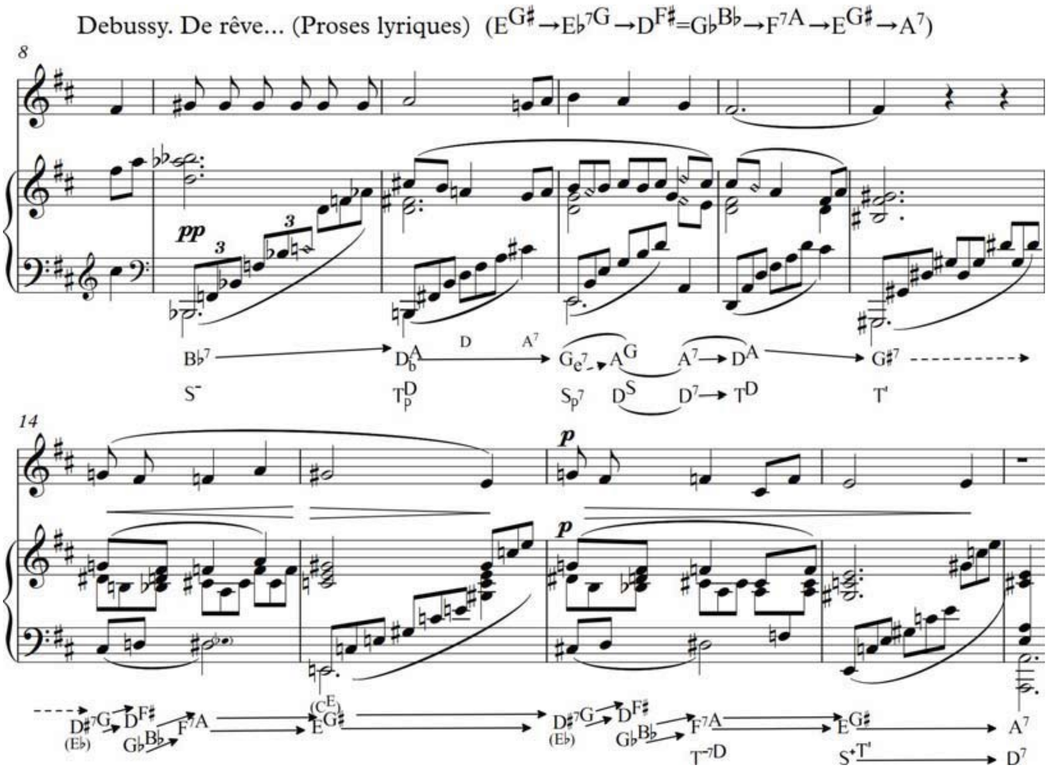
In the first song (*De rêve…*, example 7-39) we also have a long homotonic progression, in this case, from the Phrygian dominant (B♭) of the dominant (bar 9) to the dominant (A^7) (bar 18).

In bar 9 we have an example of the double function of the augmented sixth in the chord (G♯), as leading-tone of the dominant and as minor seventh of the fundamental. Debussy seems to be aware of this duality and in the melody places a G♯ and a A♭ on the piano.

In bars 10-12 we see combinations of fundamentals corresponding to the tonic (D), the subdominant (G) and the dominant (A), the latter being the main fundamental of these bars until in compass 13 it resolves Phrygianly to $G\sharp^7$, chord that prepares an atonal fragment in bars 14-17 (there are the 12 notes of the chromatic scale every two bars) but, as we shall see, consistently *homotonically* speaking. In fact, bars 14-15 and 16-17 are harmonically the same and basically are formed by chords of the family of augmented chords (augmented fifth) and their fundamentals are joined by Phrygian homotonic relaxions. As we have seen in 3.1.4 and 6.2.4, the chords of this family have three fundamentals at M3 distance, but for simplicity we only show two (those that correspond most to the written notes). In this example in some cases we put the three to see more clearly this Phrygian homotonic chain. If we place the three fundamentals, we would see that each is Phrygian relaxion of some fundamental of the previous chord.

In bar 18 we return home with the htonal relaxion towards the dominant and also by the *sonance* resolution.

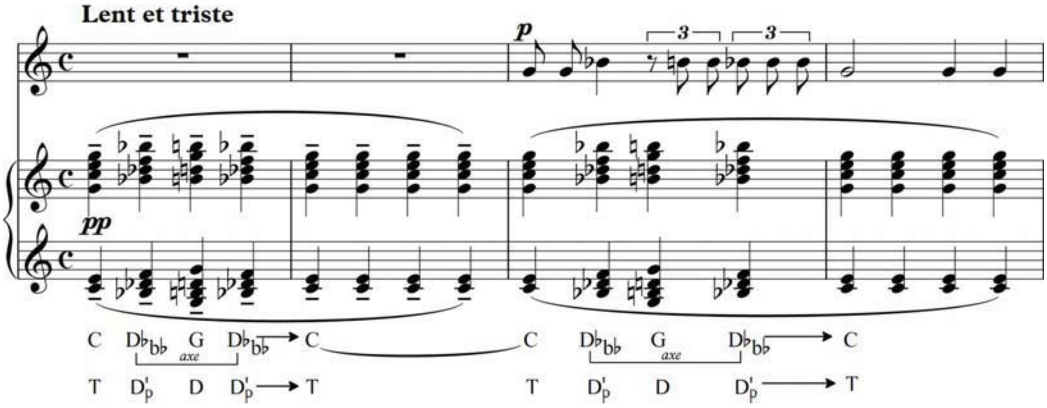
Ej. 7-39



At the beginning of the third song (*De fleurs...*, Example 7-40) we see again what we have already analyzed in other examples: the combined use of the Phrygian (D') and honal (D) dominants to go towards the tonic or the dominant (remember that the Phrygian dominant also has something of subdominant); in our example they go towards the tonic using a variant of the Phrygian dominant (D'_p). Recall also that D, D', D_p and D'_p are in the same tonal axis and in the theory their succession is harmonically neutral (there is no tension or homotonic distension between the chords), although there is a difference of *sonance* between major and minor chords.

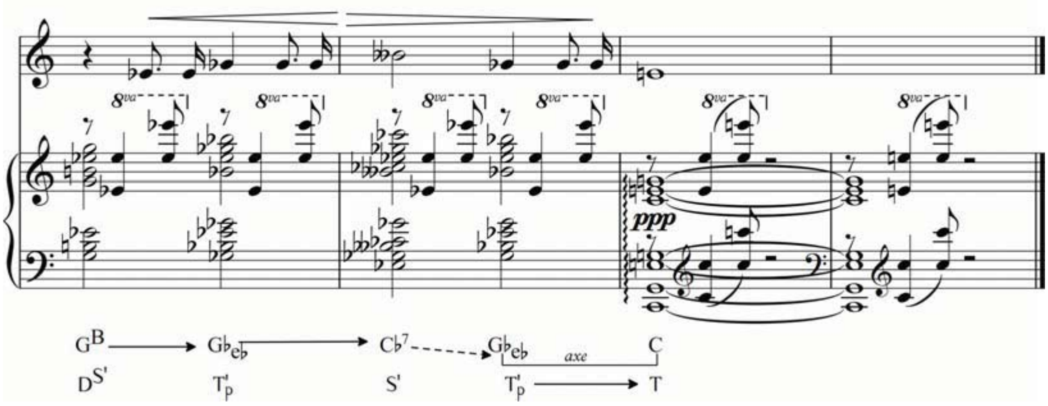
Ej. 7-40

Debussy. De fleurs... (Proses lyriques) (C $D\flat_{bb}$=G=$D\flat_{bb}$→C)



Ej. 7-41

Debussy. De fleurs... (Proses lyriques) (G^B→$G\flat_{e\flat}$→$C\flat^7$⇢$G\flat_{e\flat}$=C)

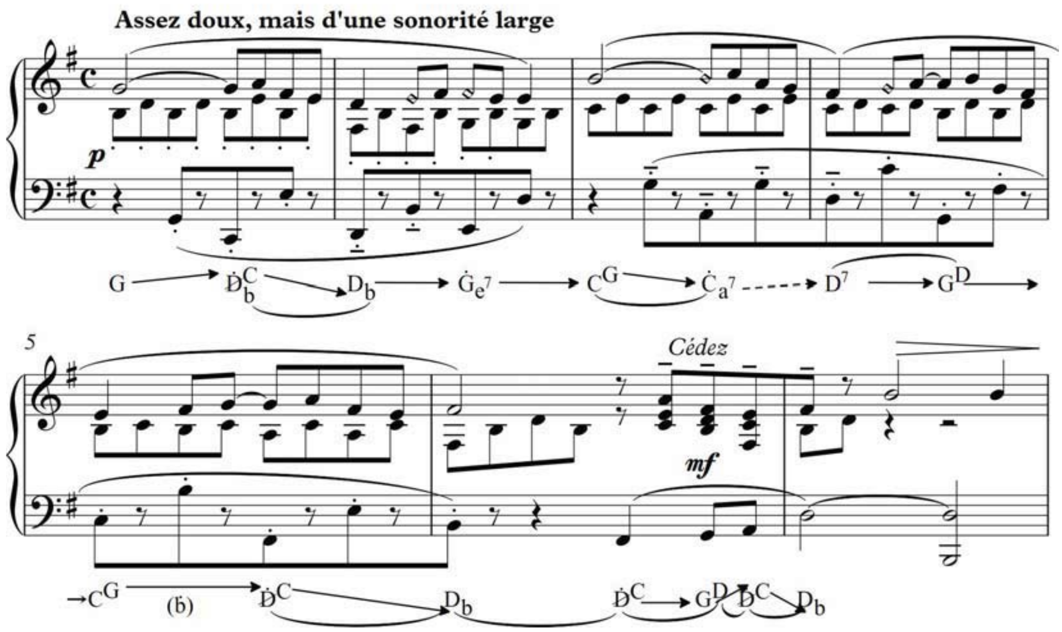


At the end of this song (Example 7-41), Debussy seems to take advantage of tonal axes functional equivalence to go to the tonic. He does something similar to the previous example, but instead of using the dominant (D'_p = D), we see the direct use of the tonic (T'_p = T). We also have, in last chord, a relaxation of *sonance* (the chord follows the overtones trail). In order to reach $G\flat_{e\flat}$ a homotonic chain is used (Phrygian + htonal + Locrian relaxions) (G^B→$G\flat_{e\flat}$→$C\flat$↘$G\flat_{e\flat}$).

Ej. 7-42

Ravel. Pavane pour une infante défunte

$(G \nearrow \dot{Đ}^C \searrow D_b \rightarrow \dot{G}_{e}{}^{7} \rightarrow C^G \searrow \dot{C}_{a}{}^{7} \cdots D^7 \rightarrow G^D \rightarrow C^G \nearrow \dot{Đ}^C \searrow D_b\, \dot{Đ}^C \rightarrow G^D \nearrow \dot{Đ}^C \searrow D_b)$



In Example 7-42 we have a very long homotonic link in the beginning of *Pavane pour une infante défunte* by Ravel. All the functional fundamentals (real or virtual) correspond to the three functional fundamentals of G major (G, C, D = T, S, D), in different combinations. We do not have any tonal deviation in these initial measures, although the Phrygian resolutions at the beginning of bars 2, 6 and 7 ($C \searrow D_b$) give a touch of B Phrygian mode.

From mid-bar 5 (or even from mid-bar 4) to bar 7 we have the dominant function present in different chord structures ($Đ^C$, G^D and D_b), but we never hear the dominant chord in its usual triadic form. We can say the same thing with the subdominant triad and the tonic triad (except at the beginning).

In the following example (Bartók's Bagatelle No. 4, Example 7-43) we also have (almost) all chords (except in three) using the functional fundamentals of a tonality (in this case of F —F,B♭,C—, although this work is not properly in F major).

The work rests every four bars in the D minor seventh chord, i.e. the triad of the tonic F (FAC) plus the D, which gives a modal eolian air. The fact that the final chord (and the fermata of bar 2) has F as functional fundamental allows this rest.

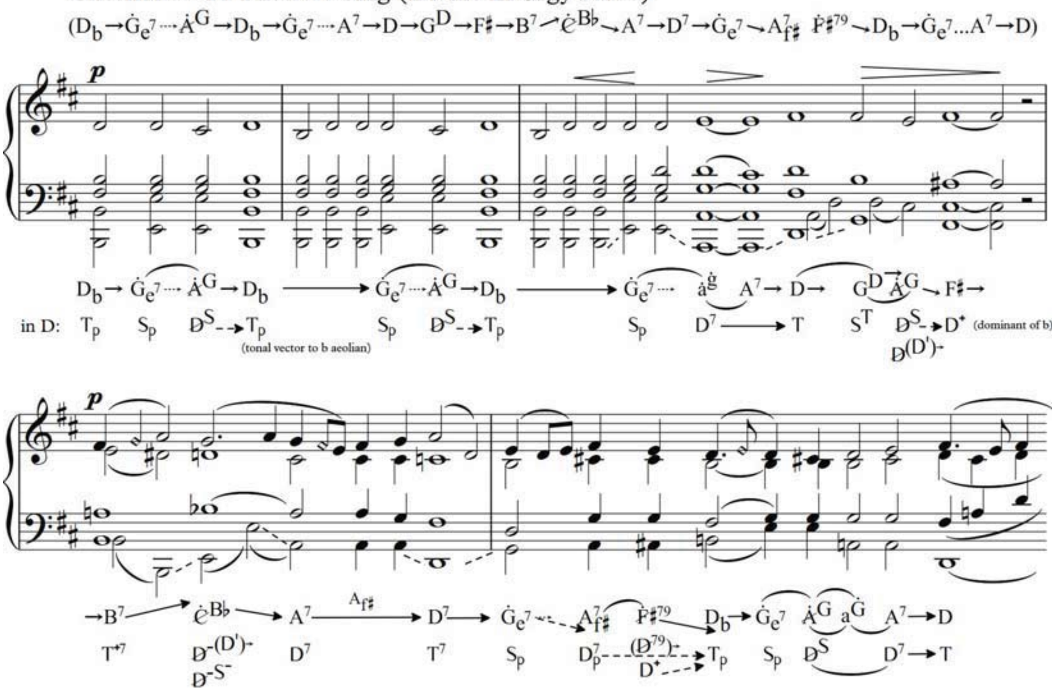
Most of these chords are also homotonically linked. The ending bars does not have these diatonic fundamentals (bars 8 or 12, because the third system is a copy of the second). In the second beat of this ending measure Bartók puts a dissonant chord (from the symmetrical dominant chord family: $G\#_D$) which contrasts sharply with the diatonic chords we have been hearing so far, but it is a chord that follows the tonal axes theory of Lendvai, that is, before the final chord of D minor seventh, which functionally (seen from F) is type T_p, we find other tonic tones (fundamentals) of the same tonal axis (D, F, G♯/A♭ , respectively T^+, T, T^-).

Ej. 7-43

Almost all chords are linked with homotonic relaxions in Example 7-44 (*To Thee We Sing* by Chesnokov).

Ej. 7-44

Chesnokov. To Thee We Sing (Divine Liturgy No. 6)



In the first two bars we have resolution towards the functional fundamental (D) of the B minor chord ($A^G \to D_b$) and a Locrian one from the second fundamental (G→b), which gives (as in the previous example) a modal air (B Eolian). In fact, in this musical fragment, two tonal vectors alternate towards B and D.

Since we have put all the functional symbology with regards to tone D, the symbols (functional symbology) that indicate redirection of the tonal vector towards B are T_p (tonic of the relative key) and D^+(dominant of the relative key). The fundamental corresponding to the dominant of the relative tone/key B (F♯) is usually reached by a Phrygian relaxion of the fundamental G, corresponding to the subdominant of main key D (G→F♯).

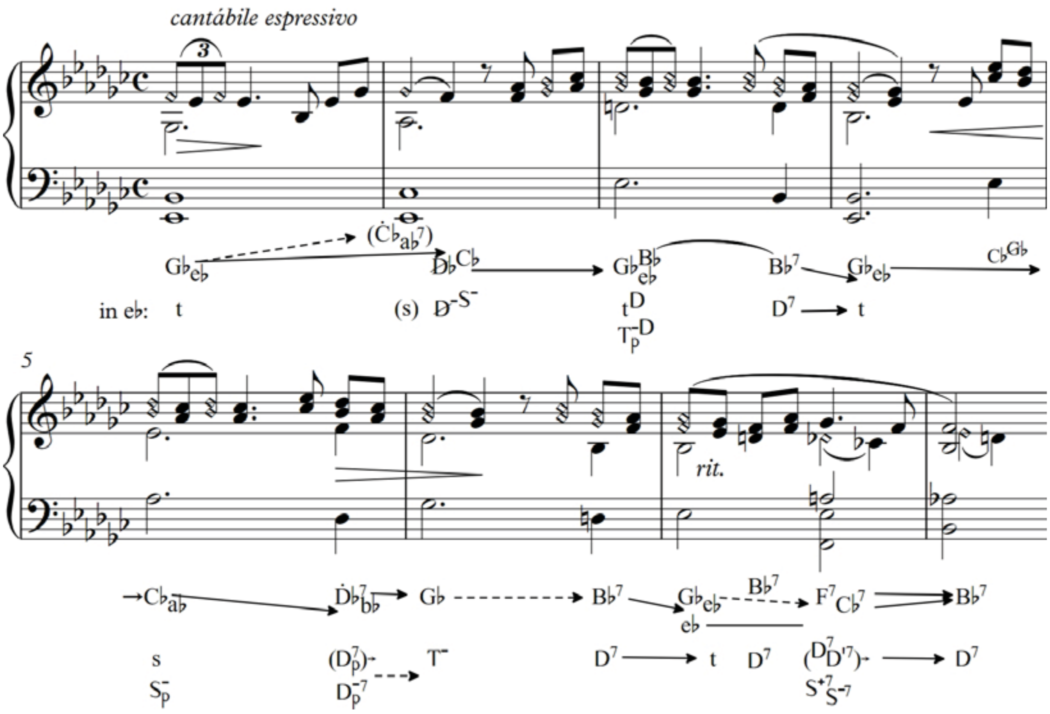
We can see a long homotonic chain in the *Cançó i Dansa n.º 6* (Example 7-45) by Mompou, in this case coloured with numerous appoggiaturas and passing tones.

In bar 8 we have, again, the combined use of (D) and (D') to resolve htonally and Phrygianly towards the dominant.

Ej. 7-45

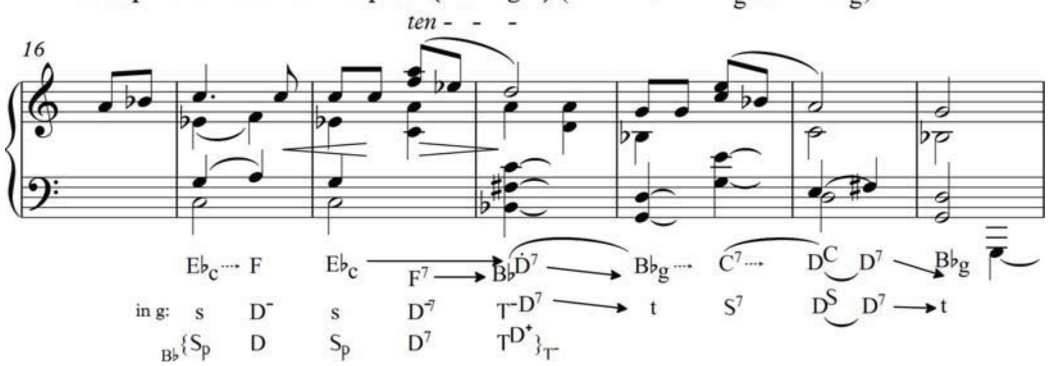
Mompou. Cançó i dansa No. 6

$(G\flat_{e\flat} \nearrow \dot{D}\flat^{C\flat} \to G\flat^{B\flat}\ B\flat^7 \searrow G\flat_{e\flat} \to C\flat_{a\flat} \searrow \dot{D}\flat^7_{b\flat} \to G\flat \cdots B\flat \searrow G\flat_{e\flat} \to F^7_{C\flat^7} \to B\flat)$



Ej. 7-46

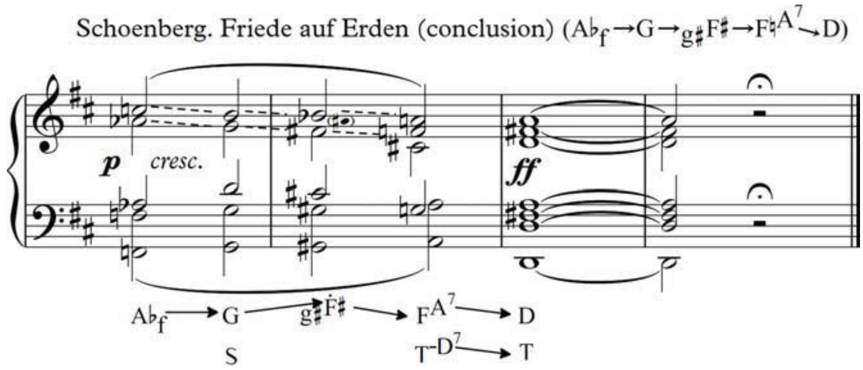
Mompou. La font i la campana (Paisatges) $(F^{E\flat} \to B\flat^{D} \searrow B\flat_g ... D^7 \searrow B\flat_g)$



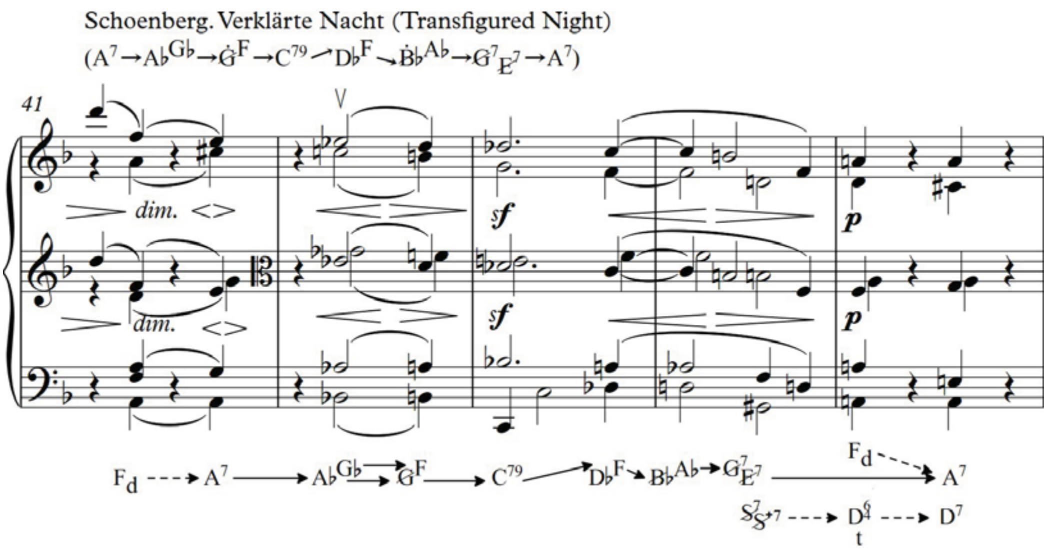
In *La Font i la Campana*, also by Mompou (Example 7-46), apart from the Dorian and htonal sequences we have a link that sweetens the arrival of the dissonant chord of bar 19 ($B\flat^{D^7}$, a mixture of the families of augmented and dominant chords). We find a double relaxion between fundamentals: a Phrygian one to the incipient future dominant ($E\flat \to D^7$) and the other htonal towards the local tonic ($F^7 \to B\flat$). Indeed, this chord marks the end of the tonal vector towards B♭ to redirect it to G (minor). It is a mixture of authentic cadence and Phrygian cadence to the dominant, for that reason its dissonance is sweetened.

A very similar chord is found in Schoenberg's *Friede auf Erden* (F^{A^7}, in other inversion) (Example 7-47), but here the harmonic situation is different. We see a chromatic sequence of chords consisting of three consecutive Phrygian relaxions and a final htonal distension at the end of the work.

Ej. 7-47



Ej. 7-48



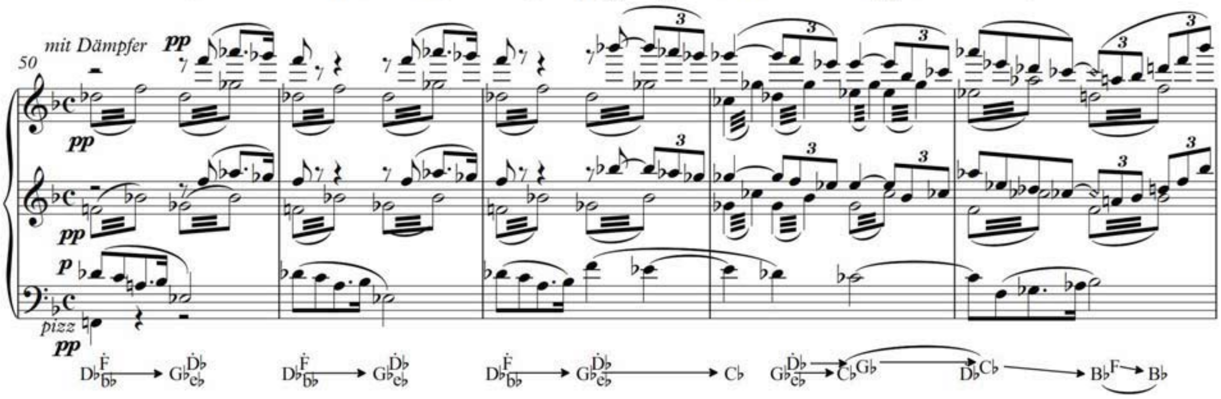
In Schoenberg's *Transfigured Night* (Example 7-48) we can also find many homotonic relaxions. In bars 41-45 we have an example. It is a fragment with a mixture of htonal and Phrygian relaxions between fundamentals (with some Locrian ones) to finish in the dominant.

The diminished seventh chord of the second minim of bar 44 (be aware of the C-clef from bar 42) have been symbolized in the form $\not{G}^{7}{}_{\not{E}^{7}}$ (one could also write $\not{E}^{79}$, see 3.1.5) to show that the virtual fundamental G is a Phrygian relaxion from the previous chord (A♭→$\not{G}^{7}$) and that the virtual fundamental E creates the «dominant of the dominant» tension that resolves in the dominant ($\not{E}^{7}$→A^{7}). Between these two chords we have a cadential $^{6}_{4}$, which can be considered as an appoggiatura chord to the dominant chord, although in the minor mode it implies a Locrian relaxion between functional fundamentals (F_{d}→ A^{7}).

A few bars later (Example 7-49) we find several chords with two functional fundamentals that are also linked using homotonic relaxions.

Ej. 7-49

Schoenberg. Verklärte Nacht (Transfigured Night) (D♭$^{F}_{b\flat}$→G♭$^{D\flat}_{e\flat}$→C♭ G♭$^{D\flat}$→C♭$^{G\flat}$→$_{D\flat}$C♭→B♭^{F}→B♭)



We could continue to give many more examples of the use of homotonic relaxions (basically the htonal and Phrygian ones), as a way to achieve relaxed successions between chords or harmonically fluid progressions, with or without the participation of tonal distensions, which we have shown in a lower line in the examples, although, when the tonality is well established (strong tonal field), they are more powerful than the homotonic forces. When the tonal field is weaker, homotonic relaxions acquire more significance, as we have seen in some previous examples.

We have also seen the utility of separating the chords in their fundamentals that define them tensionally, the capital letters indicating functional fundamentals that manifest the «quasi-fifth» tension of the M3 they contain or, when ^{7} also appears, the tension of its tritone, being the fundamental crossed (virtual fundamental) or not (fundamental that exists in the chord). These fundamentals give us a lot of information about the harmonic move and the tonal vectors that are created. For example, to find a tonal vector we simply have to look a capital letter with a ^{7} or a

M2 interval between fundamentals, the first may be lower case or upper case, but the second should be functional (represented with a capital letter). For example, the fundamentals **a** and B, or A and B, create a tonal vector towards E, which will define a major or minor mode depending on whether its major or minor third appears in that fragment. Also B^7 or B^A directly defines a tonal vector towards E, and in a weaker way also $\dot{B}^7$ o $\dot{B}^A$.[2]

[2] If one uses a computer, this work of finding the fundamentals could be done by the same score editor software, such as the plugin I made for the Sibelius program. I can provide it free to any reader interested in it.

Annex 1

8. Morphogenesis of chords and scales

This annex is almost identical to the chapter entitled «*Morfogènesi dels acords i de les escales musicals*» in my book *La convergència harmònica*, published in 1994.

At the end of this annex we have written up harmonic tables where are represented all (!) the chords and scales that could be formed with the 12-tone equal tempered system, gruped according the number of notes they contain (eliminating repeated notes in other octaves) and according to their harmonic structure. That is, we have represented all ‘chord-classes’ and all ‘scale-classes’.

What appears in the tables are chords and scales as representantives of a wide family of musical statements, what is known as ‘chord classes’ or ‘scale classes’ (defined in 8.1 and 8.2). Depending on the horizontal or vertical musical thought each group of notes could be considered as chords (even if they are written as scales) or arranged as scales (even if they are written as chords). It is only for reasons of clearity that the groups of up to six notes are printed as chords while those of seven note or more appear as scales.

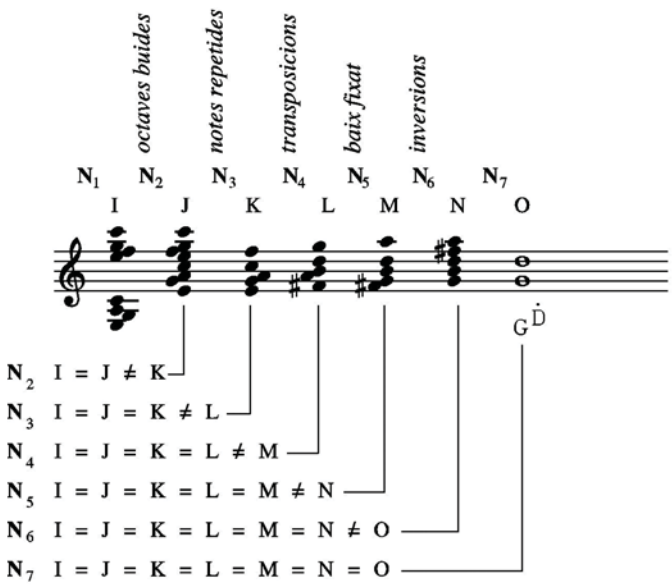
We will see that at certain equivalence level between inversions and modes the number of chords and scales that could be formed is the same so they could have the same harmonic representative which is the one that appears in the chord/scale class tables.

8.1 Equivalence level between chords

In musical theory or analysis, to say that two chords or two scales are the same could have many different interpretations. Normally, one trusts the reader’s own musical judgement to deduce the correct equivalence level of the two chords. On a more precise level, two chords are the same only if they are in the same octave, have the same notes and layout. On another level, two chords are equivalent if they have the same notes but this equivalence is only stated by the bass. Also, if one chord is the transposition of the other, if it is the inversion, and so forth.

The different equivalence levels and the number of (different) possible chords in each equivalent group (each level includes the previous equivalence level) are shown in Figure 73.

Fig. 73



Nombre d'acords segons els nivells d'equivalència

núm. notes	N_2	N_3	N_4	N_5	N_6
n	12^n	$\frac{12!}{(12-n)!}$	$\frac{11!}{(12-n)!}$	$\frac{11!}{(12-n)!(n-1)!}$	$\frac{N_5 + c(n)^*}{n}$
1	12	12	1	1	1
2	144	132	11	11	6
3	1.728	1.320	110	55	19
4	20.736	11.880	990	165	43
5	248.832	95.040	7.920	330	66
6	2.985.984	665.280	55.440	462	80
7	$3{,}58 \cdot 10^7$	3.991.680	332.640	462	66
8	$4{,}30 \cdot 10^8$	19.958.400	1.663.200	330	43
9	$5{,}16 \cdot 10^9$	79.833.600	6.652.800	165	19
10	$6{,}19 \cdot 10^{10}$	$2{,}39 \cdot 10^8$	19.958.400	55	6
11	$7{,}43 \cdot 10^{11}$	$4{,}79 \cdot 10^8$	39.916.800	11	1
12	$8{,}91 \cdot 10^{12}$	$4{,}79 \cdot 10^8$	39.916.800	1	1
Total	$9{,}73 \cdot 10^{12}$ †	$1{,}30 \cdot 10^9$ †	108.505.112	2048	351

* *c(n)=nombre d'acords cíclics de n notes*

† *continua per n>12*

When working with the 12 tone equal tempered gamma a first equivalence already assumed and not specified in figure 73 is the enharmonic equivalence. That doesn't mean indifference when representing the same note in a way or another, on the contrary, following the thesis of our research, the separation of chords into fundamentals implies a precise harmonic notation, although sometimes a same pitch can be written differently, according to the harmonic or melodic context.

On the N_1 level, two chords are equivalent only if they are exactly the same; at this level, the total number of chords is not specified because it is almost unlimited.

On the N_2 level, two chords are equivalent if they coincide with the removal of empty octaves or doing strict transpositions of octave (of the whole chord). The number of possible chords in this group is still very wide (thousands of billions).

On the N_3 level, the previous equivalence is included and the duplicated upper notes can be eliminated or upper octaves of individual notes added without affecting the chord's identity. The number of chords in this case is more that a thousand million (1.302.061.344 chords).

On the N_4 level, the previous equivalences are included and two chords can be identified if one is the strict transposition of the other. The number of possible chords in this group is still very high (108.505.112 chords).

On the N_5 level, the previous equivalences are included and two chords can be considered equivalent with any internal order of notes except for the bass. In this group we could include the triadic chords of traditional harmony and their inversions (as non-equivalent chords). The number of possible chords in this level is considerably reduced (2048).[1]

On the N_6 level, the previous equivalences are included and two chords can be identified if one is the inversion of another one. In fact, at this level, a chord (also known as 'chord class') can be considered as a non-ordered collection of notes or, better to say as a collection of intervals (bearing in mind the N_4 equivalence). The number of different chords is only 351 which are the ones represented on the Chord/scale classe tables. They would be the chords in any arrangement/order and in any inversion. It is curious that the numbering is symmetrical with respect to the number of notes they contain, the chords of 6 notes as central axis and more numerous (80). Sixty for the 5 and 7 notes, 43 for the 4 and 8 notes, etc. **When we talk about 'chord class' we are referring to this chord group at this equivalence level.**

Moreover, a N_7 level could be considered, made up of the *"chords"* that constitute the fundamentals. The number of functional chords is not precise because on a certain level of complexity the chords can be decomposed into different convergent chords (fundamentals).

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[1] Are those that appear numerically in Table 5.

In general, we could classify chords into eight large functional families (see 3.1) and the symbology to represent them can be summarized in the formula $X^{Y}{}_{Z}$ (see 3.1 and 8.3), where X, Y and Z represent the fundamentals.

8.2 Equivalence level between scales

Taking into account that if we put the notes of a chord in horizontal form, we obtain a scale, the equivalence levels that we have seen in 8.1 also apply to scales and modes.

There is an equivalence relation between the concepts ‘inversion of a chord’ with the ‘mode of a scale’.

We have 351 ‘chord classes’ and 351 ‘scale classes’. If in a scale we fix a start note, we get one mode of this scale (just like if we change the bass of a chord, we get the different inversions of this chord). For example, the white keys of a piano (and their transpositions) form a ‘scale class’, if to this ‘scale class’ we fix different starting notes, we obtain different modes of this scale (known as medieval modes). A scale class has as many different modes as different notes have the scale (with the exception of cyclical scales, see Annex 3). A 7-note scale class has 7 modes. As there are 66 different kinds of 7-note scale class we will have a total of 462 different modes of 7 notes (66 x 7). Regardless of the number of notes, we have a total of 2 048 different possible modes, which coincides with the different possible chords in the equivalence level N_6, formed by ‘chord classes’.

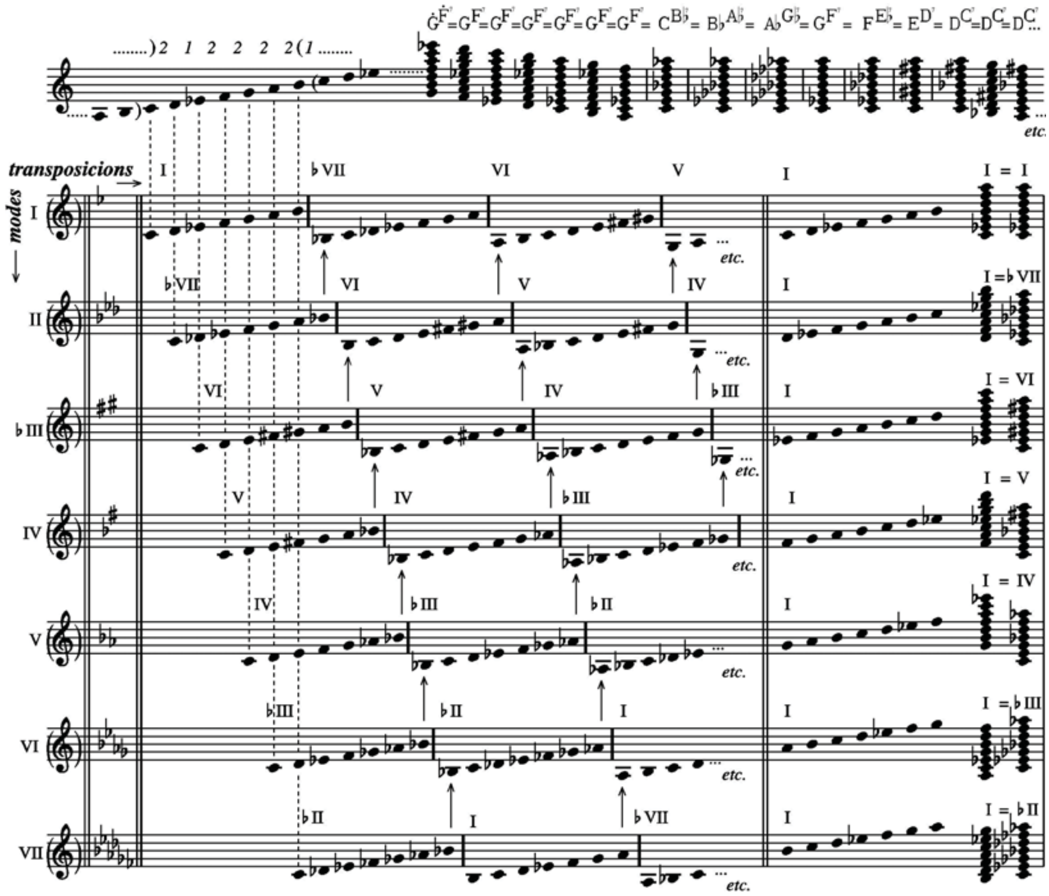
In Annex 2 we can see a detailed study of the 56 modes of the first eight ‘scale classes’ of seven notes, which are the ones most used in the musical history of all human cultures.

In Figure 74 we give an example of a ‘chord/scale class’ of seven notes with their different manifestations as scales, modes and chords (and inversions); all set of notes that appear in figure 74, seemingly distinct, correspond to a single ‘chord/scale class’ (the one corresponding to number 2 in the table of 7-note chord/scales classes), represented (in the tables) by the chord $\dot{C}^{\dot{B}\flat 7}$.

We could say that different note sets form part of the same ‘chord class’ (or ‘scale class’) if they can be ordered or reduced to the same intervallic pattern. All collections of notes belonging to the same ‘chord class’ can be decomposed into the same sub-chords, represented by the fundamentals, and in most chords up to six notes, are determined by their characteristic fundamental symbology (remember 3.1). It could also be said that a ‘chord/scale class’ is formed by a chord with all its inversions and transpositions in any vertical or horizontal combination. In figure

74 we have a lot of chords and scales but only one ‘chord/scale class’ (the number 2 of those with 7 notes) and can be decomposed into two dominant seventh chords at M2/m7 distance, that is, in two main fundamentals. Tonally one fundamental represents the function of dominant and the other one (a lower M2) the subdominant function, creating a clear tonal vector.

Fig. 74



The decomposition/separation of scales and chords into fundamentals (which basically represent the internal tensions of the ‘quasi-fifths’) gives us a lot of information on the resolutive properties of the chord or scale.

For example, as we have said, the scale in figure 75 (chord class 7-2) has a tendency to resolve towards A♭ or, in other words, A♭ is its tonic because the fundamentals of this chord/scale class are precisely the dominant and the subdominant of A♭ (enharmony C♭=B). But, at first sight, it may appear that the melody belongs to C minor. In fact, this tendency to C exists but it is only secondarily as a Phrygian homotonic resolution of the secondary fundamental (D♭^{7}→c), like the secondary resolution to G♭ (htonal homotonic relaxion from D♭) (D♭^{7}→g♭), even if it is not a note of the scale (Figure 15).

Fig. 75

A similar thing can be said about this scale in chord form. By its tonal tendency (and by the homotonic resolution of the main fundamental) this chord, as it appears in figure 76, would have a main resolution to a chord based on a fundamental E, but, as we have seen in previous chapters, would also have other harmonic relaxions between fundamentals. The precisest harmonic notation for this chord (isolated) would have to be with D♯ instead of E♭ (I have put E♭ just to show how the convergent decomposition can help to find the melodic enharmonies). But this chord, for melodic reasons, can perfectly appear with E♭ when the resolution is due to the relaxion of the secondary fundamental on a chord based on D. For ease of understanding of Figure 76 the same notation has been used for all the examples, but in cases (a), (c), (d) and (e) it would be more correct to put D♯.

Fig. 76

Obviously, the resolutions in figure 76 are not conclusive; they merely reveal a locally harmonic relaxion[2] –in some of them, for example (e), there is *sonance* tension. I insist, once more, on the importance of distinguishing between harmonic (homotonic) local tensions, the tensions of *sonance* and the tonal tensions.

These examples, centered on this chord/scale class manifest us the utility of knowing the internal tensions that the fundamental symbology shows.

The term 'chord class' or 'scale class' should not be confused with the term PC-Set (Pitch Class Set), introduced by Milton Babbitt and developed by Allen Forte in his book The Structure of Atonal Music. Forte adds a new relation of equivalence between chords: he considers chords of the same group the symmetrical chords (the symmetrical inversion of chords), that is, for example, he considers within the same PC-Set the major triad and the minor triad or the dominant seventh chord and the Tristan chord. See more information on symmetrical chords and modes (and the relation with PC-Set) in Annex 4.

8.3 'Chords classes' and 'Scale classes' tables.

Once we have studied the meaning and utility of the 'chord/scale classes' we will explain the meaning of the chords and scales that appear in Table 4: Chord/ Scale classe Tables.

On these tables there are represented all the possible (different) chord/scale classes that could be formed with the equal tempered palette of 12 tones (that is, all possible chords –or scales– at the equivalence level N_6). They are grouped according on the number of different notes in each chord/scale class. In total there are 351 classes[3], but each group starts with an independent numbering.

The chord/scale classes up to six notes have been written vertically in chord form and from seven notes have been placed horizontally, in scale form.

The symbology chosen to represent the chords is the one already introduced in 3.1 and we call it fundamental symbology.

That is to say, the symbology takes the form X_Z^Y, where X, Y and Z are fundamentals of the chord (in capital letter if they are functional —the fundamental has its M3— or in lowercase if they are not). Y represents the possible fundamentals forming intervals of fifth, major third and minor seventh (and m2) with respect to X ('harmonic' fundamentals), and Z are the possible fundamentals forming intervals

[2] Bear in mind that if we play the examples in Fig. 76 in a continuous way, we can create a tonal field towards E, which would distort the local homotonic sensation of relaxion that we want to show.

[3] Counting also the groups of 1, 11 and 12 notes, not represented in the tables.

of lower minor third and tritone with respect to X (fundamentals in the same tonal axis).

Almost all chords up to six notes can be expressed with this symbology, but not all.

When the fundamental has its minor seventh in the chord, we put a 7 above (except when Y is precisely the minor seventh of X or Z). A functional fundamental (uppercase) with its minor seventh has a significant tonal force in the chord or scale since it implies that it contains the 7M3 structure. Complex chords, or with many notes, can sometimes be symbolized (applying $X_Z{}^Y$) in different ways, but what is really important is to find the significant functional fundamental(s) of the chord (such as those with their minor seventh).

When the fundamental has its (upper) fifth in the chord, we put a point above it (in the tables). With two exceptions: one, when Y is precisely the fifth of X and second, when Z is the lower minor third of X and X is in uppercase, in this case we will not put a point in Z because this configuration always implies that Z has its fifth and will save us work by writing the symbology of minor chords. We put a point here in these tables but it is optional in the analyzes.

When the chord has other functional fundamentals that are not reflected in the main symbol, we put them below (in lower rows if they are several). This usually happens from 5-note chords.

As representative of each chord class we have placed the one that has tone C as central fundamental (the X). Being central does not mean that it is the most significant fundamental of the chord since Y or Z can have the structure 7M3 and X does not, but in general it is so.

From 7 notes we have put the group in scale form. We have not look for functional fundamentals since they grow exponentially in number and complexity (except in the first eight of 7 notes).

Finding the functional fundamentals of a scale consist to look for the M3 and tritone intervals in it. Only in the first eight scales —which with their respective modes have been those most used in the history of music (see Annex 2)— we have specified the functional fundamentals they contain; in addition, in the inferior part, we have placed the characteristic tonal cadence of the scale towards its main tonic, using all its notes.

In the tables we have not shown groups of 1, 11 and 12 notes since there is only one chord class for these groups or, in other words, all possible "chords" (or scales) of one, eleven or twelve notes are the "same" at the equivalence level N_6.

In order not to create confusion I use the same numbering that appears in my book written in 1994, although I would now use another numbering because the first one was based on the logic of two fundamentals (in the first book I only used binary symbology type X^{Y} and X_{Z}) and now I use the form X_{Z}^{Y} in order to be able to code more chords, but, as I have said, the symbols used are of no more importance if the functional fundamentals of the chords are determined. For most chords of three and four notes (and even of five) the ‘fundamental symbology’ coincides in the two books.

8.4 How to find the fundamentals or the fundamental symbology of the chords

If there is a set of notes in the form of a chord or scale and we want to make a homotonic functional analysis with respect to the groups of notes that precede and follow it, then, the first step is to analyze their fundamentals and, therefore, know the internal tensions of the chord and its tonal influence.

The most practical way to do this is by directly analyzing the interval relationships between notes. It will be sufficient to look for the intervals of M3 and tritone. Indeed, in many cases it will be sufficient to find the M3. Once these intervals are found it is easy to deduce their functional fundamentals.

For example, if we have the chord EB♭C♯F♯A (Figure 77(a)): M3 intervals. (including enharmonic changes) are: A-C♯ and F♯-A♯(B♭). Indeed, having A and F♯ as a fundamentals, the chord is separated into two convergent (harmonic) chords. The fundamental symboly will, then, be: $A_{F\#7}$

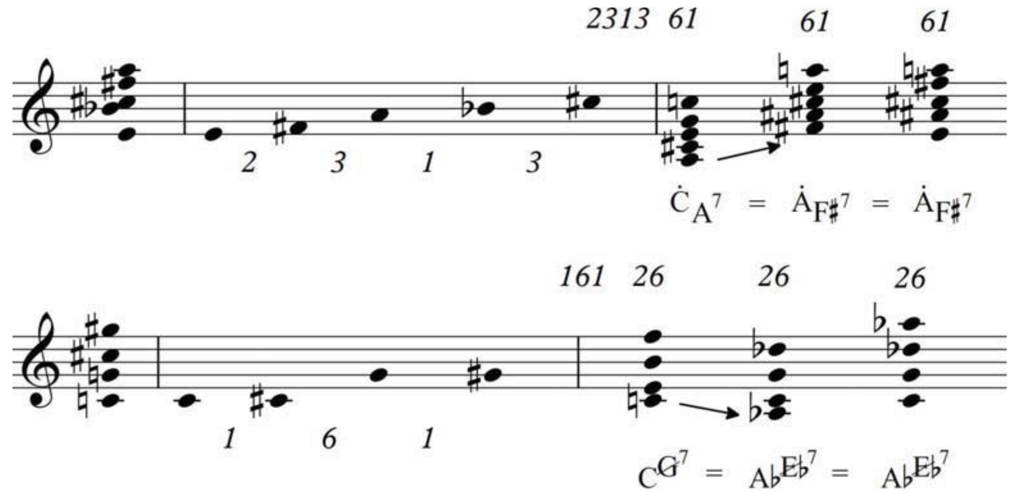
Fig. 77

Another example, if we take the chord: CGC♯G♯ (Figure 77b), it could be thought that, as the chord contains two P5, the fundamentals would be C and C♯. Actually this example is a trap that hides the true fundamentals since, as it was stated on 1.3, the strength of fifths to support a fundamental is very weak. In this chord there is only a M3 A♭(G♯)-C, the rest of the function is given by the tritone G-C♯(D♭) in its convergent direction towards E♭ (because E♭ is the fifth of A♭). In this way, the two fundamentals are A♭ and E♭ —this latter one virtual— and the fundamental symbology: $A\flat^{E\flat^{7}}$.

Only if the intervalic analysis is complicated or one is not sure of the results (or, why not, for curiosity) it is possible to look up the numerical tables (Table 5) at the end of this appendix where for every group of notes it will be easy to find their representative in the chord/scale class tables and deduct their fundamentals.

Taking as an example the same chords as before (Figure 78):

Fig.78



First of all it is necessary to put the chord in scale form (into an octave according to the order of the chromatic scale, if there are any repeated notes, they are eliminated). It is possible to start with any note of the chord (here, we show the bass as first degree of the scale to be clearer). Then one looks at the number of semitones between the different degrees in the scale just formed; this gives us one digit numbers which are considered as digits of one whole number (in this case 2313). Beside number 2313 in this Table 5 number 61 is found. This number indicates that the representative of the chord is num. 61 in the Chord/scale class tables (from the five notes group). Indeed, you just have to do a major sixth transposition of the representative in the tables to get the notes of our chord. The process for the other chord is similar, in this case we get number 26 and looking at the representative in the tables we deduce the fundamentals. A plugin for the Sibelius program that does this and other analyzes is also available.[4]

January 2020: See 'Chord analyzer (online)' :

http://www.lamadeguido.com/fundamentos/chords.htm

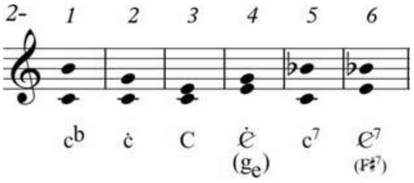
[4] The interested reader can request it free of charge to lbalsach@gmail.com

Chord/Scale class

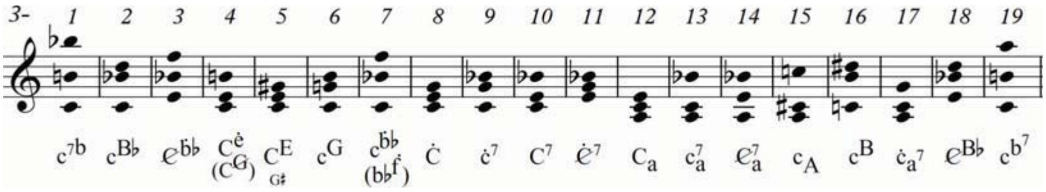
Tables

Table 4. Chord/Scale class tables

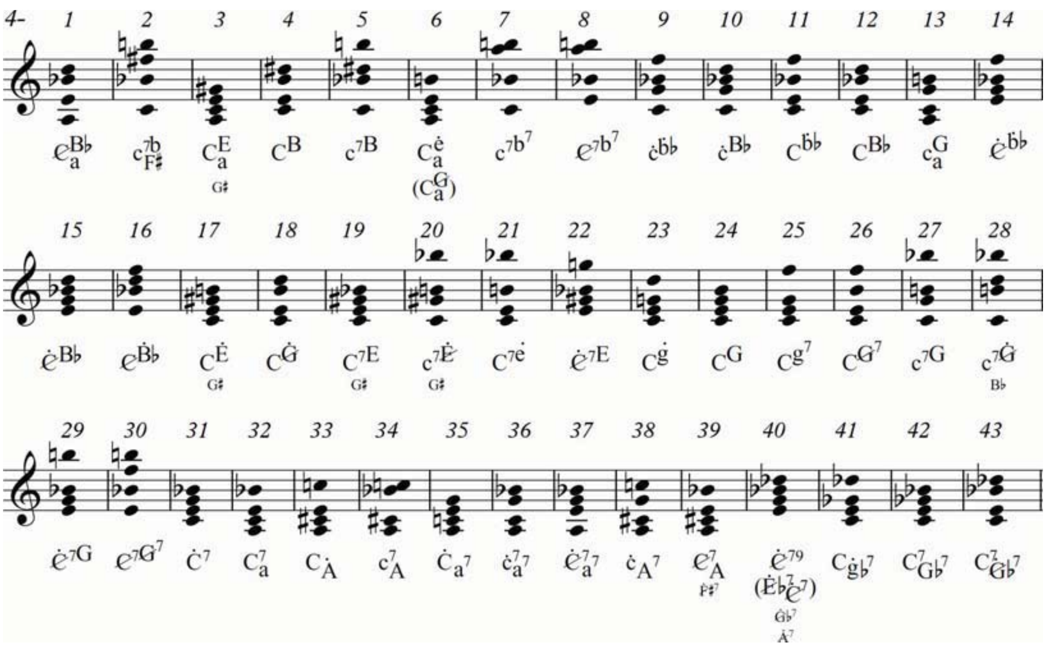
2-note chords/scales



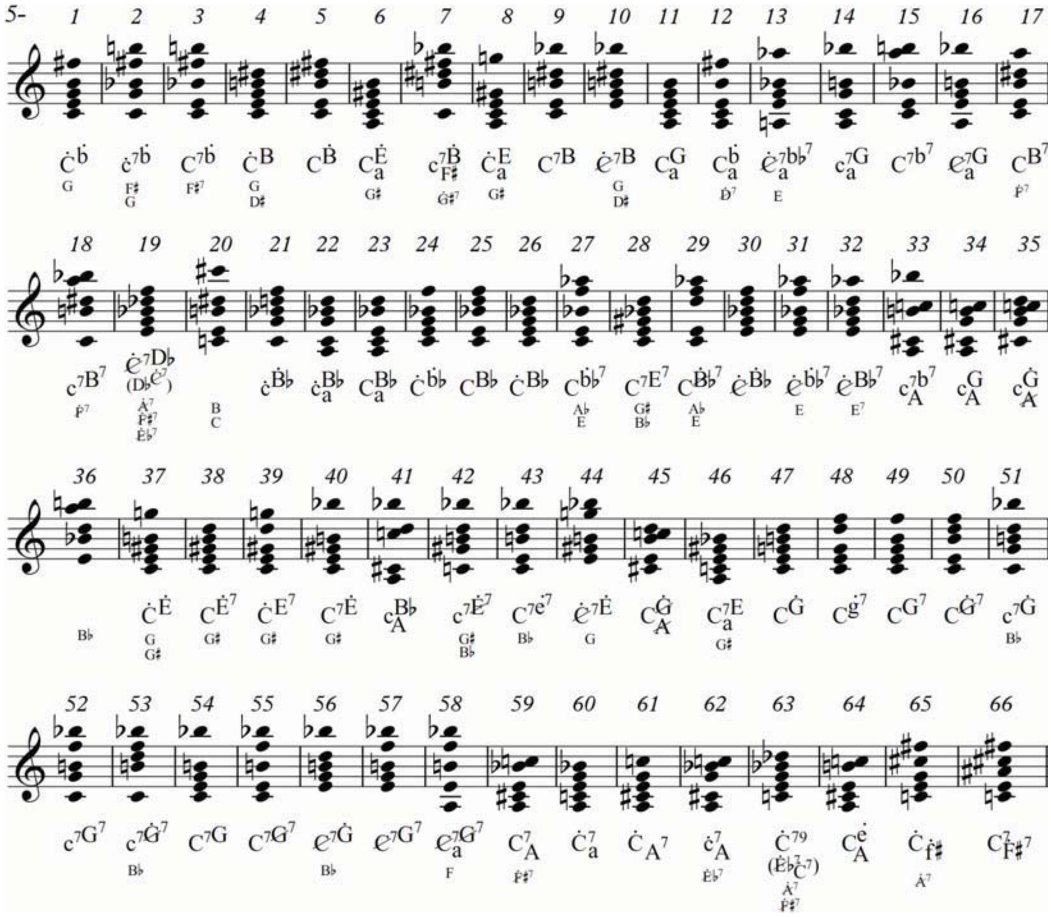
3-note chords/scales



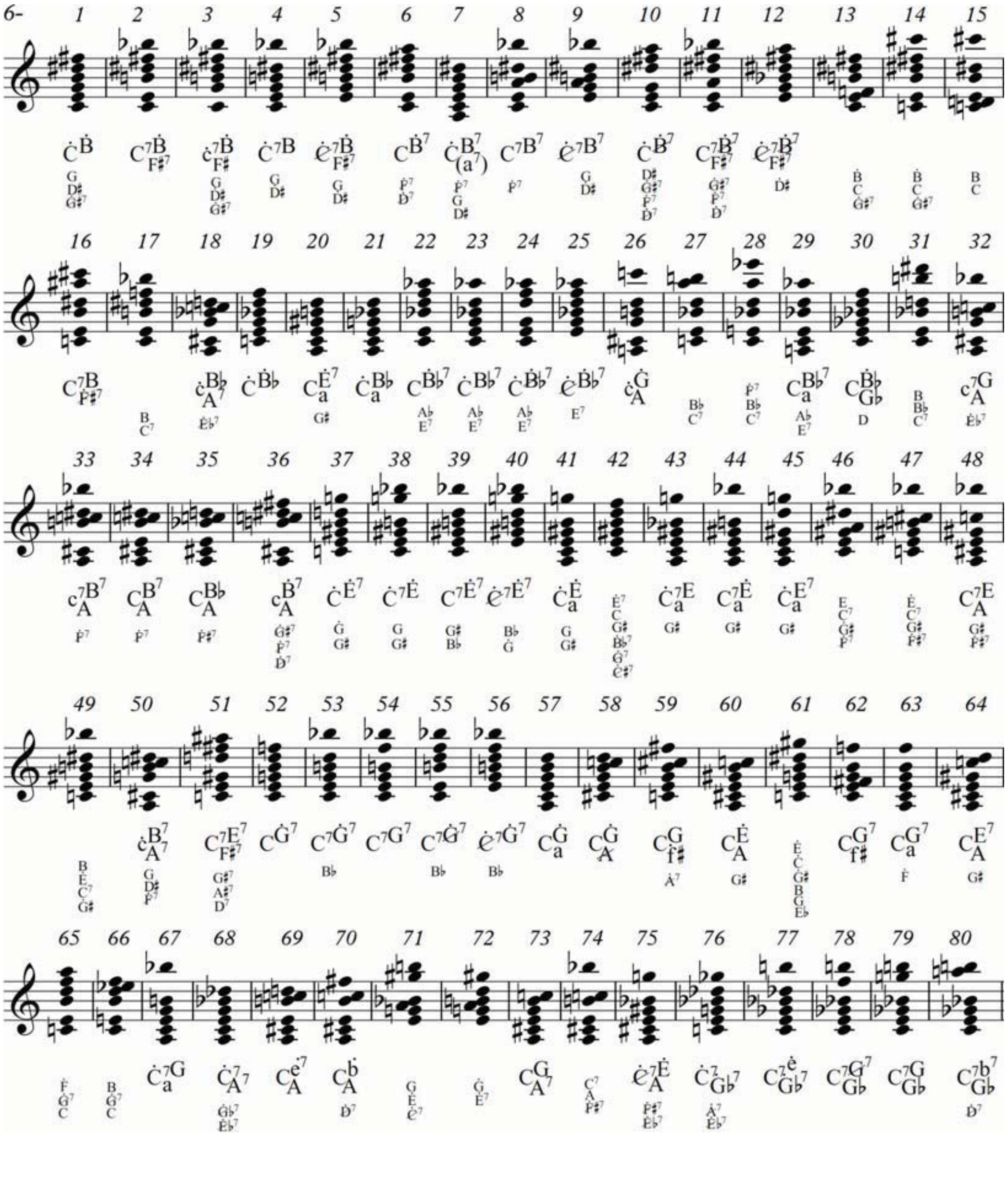
4-note chords/scales



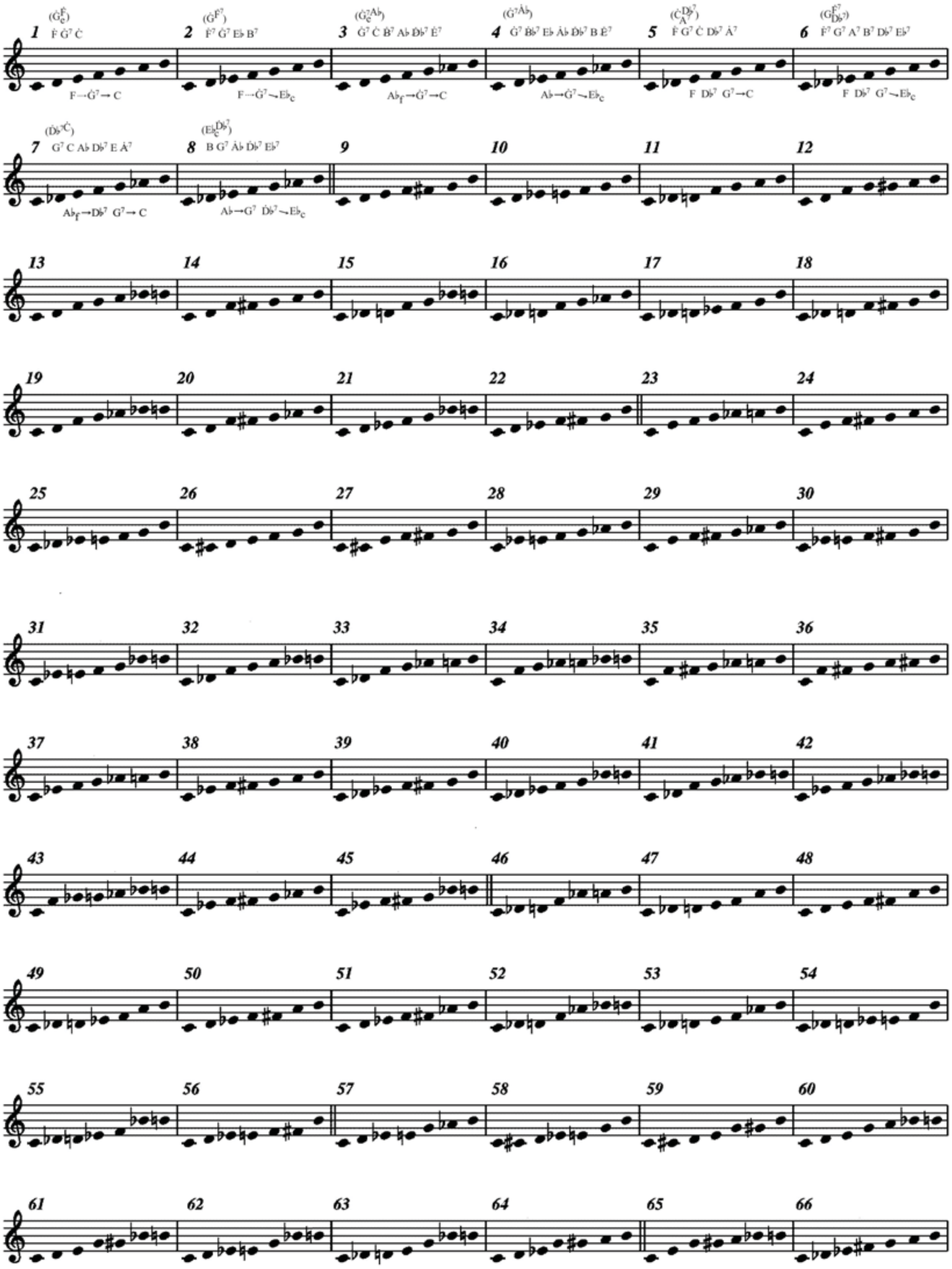
5-note chords/scales



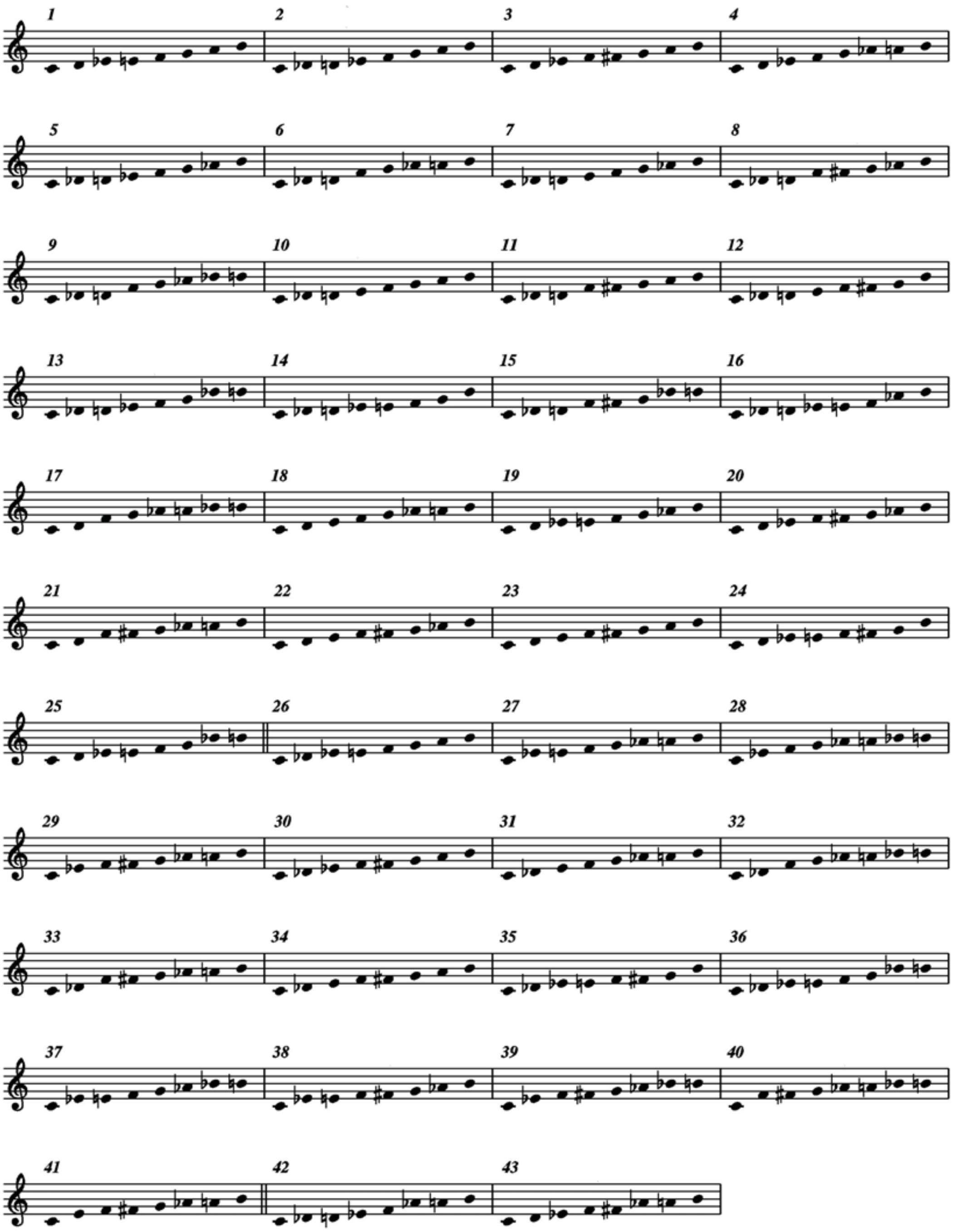
6-note chords/scales



7-note chords/scales



8-note chords/scales



9-note chords/scales

10-note chords/scales

Table 5

1	1	54	8	151	30	263	22	432	35
2	5	55	7	152	11	271	18	433	31
3	4	56	3	153	29	272	36	434	24
4	3	61	3	154	16	281	28	441	3
5	2	62	10	155	8	311	27	442	19
6	6	63	11	161	26	312	22	443	17
6	6	64	18	162	38	313	33	451	32
7	2	65	14	163	37	314	17	452	6
8	3	71	4	164	2	315	29	461	21
9	4	72	17	171	4	316	38	511	8
11	1	73	9	172	13	317	4	512	32
12	13	74	6	173	27	321	37	513	39
13	16	81	16	181	34	322	10	514	1
14	4	82	2	182	20	323	35	515	30
15	3	83	15	191	7	324	31	515	30
16	14	91	13	211	20	325	9	521	6
17	6	92	19	212	36	326	41	522	23
18	15	111	7	213	43	331	29	523	9
19	19	112	28	214	6	332	31	524	11
21	19	113	5	215	16	333	40	531	33
22	2	114	21	216	37	333	40	532	10
23	17	115	8	217	13	333	40	533	29
24	10	116	2	218	34	333	40	541	25
25	7	117	27	221	13	334	15	542	16
26	18	118	20	222	12	335	39	551	8
27	9	119	7	223	23	341	24	611	21
28	2	121	34	224	19	342	15	612	14
29	13	122	18	225	10	343	35	613	41
31	15	123	14	226	12	344	3	614	26
32	9	124	32	227	18	351	39	621	43
33	11	125	25	231	38	352	23	622	12
34	12	126	22	232	9	353	33	623	38
35	8	127	36	233	15	361	14	631	22
36	11	128	28	234	35	362	43	632	37
37	17	131	4	235	23	371	5	641	2
38	16	132	41	236	14	411	2	711	5
41	6	133	39	241	11	412	25	712	18
42	18	134	3	242	42	413	3	713	4
43	8	135	33	242	42	414	24	721	36
44	5	136	43	243	31	415	11	722	13
44	5	137	5	244	19	416	26	731	27
44	5	141	26	245	32	421	16	811	28
45	12	142	1	251	1	422	19	812	34
46	10	143	24	252	9	423	15	821	20
47	4	144	17	253	10	424	42	911	7
51	14	145	6	254	25	424	42	1111	33
52	7	146	21	261	41	425	1	1112	45
53	12	151	30	262	12	431	17	1113	18

1114	15	1233	19	1451	15	2171	20	2512	23
1115	35	1234	30	1511	55	2211	34	2513	5
1116	13	1235	53	1512	57	2212	48	2521	22
1117	14	1241	49	1513	65	2213	29	2522	48
1118	33	1242	32	1514	58	2214	11	2531	51
1121	20	1243	60	1521	17	2215	25	2611	43
1122	43	1244	46	1522	25	2216	50	2612	31
1123	53	1251	5	1523	52	2221	25	2621	42
1124	46	1252	48	1531	44	2222	28	2711	45
1125	51	1253	51	1532	16	2223	26	3111	13
1126	42	1261	31	1541	35	2224	28	3112	51
1127	45	1262	42	1611	9	2225	23	3113	8
1131	9	1271	45	1612	50	2231	39	3114	54
1132	36	1311	2	1613	2	2232	21	3115	52
1133	7	1312	5	1621	62	2233	26	3116	2
1134	8	1313	4	1622	34	2234	47	3121	44
1135	64	1314	37	1631	13	2241	27	3122	38
1136	18	1315	17	1711	20	2242	28	3123	19
1141	55	1316	9	1712	41	2243	38	3124	32
1142	3	1321	65	1721	14	2251	23	3125	5
1143	54	1322	27	1811	33	2252	22	3131	37
1144	40	1323	56	2111	14	2261	43	3132	56
1145	15	1324	66	2112	42	2311	52	3133	61
1151	58	1325	36	2113	64	2312	32	3134	4
1152	52	1331	10	2114	40	2313	61	3141	10
1153	16	1332	61	2115	16	2314	39	3142	39
1154	35	1333	63	2116	34	2315	57	3143	37
1161	2	1334	7	2117	41	2321	12	3151	57
1162	34	1341	4	2121	62	2322	21	3152	17
1163	13	1342	29	2122	22	2323	21	3161	9
1171	41	1343	8	2123	30	2324	24	3211	16
1172	14	1351	59	2124	60	2331	56	3212	60
1181	33	1352	64	2125	48	2332	26	3213	63
1211	41	1361	18	2126	31	2333	19	3214	12
1212	31	1411	58	2131	17	2341	47	3215	65
1213	59	1412	49	2132	66	2342	30	3221	11
1214	6	1413	10	2133	63	2351	53	3222	26
1215	44	1414	1	2134	29	2411	3	3223	21
1216	62	1415	55	2135	59	2412	24	3224	27
1217	20	1421	1	2141	1	2413	27	3231	61
1221	50	1422	39	2142	12	2414	49	3232	21
1222	23	1423	12	2143	11	2421	66	3233	56
1223	47	1424	3	2144	6	2422	28	3241	24
1224	38	1431	37	2151	65	2423	32	3242	66
1225	22	1432	11	2152	25	2431	38	3251	36
1226	43	1433	54	2153	44	2432	60	3311	54
1231	57	1441	6	2161	50	2441	46	3312	19
1232	24	1442	40	2162	62	2511	36	3313	56

3314	10	4411	46	11123	58	11324	77	12161	16
3321	63	4412	6	11124	43	11331	3	12211	28
3322	26	4421	40	11125	69	11332	73	12212	52
3323	61	4511	15	11126	33	11333	75	12213	42
3331	19	5111	15	11131	8	11341	60	12214	65
3332	63	5112	53	11132	77	11342	47	12215	55
3341	7	5113	36	11133	75	11351	74	12221	63
3411	7	5114	55	11134	47	11411	78	12222	23
3412	47	5121	59	11135	74	11411	78	12223	21
3413	4	5122	23	11141	62	11412	54	12224	29
3421	30	5123	57	11142	80	11413	9	12231	37
3422	29	5131	5	11143	67	11414	62	12232	57
3431	8	5132	65	11144	44	11421	79	12233	53
3511	53	5141	58	11151	13	11422	30	12241	20
3512	59	5211	64	11152	26	11423	80	12242	39
3521	64	5212	22	11153	71	11431	38	12251	27
3611	18	5213	17	11161	66	11432	67	12311	54
4111	35	5221	48	11162	32	11441	44	12312	11
4112	46	5222	25	11171	15	11511	17	12312	11
4113	7	5231	52	11211	31	11512	28	12313	56
4114	3	5311	51	11212	14	11513	13	12314	46
4115	58	5312	44	11213	48	11521	18	12321	6
4121	6	5321	16	11214	41	11522	26	12322	19
4122	47	5411	35	11215	34	11531	71	12323	72
4123	24	6111	18	11216	16	11611	31	12331	10
4124	49	6112	43	11221	55	11612	66	12332	36
4131	4	6113	9	11222	29	11621	32	12341	58
4132	27	6121	31	11223	53	11711	15	12411	2
4133	10	6122	50	11224	39	12111	66	12412	52
4141	49	6131	2	11225	27	12112	35	12413	64
4142	1	6211	42	11231	46	12113	60	12421	25
4151	55	6212	62	11232	72	12114	38	12422	50
4211	40	6221	34	11233	36	12115	18	12431	43
4212	30	6311	13	11234	58	12116	31	12511	14
4213	66	7111	45	11241	64	12121	12	12512	35
4214	1	7112	20	11242	50	12122	20	12521	69
4221	29	7121	41	11243	43	12123	10	12611	33
4222	28	7211	14	11251	35	12124	25	13111	13
4223	39	8111	33	11252	69	12125	14	13112	64
4231	32	11111	15	11261	33	12131	5	13113	3
4232	12	11112	33	11311	17	12132	40	13114	79
4241	3	11113	74	11312	2	12133	68	13115	17
4311	8	11114	44	11313	4	12134	48	13121	1
4312	38	11115	71	11314	49	12141	1	13122	37
4313	37	11116	32	11315	8	12142	24	13123	6
4321	60	11117	15	11321	59	12143	41	13124	2
4322	11	11121	16	11322	45	12151	12	13131	61
4331	54	11122	27	11323	70	12152	34	13131	61

13131	61
13132	7
13133	4
13141	5
13142	49
13151	8
13211	9
13212	56
13213	76
13213	76
13214	59
13221	7
13222	22
13223	45
13231	40
13232	70
13241	77
13311	4
13312	42
13313	3
13321	68
13322	73
13331	75
13411	48
13412	60
13421	47
13511	74
14111	62
14112	46
14113	59
14114	78
14114	78
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Annex 2

9. Modes of the first eight 7-note scale classes

The first eight 7-note scale classes have in common that the notes of their modes can be written as different chromatic alterations of the major mode diatonic scale. In fact, this could be done for the majority of 7-note scale classes (forcing the enharmony to the extreme) but, except for some cases, only with the first eight scale classes can this be done without adversely affecting the harmonic significance of the notes.

For example, the C-D-F-G-A♭-B♭-B♮ scale (chord/scale class #19) could be written as a chromatism of the diatonic scale as follows: C-D-E♯-F**x**-G♯-A♯-B or, another example, C-D♭-D♮-F-G-A-B (chord/scale class #11) could be as well written as C-D♭-E♭♭-F-G-A-B. But in both cases there could be an inaccuracy regarding the harmonic meaning of the notes because both scales contain the convergent structure (GBDF) which is the one that establishes the tonic C. So, in the first case E♯ and F**x** should be F and G and in the second case E♭♭ has to be D♮ (Figure 79).

Fig.79

The fact that the first eight scale classes (and their modes and transpositions), could be written as major diatonic scale chromaticisms is very useful because there is no need to put notes to show the scale classes modes, it is enough to indicate the key signature on the stave. This saves space and allows to express the harmonical relationships between all the modes of the eight scale classes in a clear way.

The meaning of the Modes Table 6 in this appendix is as follows:

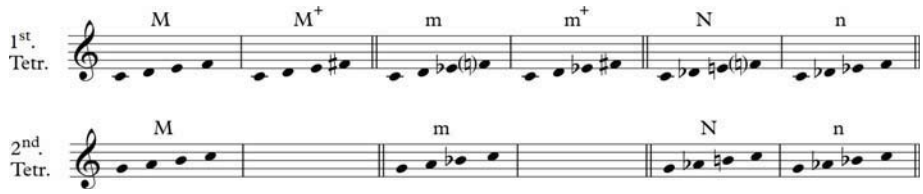
Each column belongs to a scale class and each row to a mode. Even if there are 13 rows, there is only 7 authentically different modes for each column (those marked with *), the rest are their chromatical transpositions (all those having C♯ or C♭ on the key signature –the roman number in parentheses shows the original mode–).

On the central row, belonging to I, there are the representatives of each scale class as shown in the Chord/scale class tables. These are the most stable modes in the sense that they start with the most tonicizated note. The left roman numbers show the mode degree in relation to their position in the representing scale. The right degrees show the transposition of the modes.

To know a mode, it is enough to form a heptatonic scale, starting by C and put the accidentals indicated on the key signature. For example, 7th. column, row 6 is the mode of the degree IV from scale class #7 and has as a key signature F♯, E♭ and A♭. The scale C-D-Eb-F♯-G-A♭-B will represent the structure of this mode (named gipsy or hungarian mode). The transpositions of this mode to another tone could be easily known looking at the degrees that appear on the right hand side of the table, bearing in mind that each row represents a 5th. interval. Like that, if one wants to transpose this mode a rising major 2nd. it will be sufficient to look two rows higher (or find the degree V+M2=VI –on the right–) and see the key signature C♯, G♯ and B♭. But now, instead of starting by C it is necessary to start obviously by D. So the scale D-E-F-G♯-A-B♭-C♯ is the same mode applied to D (C+M2) (See also Figure 74 on Annex 1).

The modes that have the perfect fifth from the initial note can also be studied and classified from the point of view of the two tetrachords:

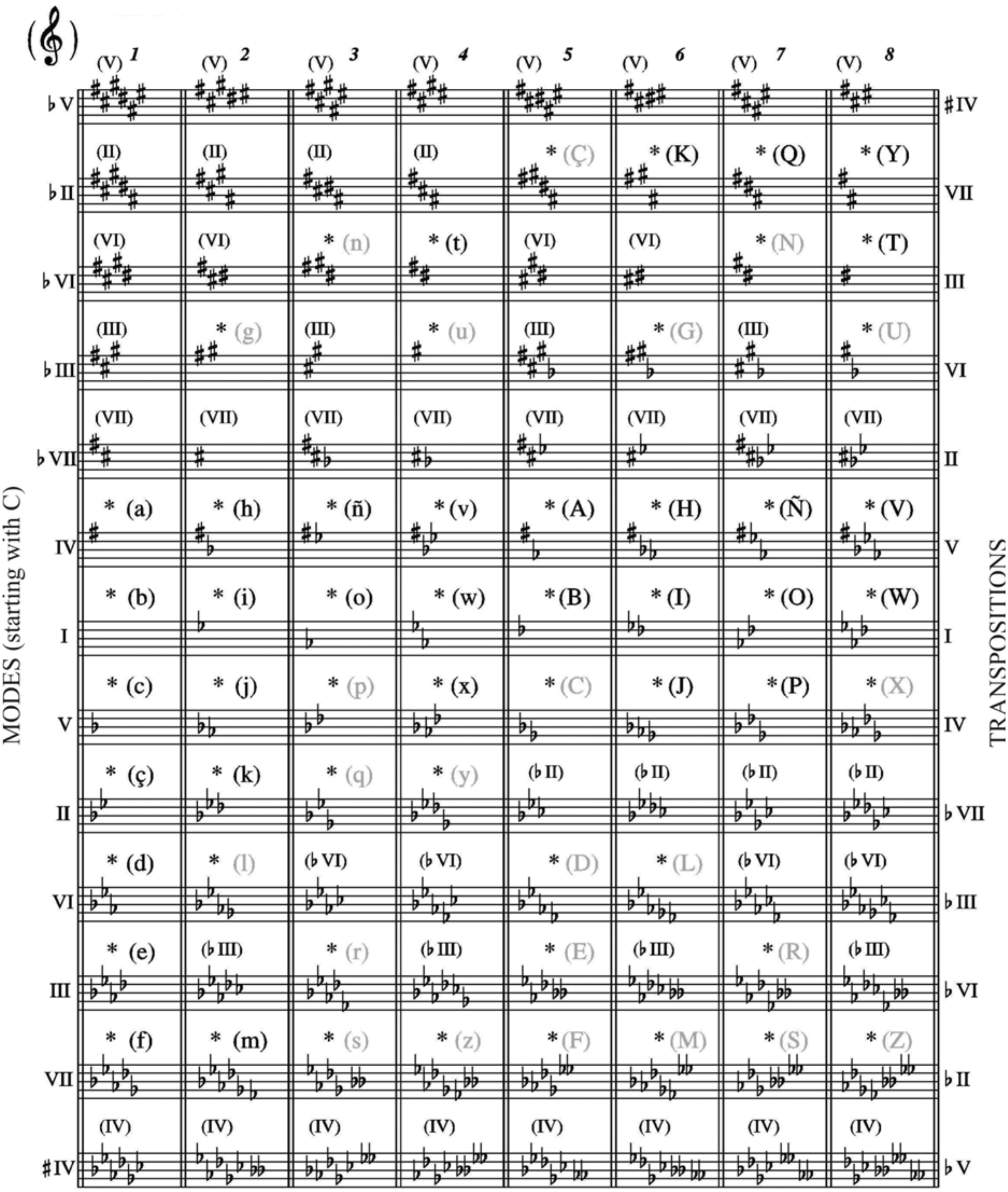
Fig. 79bis



The division of the octave into seven sounds or intervals has been one of the most common in all musical cultures. Using only the modes of the first eight 7-note scale classes we obtain a wide range of expressions and musical colours. The names of some of these modes are listed on Table 7 (assuming the needed tune adjustments in each case) according to the musical theories of different countries.[1] Other known modes, particularly the Hindus, don't appear because they are in other chord/scale classes. We also indicate in this table its separation in tetrachords according to figure 79bis.

[1] As far as the ecclesiastical modes after the *Oktoechos* are concerned, the names appearing in the *Dodecachordon* (1547) of Glareanus have been used, grouped according to the criterion of *finalis*=tonic (1st degree). The same criterion has been followed in Iranian-Arabic music over the 12 *dastgah-ha*. Hindu modes have been extracted from the 72 *malakartas* of the (south) carnatic theory according to Venkatamakhi and the 10 *thate* of the Hindu (north) theory according to Bhathhande. All this information has been collected from The New Grove Dictionary of Music (London), *Encyclopedie de la Musique* (Paris), Danielou (1943) and sporadically from other sources. There are small divergences, which are manifested with a question mark.

Table 6. Modes of the first eight 7-note scale classes



* true mode stating with C♮

(VI) chromatic transposition of true mode VI

Tabla 7

1-IV	(a)	$M^{+}M$	Modo de Fa, Hipolidio griego, Tritus, Lidio (plagal hipolidio), *Méshakalyâni, Kalyana, Gaur-Sàrang*, ***ichikotsucho (ryo), kung tiao***
1-I	(b)	MM	Modo mayor, Lidio griego, Jónico (plagal hipojónico), *Dhira-shankarâbharana, Bilaval*, **segah**, ***chin tiao***
1-V	(c)	Mm	Modo de Sol, Hipofrigio griego, Tetrardus, Mixolidio (plagal hipomixolidio), *Hari-kâmbhoji, Matsaríkrta, Khammaj(?),* **bayat-e tork (mahur?), rast**, ***shang tiao***
1-II	(ç)	mm	Modo de Re, Frigio griego, Protus, Dórico (plagal hipodórico), ruso menor, *Kharahara priya, Sudda Sadja, Kâfi(?),* ***hyojo (ritsu), yü tiao***
1-VI	(d)	mn	Modo menor natural o descendente, Hipodórico griego, Eolio (plagal hipoeolio), *Nata-bhairavi, Asâvari,* **Isfahân (afshari?),** ***chüeh tao***
1-III	(e)	nn	Modo de Mi, Dórico griego, Deuterus, Frigio (plagal hipofrigio), *Hanumat-todi, Bhairavi*, **shur (nava),** ***zokuso, pien kung tiao***
1-VII	(f)		Modo de Si, Mixolidio griego, Locrio, **Shahnaz (dashti?),** ***pien chih tiao***
2-IV	(h)	$M^{+}m$	Modo de los armónicos II (Scriabin, Albrecht, Szymanowski), Modo de Podhale (Polonia), Acústica (Bartók), *Vâchaspati*
2-I	(i)	mM	Menor ascendente, “Hawaiano”, *Gauri-manohari*
2-V	(j)	Mn	Mayor-menor, *Châru-késhi*
2-II	(k)	nm	*Nâtaka-priya*
2-VII	(m)		Super Locrio
3-IV	(ñ)	$m^{+}M$	*Dharmavati*
3-V	(p)	Nm	Menor armónico inverso, *Chakravâka*
3-I	(o)	MN	Mayor armónico, Modo de Hauptmann, *Saransángi*
4-bVI	(t)		*Kosala*
4-IV	(v)	$m^{+}m$	Dórico ucraniano, *Haimavati*, **Homayun**
4-I	(w)	mN	Menor armónico, Gitano español, Andaluz, *Kiravâni*, **bayat-e Esfahan**
4-V	(x)	Nn	Frigio dominante, *Vakulâbharana*, **Shad Araban, Alhijaz**
4-VII	(z)		Ultra locrio
5-IV	(A)	$M^{+}N$	*Latângi*
5-I	(B)	NM	*Sûrva-kânta*
5-III	(E)		*Senâpati*
5-VI	(D)		Sabach (Grecia)
6-IV	(H)	$M^{+}n$	Modo de los armónicos I, *Rishabha-priya*
6-I	(I)	nM	Napolitano mayor, Círculo cerrado de cuartas según J.Darias, *Kokila-priya*
6-V	(J)		Locrio mayor
6-bII	(K)		Escala de tonos con sensible
6-VII	(M)		Escala de tonos con sensible descendente
7-IV	(Ñ)	$m^{+}N$	Gitano menor, Húngaro menor, Doble armónico menor, Niavent (Grecia) *Simhendra-madhyama*
7-I	(O)	NN	Doble armónico, Bizantino, Gitano mayor, *Mâyâ-malava-gaula, Bhairav*, **Tchahârgah**
7-V	(P)		Modo de Wollet, Oriental, Tsinganikos (Grecia), *Raga Ahira-Lalita*
7-bII	(Q)		*Râsika priya*
8-bVI	(T)		*Shûlini*
8-IV	(V)	$m^{+}n$	Modo gitano húngaro (?), *Shanmukha priya*
8-I	(W)	nN	Napolitano menor, *Dhenukâ, Todi (?)*
8-bII	(Y)		*Chitrâmbari*

Redonda: Modos occidentales
Cursiva: Modos indios/hindús
Negrita: Modos irano-árabes
Negrita-cursiva: Modos japoneses o chinos

Annex 3

10. Cyclical chord/scale classes

It would seen logical to think that the number of possible inversions of any chord must be the same as to the different number of notes it contains, or that the number of modes of a scale would also be the same as the number of notes of the mode. This is not always true on account of the internal cyclical chord/scale classes; in those cases there are some inversions or modes that coincide. For this reason the number of chords in the N_5 level of equivalence (see Figure 73) is not always the same as the number of chord classes in N_6 level multiplied by the number of notes. This is only true for the chord/scale classes which never have a cyclical structure, i.e. the chords/scales of 5, 7 and 11 notes.

In this annex (Table 8) there are represented all the cyclical and semi-cyclical chord/scale classes (the left number means its classification in the chord/scale Tables), that is to say, all the chords or scales that have some or all of their inversions or modes with an identical structure –concerning the internal intervals–.

The *modes à transpositions limitées* introduced by O. Messiaen in his book *Technique de mon langage musical* (1944) are in fact some of these cyclical chord/ scale classes. Messiaen named these modes from 1 to 7. Mode 1 is equivalent to 6-notes chord/scale class #51 (scale of tones), mode 2 is related to 8-notes chord/ scale #43 (octatonic scale), mode 3 is related to 9-notes chord/scale #19, mode 4 to 8-notes chord/scale #8, mode 5 to 6-notes chord/scale #78, mode 6 to 8-notes chord/ scale #30 and mode 7 to 10-notes chord/scale #3.

Messiaen named these scales as modes of limited transposition because if they are successively transposed by semitones there is always a moment where the same scale (the same notes) is found –obviously, before getting to the transported octave– . For example, if the transposition of scales of mode 2 (8-43) is made by minor third the result is always the same notes or mode 1 (6-51) by major 2nds., and so on. All this is also true for all the modes in Table 8.

Tabla 8

2 notas

3 notas

4 notas

6 notas

8 notas

9 notas

10 notas

12 notas

Annex 4

11. Symmetrical modes and chords

Two study groups are distinguised: the symmetry between chord/scale classes and the symmetrical internal structures in one chord or scale.

11.1 Symmetry between chords

For simplicity, when referring to chords, I mean 'chord classes'.

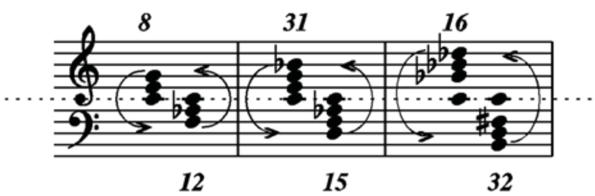
It is commonly used the term 'inversion' to speak of the symmetrical chord (or mode); but we know that this word also has another meaning to differentiate chords. This duplicity of meanings can cause confusion; in this book whenever the word 'inversion' appears we are referring to the traditional term used in books of harmony to distinguish the different dispositions of the chords according to the note that appears in the bass. Here I will use the term 'symmetric' instead of 'inversion'.

In figure 80 we have some examples of symmetrical chords (symbolically it would be like a chord and its image in a mirror), they are formed by the same intervals in inverse order:

Fig. 80

A characteristic of symmetrical chords is that the symmetry of any inversion of a chord class is at the same time an inversion of the symmetric chord class. So, a chord class has only one symmetrical chord class, despite the notes layout.

Table 9 shows the relation between symmetric chord classes –following the numeration on the Chord/scale class tables (two first numbers)–. The third numeration (third column, from 3-note group) is the correspondence with the Set Theory, specifically with the numbering given by Allen Forte (PC-Set) in his book The



Structure of Atonal Music (1973). In brackets there is the real number of different symmetries –shown by an asterisk– (the symmetries that do not have an asterisk are just a repetition of a symmetry already marked –in order to facilitate their searching–). The chord classes with two asterisks are doubly symmetric, they have internal symmetry and so the symmetric chord is itself. In fact they are union of two symmetrical chords (with respect to a common note).

Set Theory, which enjoys a certain diffusion, could be considered as a dualistic theory carried to its extreme since it considers a chord and its symmetrical one as a «same class» of chord. For example, it establishes an equivalence or parallelism between the major triad and the minor triad, between the dominant seventh chord and the Tristan chord, etc. This equivalence does not fit with the functionalities of the chords in our theory (since the symmetry of harmonics does not appear in nature) nor do I think it could be applicable to tonal music but it can be an interesting system of analysis and a method to compose atonal music, for which it was thought. Because, although it does not come from a natural phenomenon, we do not rule out that the brain is capable of perceiving these relations (although we do not affirm it *a priori*).

If we consider this new equivalence between chords (equivalence between symmetric chords), a new level of equivalence could be added in figure 73, which we could call N_8 (equivalent chords discarding order between notes, transpositions, inversions and symmetries), and the possible numbers of chords —for each group of notes— are those that appear in parentheses in Table 9, i.e.: 1, 6, 12, 29, 38, 50, 38, 29, 12, 6, 1, respectively, for the groups of 1 to 11 notes.

Curiously, as happens at the N_5 and N_6 levels of equivalence (see Figure 73 of Annex 1), the number of symmetric chord classes (or PC-Sets) is also symmetrical (1, 6, 12, 29, 38, 50, 38, 29, 12, 6, 1), and the same symmetry is obtained in the number of double symmetrical chord classes (1, 6, 5, 15, 10, 20, 15, 5, 6, 1). Among other things this fact is a manifestation that the symmetric chord class of the complementary chord class is the complementary chord class of the symmetric chord class. Understanding as a complementary chord of a chord **x** the new chord that has (only) all the notes that are missing to the chord **x**.

January 2020: See 'Chord analyzer (online)' :

http://www.lamadeguido.com/fundamentos/chords.htm

Table 9

1-note (1)

**1	1

2 notes (6)

**1	1
**2	2
**3	3
**4	4
**5	5
**6	6

3 notes (12)

**1	1	3-1
**2	2	3-6
*3	14	3-5
*4	6	3-4
**5	5	3-12
6	4	
**7	7	3-9
*8	12	3-11
*9	17	3-7
*10	18	3-8
**11	11	3-10
12	8	
*13	19	3-2
14	3	
*15	16	3-3
16	15	
17	9	
18	10	
19	13	

4 notes (29)

*1	11	4-16
*2	21	4-5
*3	17	4-19
**4	4	4-7
*5	27	4-4
*6	25	4-14
**7	7	4-1
**8	8	4-6
**9	9	4-23
*10	23	4-22
11	1	
**12	12	4-21
*13	18	4-11
*14	37	4-13
*15	31	4-27
*16	32	4-Z29
17	3	
18	13	
**19	19	4-24
*20	28	4-2
21	2	
*22	43	4-12
23	10	
**24	24	4-20
25	6	
**26	26	4-8
27	5	
28	20	
*29	39	4-18
**30	30	4-9
31	15	
32	16	
**33	33	4-17
**34	34	4-3
**35	35	4-26
**36	36	4-10
37	14	
*38	41	4-Z15
39	29	
**40	40	4-28
41	38	
**42	42	4-25
43	22	

5 notes (38)

*1	49	5-20
*2	9	5-6
**3	3	5-15
*4	37	5-21
*5	17	5-Z18
**6	6	5-Z17
*7	54	5-Z38
**8	8	5-Z37
9	2	
**10	10	5-22
*11	47	5-27
*12	24	5-29
*13	18	5-4
*14	45	5-2
*15	35	5-5
*16	53	5-Z36
17	5	
18	13	
*19	63	5-31
*20	41	5-3
**21	21	5-35
*22	48	5-23
*23	25	5-24
24	12	
25	23	
**26	26	5-34
*27	39	5-30
**28	28	5-33
*29	38	5-26
*30	60	5-25
*31	62	5-10
*32	66	5-28
**33	33	5-1
*34	43	5-9
35	15	
*36	52	5-14
37	4	
38	29	
39	27	
*40	46	5-13
41	20	
**42	42	5-8
43	34	
*44	59	5-16
45	14	
46	40	
47	11	
48	22	
49	1	
**50	50	5-Z12
*51	64	5-11
52	36	
53	16	
54	7	
*55	58	5-7
*56	61	5-32
*57	65	5-19
58	55	
59	44	
60	30	
61	56	
62	31	
63	19	
64	51	
65	57	
66	32	

6 notes (50)

*1	5	6-Z19
*2	79	6-Z43
*3	4	6-Z44
4	3	
5	1	
**6	6	6-Z29
*7	37	6-31
*8	13	6-5
*9	46	6-Z17
*10	68	6-27
*11	76	6-30
**12	12	6-Z13
13	8	
*14	18	6-Z11
**15	15	6-1
*16	66	6-Z3
**17	17	6-Z6
18	14	
*19	21	6-33
*20	24	6-Z24
21	19	
*22	23	6-34
23	22	
24	20	
**25	25	6-Z23
*26	27	6-9
27	26	
*28	55	6-Z12
*29	30	6-22
30	29	
**31	31	6-Z4
*32	33	6-2
33	32	
*34	35	6-Z10
35	34	
**36	36	6-Z45
37	7	
*38	48	6-15
*39	50	6-21
**40	40	6-Z49
*41	60	6-14
**42	42	6-Z28
*43	47	6-Z39
**44	44	6-Z37
**45	45	6-Z48
46	9	
47	43	
48	38	

*49	64	6-16
50	39	
**51	51	6-35
*52	65	6-Z25
*53	73	6-Z46
*54	59	6-18
55	28	
**56	56	6-Z50
**57	57	6-32
*58	67	6-Z40
59	54	
60	41	
**61	61	6-20
**62	62	6-Z38
**63	63	6-Z26
64	49	
65	52	
66	16	
67	58	
68	10	
**69	69	6-8
*70	72	6-Z47
*71	74	6-Z36
72	70	
73	53	
74	71	
**75	75	6-Z42
76	11	
*77	80	6-Z41
**78	78	6-7
79	2	
80	77	

7 notes (38)

**1	1	7-35
**2	2	7-34
*3	4	7-32
4	3	
*5	8	7-30
**6	6	7-33
**7	7	7-22
8	5	
*9	24	7-14
*10	47	7-11
*11	40	7-24
*12	19	7-25
*13	60	7-23
*14	48	7-29
**15	15	7-Z12
*16	20	7-19
*17	32	7-9
*18	27	7-7
19	12	
20	16	
*21	42	7-27
*22	64	7-Z38
**23	23	7-Z37
24	9	
*25	33	7-13
*26	41	7-Z36
27	18	
*28	57	7-21
*29	30	7-6
30	29	
*31	45	7-20
32	17	
33	25	
*34	55	7-2
*35	56	7-4
*36	43	7-5
*37	61	7-26
*38	66	7-28
**39	39	7-15
40	11	
41	26	
42	21	
43	36	
*44	59	7-Z18
45	31	
*46	53	7-16
47	10	
48	14	
**49	49	7-8
*50	51	7-31
51	50	
*52	63	7-10
53	46	
**54	54	7-1
55	34	
56	35	
57	28	
*58	65	7-3
59	44	
60	13	
61	37	
**62	62	7-Z17
63	52	
64	22	
65	58	
66	38	

8 notes (29)

*1	10	8-22
**2	2	8-21
*3	4	8-27
4	3	
*5	22	8-Z15
*6	36	8-Z29
*7	20	8-18
**8	8	8-9
*9	21	8-13
10	1	
*11	34	8-16
**12	12	8-6
*13	28	8-11
*14	32	8-2
**15	15	8-8
**16	16	8-3
**17	17	8-10
**18	18	8-26
**19	19	8-17
20	7	
21	9	
22	5	
**23	23	8-23
*24	41	8-4
*25	39	8-14
**26	26	8-24
*27	31	8-19
28	13	
*29	42	8-12
**30	30	8-25
31	27	
32	14	
*33	35	8-5
34	11	
35	33	
36	6	
**37	37	8-20
**38	38	8-7
39	25	
**40	40	8-1
41	24	
42	29	
**43	43	8-28

9 notes (12)

**1	1	9-1
*2	6	9-8
**3	3	9-6
*4	12	9-11
*5	15	9-7
6	2	
**7	7	9-10
*8	18	6-2
*9	11	9-5
*10	16	9-3
11	9	
12	4	
*13	17	9-4
**14	14	9-9
15	5	
16	10	
17	13	
18	8	
**19	19	9-12

10 notes (6)

**1	1
**2	2
**3	3
**4	4
**5	5
**6	6

11 notes (1)

**1	1

12 notes (1)

**1	1

11.2 Internal symmetries inside an octave

There are four different kinds of symmetry in an octave (a, b, c and d in Figure 81): when the symmetric core goes through one or two notes and when the symmetry contemplates (or not) the closing of the octave. Each symmetry generates in another mode of the same scale another symmetry that can be of the same class or not. The numbers –in Tables 10 and 11– show the number of notes on the scale and, separated by a hyphen, there is the numbered classified scale class according to the Chord/scale Tables.

Fig. 81

A chord/scale class (except the cyclical ones) has maximum two symmetrical chords or modes –if it has one, it also has two–.

On Table 10, there are represented all the modes with symmetry from types (a) and (b) of Figure 22, transposed to C, that is, in the boundary on an octave. On table 11, there are all the modes till 7 notes with symmetry of type (c).[1]

[1] Of the 2048 possible modes (or chords at equivalence level N5), 184 are symmetrical internally, 63 of which are of symmetry of classes (a) or (b) and 121 of classes (c) or (d) (some cyclic modes have symmetries of the two families); the symmetries in Table 11 are of the same scale class as those in Table 10 (odd-numbered groups); therefore, the rest of symmetries up to 12 notes can be easily deduced.

Tabla 10

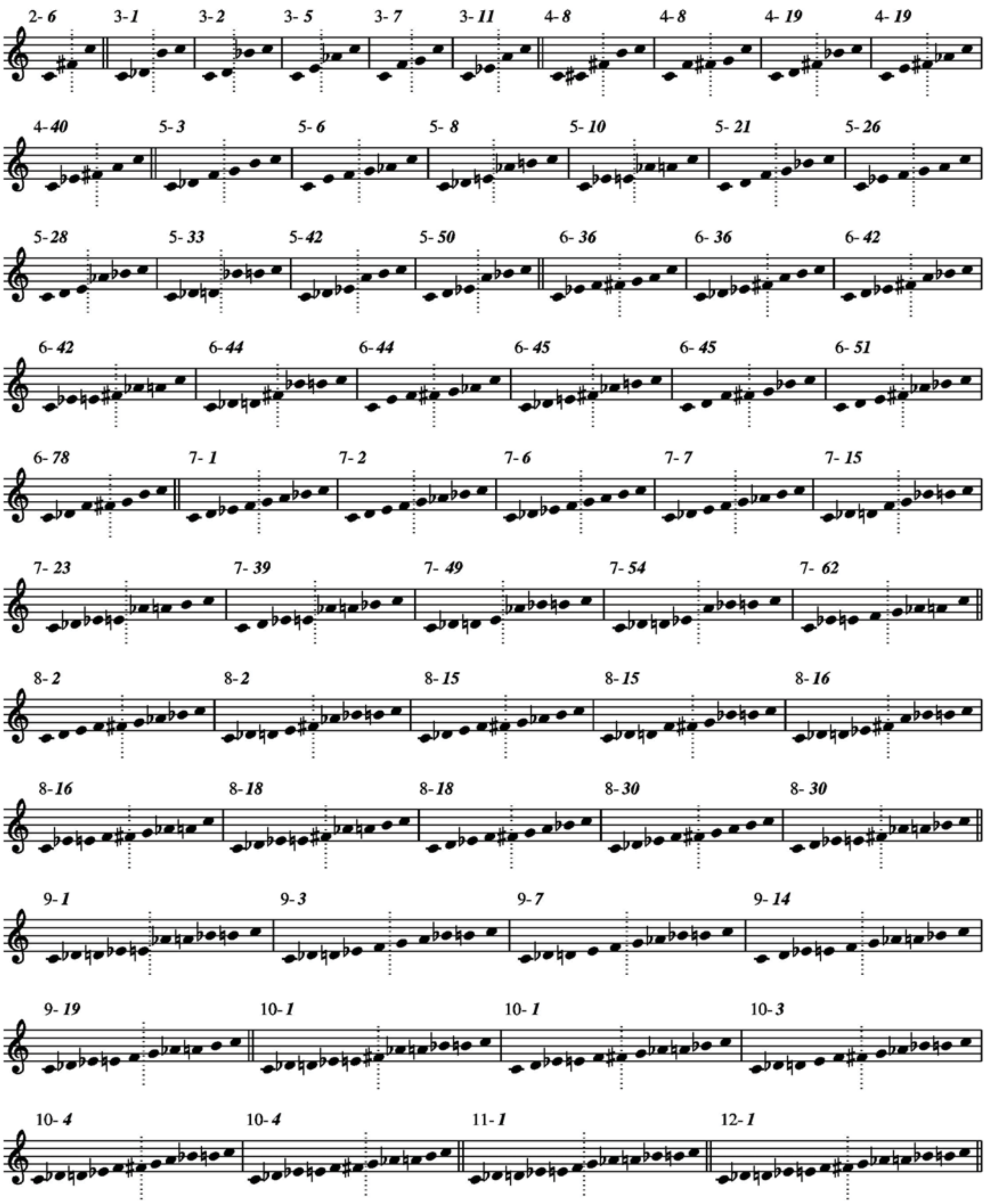
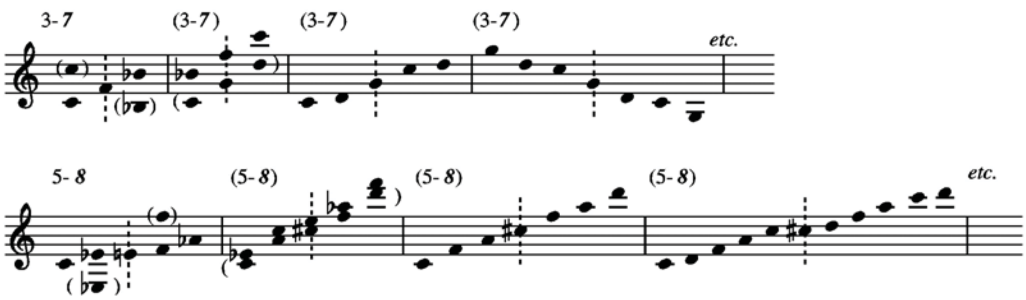

2- 6
3- 1
3- 2
3- 5
3- 7
3- 11
4- 8
4- 8
4- 19
4- 19
4- 40
5- 3
5- 6
5- 8
5- 10
5- 21
5- 26
5- 28
5- 33
5- 42
5- 50
6- 36
6- 36
6- 42
6- 42
6- 44
6- 44
6- 45
6- 45
6- 51
6- 78
7- 1
7- 2
7- 6
7- 7
7- 15
7- 23
7- 39
7- 49
7- 54
7- 62
8- 2
8- 2
8- 15
8- 15
8- 16
8- 16
8- 18
8- 18
8- 30
8- 30
9- 1
9- 3
9- 7
9- 14
9- 19
10- 1
10- 1
10- 3
10- 4
10- 4
11- 1
12- 1

Tabla 11

As it was said at the beginning the symmetry has only been established in an octave. When the range is greater than an octave every symmetrical mode generates a great number of other symmetries (always centred in the same chord/scale class). For example:

Fig. 82



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(in parenthesis, year of the first edition —if known—)

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