

Musical Mathematics

ON THE ART AND SCIENCE OF ACOUSTIC INSTRUMENTS



Cris Forster

MUSICAL MATHEMATICS

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Text and Illustrations

by Cris Forster



CHRONICLE BOOKS

SAN FRANCISCO

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In Memory of Page Smith

my enduring teacher

And to Douglas Monsour

our constant friend

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The jewel that we find, we stoop and take't,
Because we see it; but what we do not see
We tread upon, and never think of it.

W. Shakespeare

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CONTENTS

Foreword by David R. Canright	v
Introduction and Acknowledgments	vii
Tone Notation	ix
List of Symbols	xi
CHAPTER 1 MICA MASS	1
Part I Principles of force, mass, and acceleration	1
Part II Mica mass definitions, mica unit derivations, and sample calculations	14
Notes	24
CHAPTER 2 PLAIN STRING AND WOUND STRING CALCULATIONS	27
Part I Plain strings	27
Part II Wound strings	36
Notes	41
CHAPTER 3 FLEXIBLE STRINGS	44
Part I Transverse traveling and standing waves, and simple harmonic motion in strings	44
Part II Period and frequency equations of waves in strings	54
Part III Length, frequency, and interval ratios of the harmonic series on canon strings	59
Part IV Length, frequency, and interval ratios of non-harmonic tones on canon strings	69
Part V Musical, mathematical, and linguistic origins of length ratios	79
Notes	94
CHAPTER 4 INHARMONIC STRINGS	98
Part I Detailed equations for stiffness in plain strings	98
Part II Equations for coefficients of inharmonicity in cents	108
Part III General equations for stiffness in wound strings	113
Notes	115
CHAPTER 5 PIANO STRINGS VS. CANON STRINGS	118
Part I Transmission and reflection of mechanical and acoustic energy	118
Part II Mechanical impedance and soundboard-to-string impedance ratios	120
Part III Radiation impedance and air-to-soundboard impedance ratios	126
Part IV Dispersion, the speed of bending waves, and critical frequencies in soundboards	130
Part V Methods for tuning piano intervals to beat rates of coincident string harmonics	135
Part VI Musical advantages of thin strings and thin soundboards	141
Notes	143

CHAPTER 6 BARS, RODS, AND TUBES	147
Part I Frequency equations, mode shapes, and restoring forces of free-free bars	147
Part II Free-free bar tuning techniques	160
Part III Frequency equations, mode shapes, and restoring forces of clamped-free bars	174
Part IV Clamped-free bar tuning techniques	176
Notes	178
 CHAPTER 7 ACOUSTIC RESONATORS	 182
Part I Simple harmonic motion of longitudinal traveling waves in air	182
Part II Equations for the speed of longitudinal waves in solids, liquids, and gases	186
Part III Reflections of longitudinal traveling waves at the closed and open ends of tubes	189
Part IV Acoustic impedance and tube-to-room impedance ratio	196
Part V Longitudinal pressure and displacement standing waves in tubes	200
Part VI Length and frequency equations of tube resonators	203
Part VII Theory of cavity resonators	212
Part VIII Cavity resonator tuning techniques	219
Notes	223
 CHAPTER 8 SIMPLE FLUTES	 227
Part I Equations for the placement of tone holes on concert flutes and simple flutes	227
Part II Equations for analyzing the tunings of existing flutes	242
Part III Suggestions for making inexpensive yet highly accurate simple flutes	246
Notes	248
 CHAPTER 9 THE GEOMETRIC PROGRESSION, LOGARITHMS, AND CENTS	 253
Part I Human perception of the harmonic series as a geometric progression	253
Part II Logarithmic processes in mathematics and human hearing	257
Part III Derivations and applications of cent calculations	265
Part IV Logarithmic equations for guitar frets and musical slide rules	271
Notes	276
 CHAPTER 10 WESTERN TUNING THEORY AND PRACTICE	 280
Part I Definitions of prime, composite, rational, and irrational numbers	281
Part II Greek classifications of ratios, tetrachords, scales, and modes	284
Part III Arithmetic and geometric divisions on canon strings	291
Part IV Philolaus, Euclid, Aristoxenus, and Ptolemy	299
Part V Meantone temperaments, well-temperaments, and equal temperaments	334
Part VI Just intonation	365
Notes	460
 CHAPTER 11 WORLD TUNINGS	 485
Part I Chinese Music	485
Notes	504

Part II	Indonesian Music: Java	508
	Bali	522
	Notes	535
Part III	Indian Music: Ancient Beginnings	540
	South India	564
	North India	587
	Notes	600
Part IV	Arabian, Persian, and Turkish Music	610
	Notes	774
CHAPTER 12 ORIGINAL INSTRUMENTS		788
<i>Stringed Instruments:</i>		
	Chrysalis	788
	Harmonic/Melodic Canon	790
	Bass Canon	800
	Just Keys	808
<i>Percussion Instruments:</i>		
	Diamond Marimba	824
	Bass Marimba	826
<i>Friction Instrument:</i>		
	Glassdance	828
<i>Wind Instruments:</i>		
	Simple Flutes	833
CHAPTER 13 BUILDING A LITTLE CANON		834
	Parts, materials, labor, and detailed dimensions	834
	Epilog by Heidi Forster	839
Plate 1:	Chrysalis	845
Plate 2:	Harmonic/Melodic Canon	846
Plate 3:	Bass Canon	847
Plate 4:	String Winder (machine)	848
Plate 5:	String Winder (detail)	849
Plate 6:	Just Keys	850
Plate 7:	Diamond Marimba	851
Plate 8:	Bass Marimba	852
Plate 9:	Glassdance	853
Plate 10:	Glassdance (back)	854
Plate 11:	Simple Flutes	855
Plate 12:	Little Canon	856

Plate 13: Cris Forster with Chrysalis	857
Plate 14: Heidi Forster playing Glassdance	858
Plate 15: David Canright, Heidi Forster, and Cris Forster	859
Plate 16: Chrysalis Foundation Workshop	860
Bibliography for Chapters 1–9	861
Bibliography for Chapter 10	866
Bibliography for Chapter 11	871
Bibliography for Chapter 12	877
Appendix A: Frequencies of Eight Octaves of 12-Tone Equal Temperament	879
Appendix B: Conversion Factors	880
Appendix C: Properties of String Making Materials	882
Appendix D: Spring Steel Music Wire Tensile Strength and Break Strength Values	884
Appendix E: Properties of Bar Making Materials	885
Appendix F: Properties of Solids	888
Appendix G: Properties of Liquids	890
Appendix H: Properties of Gases	892
Index	895

Foreword

I met Cris Forster more than thirty years ago. Shortly thereafter, I saw him perform *Song of Myself*, his setting of Walt Whitman poems from *Leaves of Grass*. His delivery was moving and effective. Several of the poems were accompanied by his playing on unique instruments — one an elegant box with many steel strings and moveable bridges, a bit like a koto in concept; the other had a big wheel with strings like spokes from offset hubs, and he rotated the wheel as he played and intoned the poetry. I was fascinated.

Since that time, Cris has built several more instruments of his own design. Each shows exquisite care in conception and impeccable craftsmanship in execution. And of course, they are a delight to hear. Part of what makes them sound so good is his deep understanding of how acoustic musical instruments work, and part is due to his skill in working the materials to his exacting standards.

But another important aspect of their sound, and indeed one of the main reasons Cris could not settle for standard instruments, is that his music uses scales and harmonies that are not found in the standard Western system of intonation (with each octave divided into twelve equal semitones, called equal temperament). Rather, his music employs older notions of consonance, which reach back as far as ancient Greek music and to other cultures across the globe, based on what is called just intonation. Here, the musical intervals that make up the scales and chords are those that occur naturally in the harmonic series of overtones, in stretched flexible strings, and in organ pipes, for example.

In just intonation, the octave is necessarily divided into unequal parts. In comparison to equal temperament, the harmonies of just intonation have been described as smoother, sweeter, and/or more powerful. Many theorists consider just intonation to be the standard of comparison for consonant intervals. There has been a resurgence of interest in just intonation since the latter part of the twentieth century, spurred by such pioneers as Harry Partch and Lou Harrison. Even so, the community of just intonation composers remains comparatively quite small, and the subset of those who employ only acoustic instruments is much smaller still. I know of no other living composer who has created such a large and varied ensemble of high-quality just intoned acoustical instruments, and a body of music for them, as Cris Forster.

Doing what he has done is not easy, far from it. The long process of developing his instruments has required endless experimentation and careful measurement, as well as intense study of the literature on acoustics of musical instruments. In this way Cris has developed deep and rich knowledge of how to design and build instruments that really work. Also, in the service of his composing, Cris has studied the history of intonation practices, not only in the Western tradition, but around the world.

This book is his generous offering of all that hard-earned knowledge, presented as clearly as he can make it, for all of you who have an interest in acoustic musical instrument design and/or musical scales over time and space. The unifying theme is how mathematics applies to music, in both the acoustics of resonant instruments and the analysis of musical scales. The emphasis throughout is to show how to use these mathematical tools, without requiring any background in higher mathematics; all that is required is the ability to do arithmetic on a pocket calculator, and to follow Cris' clear step-by-step instructions and examples. Any more advanced mathematical tools required, such as logarithms, are carefully explained with many illustrative examples.

The first part of the book contains practical information on how to design and build musical instruments, starting from first principles of vibrating sound sources of various kinds. The ideas are explained clearly and thoroughly. Many beautiful figures have been carefully conceived to illuminate the concepts. And when Cris gives, say, formulas for designing flutes, it's not just something he read in a book somewhere (though he has carefully studied many books); rather, you can be

sure it is something he has tried out: he knows it works from direct experience. While some of this information can be found (albeit in a less accessible form) in other books on musical acoustics, other information appears nowhere else. For example, Cris developed a method for tuning the overtones of marimba bars that results in a powerful, unique tone not found in commercial instruments. Step-by-step instructions are given for applying this technique (see Chapter 6). Another innovation is Cris' introduction of a new unit of mass, the "mica," that greatly simplifies calculations using lengths measured in inches. And throughout Cris gives careful explanations, in terms of physical principles, that make sense based on one's physical intuition and experience.

The latter part of the book surveys the development of musical notions of consonance and scale construction. Chapter 10 traces Western ideas about intonation, from Pythagoras finding number in harmony, through "meantone" and then "well-temperament" in the time of J.S. Bach, up to modern equal temperament. The changing notions of which intervals were considered consonant when, and by whom, make a fascinating story. Chapter 11 looks at the largely independent (though sometimes parallel) development of musical scales and tunings in various Eastern cultures, including China, India, and Indonesia, as well as Persian, Arabian, and Turkish musical traditions. As far as possible, Cris relies on original sources, to which he brings his own analysis and explication. To find all of these varied scales compared and contrasted in a single work is unique in my experience.

The book concludes with two short chapters on specific original instruments. One introduces the innovative instruments Cris has designed and built for his music. Included are many details of construction and materials, and also scores of his work that demonstrate his notation for the instruments. The last chapter encourages the reader (with explicit plans) to build a simple stringed instrument (a "canon") with completely adjustable tuning, to directly explore the tunings discussed in the book. In this way, the reader can follow in the tradition of Ptolemy, of learning about music through direct experimentation, as has Cris Forster.

David R. Canright, Ph.D.
Del Rey Oaks, California
January 2010

Introduction and Acknowledgments

In simplest terms, human beings identify musical instruments by two aural characteristics: a particular kind of sound or timbre, and a particular kind of scale or tuning. To most listeners, these two aspects of musical sound do not vary. However, unlike the constants of nature — such as gravitational acceleration on earth, or the speed of sound in air — which we cannot change, the constants of music — such as string, percussion, and wind instruments — are subject to change. A creative investigation into musical sound inevitably leads to the subject of musical mathematics, and to a reexamination of the meaning of variables.

The first chapter entitled “Mica Mass” addresses an exceptionally thorny subject: the derivation of a unit of mass based on an inch constant for acceleration. This unit is intended for builders who measure wood, metal, and synthetic materials in inches. For example, with the mica unit, builders of string instruments can calculate tension in pounds-force, or lbf, without first converting the diameter of a string from inches to feet. Similarly, builders of tuned bar percussion instruments who know the modulus of elasticity of a given material in pounds-force per square inch, or lbf/in², need only the mass density in mica/in³ to calculate the speed of sound in the material in inches per second; a simple substitution of this value into another equation gives the mode frequencies of uncut bars.

Chapters 2–4 explore many physical, mathematical, and musical aspects of strings. In Chapter 3, I distinguish between four different types of ratios: ancient length ratios, modern length ratios, frequency ratios, and interval ratios. Knowledge of these ratios is essential to Chapters 10 and 11. Many writers are unaware of the crucial distinction between ancient length ratios and frequency ratios. Consequently, when they attempt to define arithmetic and harmonic divisions of musical intervals based on frequency ratios, the results are diametrically opposed to those based on ancient length ratios. Such confusion leads to anachronisms, and renders the works of theorists like Ptolemy, Al-Fārābī, Ibn Sīnā, and Zarlino incomprehensible.

Chapter 5 investigates the mechanical interactions between piano strings and soundboards, and explains why the large physical dimensions of modern pianos are not conducive to explorations of alternate tuning systems.

Chapters 6 and 7 discuss the theory and practice of tuning marimba bars and resonators. The latter chapter is essential to Chapter 8, which examines a sequence of equations for the placement of tone holes on concert flutes and simple flutes.

Chapter 9 covers logarithms, and the modern cent unit. This chapter serves as an introduction to calculating scales and tunings discussed in Chapters 10 and 11.

In summary, this book is divided into three parts. (1) In Chapters 1–9, I primarily examine various vibrating systems found in musical instruments; I also focus on how builders can customize their work by understanding the functions of variables in mathematical equations. (2) In Chapter 10, I discuss scale theories and tuning practices in ancient Greece, and during the Renaissance and Enlightenment in Europe. Some modern interpretations of these theories are explained as well. In Chapter 11, I describe scale theories and tuning practices in Chinese, Indonesian, and Indian music, and in Arabian, Persian, and Turkish music. For Chapters 10 and 11, I consistently studied original texts in modern translations. I also translated passages in treatises by Ptolemy, Al-Kindī, the Ikhwān al-Ṣafā, Ibn Sīnā, Stifel, and Zarlino from German into English; and in collaboration with two contributors, I participated in translating portions of works by Al-Fārābī, Ibn Sīnā, Ṣafī Al-Dīn, and Al-Jurjānī from French into English. These translations reveal that all the above-mentioned theorists employ the language of ancient length ratios. (3) Finally, Chapters 12 and 13 recount musical instruments I have built and rebuilt since 1975.

I would like to acknowledge the assistance and encouragement I received from Dr. David R. Canright, associate professor of mathematics at the Naval Postgraduate School in Monterey,

California. David's unique understanding of mathematics, physics, and music provided the foundation for many conversations throughout the ten years I spent writing this book. His mastery of differential equations enabled me to better understand dispersion in strings, and simple harmonic motion of air particles in resonators. In Section 4.5, David's equation for the effective length of stiff strings is central to the study of inharmonicity; and in Section 6.6, David's figure, which shows the effects of two restoring forces on the geometry of bar elements, sheds new light on the physics of vibrating bars. Furthermore, David's plots of compression and rarefaction pulses inspired numerous figures in Chapter 7. Finally, we also had extensive discussions on Newton's laws. I am very grateful to David for his patience and contributions.

Heartfelt thanks go to my wife, Heidi Forster. Heidi studied, corrected, and edited myriad versions of the manuscript. Also, in partnership with the highly competent assistance of professional translator Cheryl M. Buskirk, Heidi did most of the work translating extensive passages from *La Musique Arabe* into English. To achieve this accomplishment, she mastered the often intricate verbal language of ratios. Heidi also assisted me in transcribing the Indonesian and Persian musical scores in Chapter 11, and transposed the traditional piano score of "The Letter" in Chapter 12. Furthermore, she rendered invaluable services during all phases of book production by acting as my liaison with the editorial staff at Chronicle Books. Finally, when the writing became formidable, she became my sparring partner and helped me through the difficult process of restoring my focus. I am very thankful to Heidi for all her love, friendship, and support.

I would also like to express my appreciation to Dr. John H. Chalmers. Since 1976, John has generously shared his vast knowledge of scale theory with me. His mathematical methods and techniques have enabled me to better understand many historical texts, especially those of the ancient Greeks. And John's scholarly book *Divisions of the Tetrachord* has furthered my appreciation for world tunings.

I am very grateful to Lawrence Saunders, M.A. in ethnomusicology, for reading Chapters 3, 9, 10, and 11, and for suggesting several technical improvements.

Finally, I would like to thank Will Gullette for his twelve masterful color plates of the Original Instruments and String Winder, plus three additional plates. Will's skill and tenacity have illuminated this book in ways that words cannot convey.

Cris Forster
San Francisco, California
January 2010

TONE NOTATION

	32'	16'	8'	4'	2'	1'	1/2'	1/4'	1/8'
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1.	C ₀	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈
2.	C _∞	C _∞	C	c	c'	c''	c'''	c''''	c ^v
3.	C ₂	C ₁	C ₀	c ⁰	c ¹	c ²	c ³	c ⁴	c ⁵

1. American System, used throughout this text.
2. Helmholtz System.
3. German System.

LIST OF SYMBOLS

Latin

12-TET	12-tone equal temperament
a	Acceleration; in/s^2
a.l.r.	Ancient length ratio; dimensionless
B	Bending stiffness of bar; $\text{lbf}\cdot\text{in}^2$, or $\text{mica}\cdot\text{in}^3/\text{s}^2$
B'	Bending stiffness of plate; $\text{lbf}\cdot\text{in}$, or $\text{mica}\cdot\text{in}^2/\text{s}^2$
B_A	Adiabatic bulk modulus; psi , lbf/in^2 , or $\text{mica}/(\text{in}\cdot\text{s}^2)$
B_I	Isothermal bulk modulus; psi , lbf/in^2 , or $\text{mica}/(\text{in}\cdot\text{s}^2)$
b	Width; in
ϵ	Cent, 1/100 of a “semitone,” or 1/1200 of an “octave”; dimensionless
$\bar{\epsilon}$	Coefficient of inharmonicity of string; cent
c_B	Bending wave speed; in/s
c_L	Longitudinal wave speed, or speed of sound; in/s
c_T	Transverse wave speed; in/s
c.d.	Common difference of an arithmetic progression; dimensionless
c.r.	Common ratio of a geometric progression; dimensionless
cps	Cycle per second; $1/\text{s}$
D	Outside diameter; in
D_i	Inside diameter of wound string; in
D_m	Middle diameter of wound string; in
D_o	Outside diameter of wound string; in
D_w	Wrap wire diameter of wound string; in
d	Inside diameter, or distance; in
E	Young’s modulus of elasticity; psi , lbf/in^2 , or $\text{mica}/(\text{in}\cdot\text{s}^2)$
F	Frequency; cps
F_c	Critical frequency; cps
F_n	Resonant frequency; cps
\bar{F}_n	Inharmonic mode frequency of string; cps
f	Force; lbf , or $\text{mica}\cdot\text{in}/\text{s}^2$
f.r.	Frequency ratio; dimensionless
g	Gravitational acceleration; $386.0886 \text{ in}/\text{s}^2$
h	Height, or thickness; in
I	Area moment of inertia; in^4
i.r.	Interval ratio; dimensionless
J	Stiffness parameter of string; dimensionless
K	Radius of gyration; in
k	Spring constant; lbf/in , or mica/s^2
L	Length; in , cm , or mm
ℓ_M	Multiple loop length of string; in
ℓ_S	Single loop length of string; in
l.r.	Length ratio; dimensionless
lbf	Pounds-force; $\text{mica}\cdot\text{in}/\text{s}^2$
lbm	Pounds-mass; 0.00259008 mica

$M/u.a.$	Mass per unit area; mica/in ² , or lbf·s ² /in ³
$M/u.l.$	Mass per unit length; mica/in, or lbf·s ² /in ²
m	Mass; mica, or lbf·s ² /in
n	Mode number, or harmonic number; any positive integer
P	Pressure; psi, lbf/in ² , or mica/(in·s ²)
p	Excess acoustic pressure; psi, lbf/in ² , or mica/(in·s ²)
psi	Pounds-force per square inch; lbf/in ² , or mica/(in·s ²)
q	Bar parameter; dimensionless
R	Ideal gas constant; in·lbf/(mica·°R), or in ² /(s ² ·°R)
r	Radius; in
S	Surface area; in ²
SHM	Simple harmonic motion
T	Tension; lbf, or mica·in/s ²
T_A	Absolute temperature; dimensionless
t	Time; s
U	Volume velocity; in ³ /s
u	Particle velocity; in/s
V	Volume; in ³
v	Phase velocity; in/s
W	Weight density, or weight per unit volume; lbf/in ³ , or mica/(in ² ·s ²)
w	Weight; lbf, or mica·in/s ²
Y_A	Acoustic admittance; in ⁴ ·s/mica
Z_A	Acoustic impedance; mica/(in ⁴ ·s)
Z_r	Acoustic impedance of room; mica/(in ⁴ ·s)
Z_t	Acoustic impedance of tube; mica/(in ⁴ ·s)
Z_M	Mechanical impedance; mica/s
Z_b	Mechanical impedance of soundboard; mica/s
Z_p	Mechanical impedance of plate; mica/s
Z_s	Mechanical impedance of string; mica/s
Z_R	Radiation impedance; mica/s
Z_a	Radiation impedance of air; mica/s
z	Specific acoustic impedance; mica/(in ² ·s)
z_a	Characteristic impedance of air; 0.00153 mica/(in ² ·s)

Greek

Δ	Correction coefficient, or end correction coefficient; dimensionless
$\Delta\ell$	Correction, or end correction; in, cm, or mm
δ	Departure of tempered ratio from just ratio; cent
γ	Ratio of specific heat; dimensionless
θ	Angle; degree
κ	Conductivity; in
Λ	Bridged canon string length; in
Λ_A	Arithmetic mean string length; in
Λ_G	Geometric mean string length; in
Λ_H	Harmonic mean string length; in

λ	Wavelength; in
λ_B	Bending wavelength; in
λ_L	Longitudinal wavelength; in
λ_T	Transverse wavelength; in
μ	Poisson's ratio; dimensionless
Π	Fretted guitar string length; mm
π	Pi; ≈ 3.1416
ρ	Mass density, or mass per unit volume; mica/in ³ , or lbf·s ² /in ⁴
τ	Period, or second per cycle; s

Index

Internet Version: no diacritics

A

- Abul-Salt, 628, 630–632
- Acceleration
of bar, 156–158
definition, 4
dimensional analysis, 4, 6
English Gravitational System, 9ff.
consistent system, 9–10, 14–15
experiment, 8–11, 15
gravitational, 6–7, 10–12, 14
of mass
mica, 14–16
slug, 10–11
Newton, 5–6
standard gravity, 11–12
of string, 2–3
- Adiabatic bulk modulus
of air, 22
definition, 21–22
- Admittance. *See* Flutes
- Ahobala, 93, 587–591
- Al-Din, Salah, 761–762
- Al-Farabi, Abu Nasr, 93, 326, 354–355, 366, 375, 378, 401, 610, 622, 625, 628, 632–666, 669, 673–674, 676–677, 679–681, 696, 701–707, 709–710, 712, 715, 717, 720–722, 727, 733–736, 749–752, 754–755, 757–758, 771
- Al-Faruqi, Lois I., 641–642, 648, 760
- Al-Isfahani, Abul-Faraj, 619–620, 624
- Al-Jurjani, 401–403, 448, 610, 626–631, 707, 717–722, 726–727, 731–733, 744, 746, 748
- Al-Khulai, M. Kamil, 749, 755
- Al-Kindi, Ishaq, 93, 334, 611–617, 619–620, 642–643, 646, 709–710, 755
- Al-Mauaili, Ishaq, 619–620, 622
- Al-Sanusi, Al-Manubi, 402, 720
- Alexander the Great, 610
- Ambisonance, 779n.74
- Ancient length ratios (a.l.r.)
definition, 75–77, 287
Al-Farabi, 378
Needham, 487
equations, for stopped (bridged) string, 77
Euclid, 76, 81–82, 302–307
Greek ratios; *multiple*, *epimore*, and *epimere*, 286–287
Latin ratios; *superparticular* and *superpartient*, 384–385
Ptolemy, 321–322
string length of 120 parts, 316–317
Ramamatya, 568–573
Rameau, 431–434, 442–447, 452
Ssu-ma Ch'ien, 486–487
vs. frequency ratios, 75–76, 91, 93, 305, 307, 427, 431, 487, 576–577, 587
vs. modern length ratio, 305–307, 427, 440–441
vs. “vibration ratios,” 85–86, 93, 435–436
vs. “weight ratios,” 83, 85–86, 93
Zarlino, 378, 436, 452
- Antinodes. *See* Bars, rods, and tubes; Flexible strings; Just intonation, Sauveur; Resonators, tube
- Arabian musical terms. *See also* Scales, Arabian 24-TET
5 *nim* (Pers. *lowered*) notes, 756, 759
5 *tik* (Pers. *raised*) notes, 756, 759
7 fundamental notes; *naghamat* (notes), *asasiyyah* (fundamental), 756, 759
7 half notes, or “semitones”; *arabat* (sing. *arabah*, *half note*), 756, 759
24 quarter tones, *Mashaqah*, (sing. *rub*, *quarter*; pl. *arba*, *quarters*), 756, 759
- interval
Al-Farabi
awdah (whole step), 355, 647
fadlah (half step), 355, 647
Al-Jurjani
baqiyyah, *limma*, [L], 721–723
fadlah, *comma of Pythagoras*, [C], 722–723
mutammam, *apotome*, [L×C], 722–723
tanini, *double-limma plus comma*, [L×L×C], 721–723
tatimmah, *double-limma*, [L×L], 722–723
- mode
6 hierarchical functions of tones: *qarar*, *quwwah*, *zahir*, *ghammaz*, *markaz*, *mabda*, 738
46 Modern *Maqamat* (sing. *maqam*, *position* or *place*; *mode*), 738, 761–771, 786n.209
Al-Farabi, 8 *Ajnas* (sing. *jins*, *genus* or *kind*; *mode*), 640, 646–655

Arabian musical terms (*Continued*)

- Al-Munajjim
8 *majari* (sing. *majra*, *course* or *path*; *mode*), 622–625
and Philolaus' diatonic scale, 624
- Ibn Sina
12 *shudud* (sing. *shadd*), primary or principal mode, 727
Mustaqim (*direct*, *straight*, or *regular mode*), Melodic Mode 8, 681–682, 685, 727, 754
majra (*course* or *path*; *mode*), 617, 622ff.
- Safi Al-Din
6 *awazat* (sing. *awaz*; also *shubah*), secondary or derived mode, 727, 730–733, 744
12 *shudud* (sing. *shadd*), primary or principal mode, 726–731, 737, 740–743, 767–768
Rast (Pers. *direct*, *straight*, or *regular mode*), Melodic Mode 40, 727–730, 733, 740–741, 746–748
- Arabian ratios. *See also* Greek ratios; Latin ratios
- apotome*
Al-Farabi, 636, 655
Al-Jurjani, 722–723
Al-Kindi, 334, 611, 615–616
- apyknon*, Al-Farabi, 659–663
- comma of Pythagoras (ditonic comma)
Arabian theory
Al-Farabi, 655, 702–706
Al-Jurjani, 722–723
Al-Kindi, 611, 615–616, 620
of *tunbur* and *ud*, ancient, 696–697
variants of, notation, 697–701
Al-Farabi's *tunbur* of Khurasan and original *tunbur*, 696–706
Safi Al-Din, *First Ud Tuning*, 712–713
Turkish theory, modern
of *tunbur*, 733–737
variants of, notation, 734
- epimere*, Al-Farabi, 661–663
- epimore*
Al-Farabi, 661–663
Ibn Sina, 673–676, 678–679
- limma*
Al-Farabi, 633, 636–640, 643, 655–656, 702–707
Al-Jurjani, 722–723
Al-Kindi, 334–335, 611, 615–617
double-limma, 697, 699–700, 703–705, 722–723
Ibn Sina, 667–668, 671, 682–685
Safi Al-Din, 708–713, 722–723, 741–743
triple-limma, 722–723, 739–745, 767
Turkish theory, modern, 736–737
triple-limma, 739–745, 767
- “minor third,” oldest *ud* tuning, 3-limit ratio, 620–622
Persian middle finger fret, Al-Farabi's *uds*, 17-limit ratio, 634–639, 650–651, 653
“neutral second”
Arabian theory, modern
46 *Maqamat*, 763–766
of basic 16-tone scale, 770–771
Safi Al-Din, 84 Melodic Modes, *Second Ud Tuning*; in six of nine unchanged modes, 767–768
tik Zirkulah [D[♭]], 756
origins in near-equal divisions of intervals; Ptolemy, Al-Farabi, and Ibn Sina, 677–681
Persian theory, modern
of basic 17-tone scale, 770
Farhat
of 2 *tars* and 3 *setars*, two different kinds, 686–689
12 *Dastgaha*, 692–696
as tone on *ud*
Al-Farabi
Fret 5 on 12-fret *ud*, 641–643
Frets 3 and 4, *Mujannab* frets, on 10-fret *ud*, 632–635, 639, 673–674
Ibn Sina
Fret 2, *Assistant to the middle finger of Zalzal* fret, 668, 671, 673–674
two different kinds, 672
Safi Al-Din, Fret 2, *Mujannab*, *Anterior* fret, *Second Ud Tuning*, 714–716
as tone or interval of a genus or mode
Al-Farabi, *Jins 2/Jins 8*, 651–654, 676–677, 679–680
Ibn Sina
11 Melodic Modes, 681–685
Diatonic Genus 4 and 7, 674–677
Safi Al-Din, 84 Melodic Modes, *Second Ud Tuning*, 724–725, 729
“neutral seventh”
Arabian theory, modern
Awj [B[♭]], 749, 756, 758–759
of basic 16-tone scale, 770
used to justify 24-TET
D'Erlanger, 755–758
Marcus, 760–761
historical context of, Al-Farabi's and Ibn Sina's *uds*, 749–754
“neutral third”
Arabian theory, modern
of basic 16-tone scale, 770
Safi Al-Din, 84 Melodic Modes, *Second Ud Tuning*; in six of nine unchanged modes, 767–768
Sikah [E[♭]], 749, 756, 758–761

- fasilah* (genus), 761–762, 765
maqam, 767–769
 used to justify 24-TET
 D’Erlanger, 755–758
 Marcus, 760–761
 historical context of, 749, 754–755, 761
 Farmer, 716
 origins in near-equal divisions of intervals;
 Ptolemy, Al-Farabi, and Ibn Sina,
 677–681
 Persian theory, modern
 of basic 17-tone scale, 770
 Farhat
 of 2 *tars* and 3 *setars*, two different
 kinds, 686–689
 12 *Dastgaha*, 692–696
 as tone on *ud*
 middle finger of Zalzal fret
 Al-Farabi, 11-limit ratio
 Fret 8 on 10-fret *ud*, 634–640
 Fret 9 on 12-fret *ud*, 641–645
 Ibn Sina, 13-limit ratio, Fret 5, 667,
 671–674, 676–677
 Safi Al-Din, Fret 5, *Persian middle finger*
 fret, *Second Ud Tuning*, 714–717
 as tone or interval of a genus or mode
 Al-Farabi, *Jins* 2, 651–654, 676–677,
 679–680
 Ibn Sina
 11 Melodic Modes, 681–686
 Diatonic Genus 7, 675–677
 Safi Al-Din, 84 Melodic Modes, *Second Ud*
 Tuning, 724–725, 729
 Ptolemy’s classifications in
 Al-Farabi, 658–659
 Ibn Sina, 678–679
 Safi Al-Din, 711–713
 emmelic/melodic intervals
 B (*baqiyyah* or *fadlah*), 721–723, 740
 J (*tatimmah* or *mutammam*), 721–723,
 740, 744
 T (*tanini*), 721–723, 727, 740, 744
pyknon, Al-Farabi, 659–663
 “quarter-tone”
 4 modern musical symbols of, 713–714
 half-flat and half-sharp signs, Racy, 760
 koron and *sori* signs, Farhat, 640
 Arabian theory
 Al-Farabi, origin and function of ratio $\frac{33}{32}$,
 354–355
 Ibn Sina, origin and function of ratio $\frac{36}{35}$,
 679
 Marcus
 ambiguity of “quarter-tones” in musical
 practice, 760–761
 description of Mashaqah’s twenty-four
 “quarter” tones, 759
- schisma*
 definition, 373, 697
 variants of, notation, 697–701
 Al-Farabi’s *tunbur* of Khurasan and original
 tunbur, 696–707
 Safi Al-Din
 First Ud Tuning, 373–375, 710–713, 717
 Second Ud Tuning, 716
superparticular (*epimore*)
 Al-Farabi, 660–661
 Ibn Sina, 678–679
superpartient (*epimere*), Al-Farabi, 661
- Arabian scales. *See* Scales, Arabian
- Arabian tetrachords
 Al-Farabi
 8 *Ajnas*
 description of, 646–648
 fractional and integer parts of tones,
 Aristoxenian theory, 648–649
 interpretation of
 as length ratios on *ud*, 650–655
 modern, in cents, 650
 Jins 2, 676–677, 679–680
 Jins 8, 679–680
 conjunct, three standard modes, 644–645
 genera, *soft genus* and *strong genus*, 659–660
 classification of 15 tetrachords
 soft ordered consecutive and *non-*
 consecutive, 660–663
 strong doubling, *strong conjunct*, and
 strong disjunct, 661–663
 Greater Perfect System. *See also* Greek
 tetrachords
 three conjunct/disjunct systems for the
 distribution of, 662–666
 inversion of intervalic order
 Aristoxenus’ tetrachords, 648
 Philolaus’ tetrachord, 648–649
 Al-Kindi’s *ud*, 612
 description of, 613
 Philolaus’ tetrachord, three *harmoniai*,
 616–617
 classification, modern
 9 *fasail* (genera): seven tetrachords, one
 trichord, and one pentachord, 762
 46 *Maqamat*, 763–766
 Safi Al-Din, 84 Melodic Modes, *Second Ud*
 Tuning
 9 modern *maqamat*, interval patterns
 identical to, 767–768
 32 modern tetrachords and modes,
 traceable to, 763–768
fasail (sing. *fasilah*, *family* or *genus*), *maqamat*
 with similar lower tetrachords, 761–762

Arabian tetrachords (*Continued*)

jins al-asl or *jins al-jidh*, primary lower tetrachord, 761, 767–769

jins al-far, secondary upper tetrachord, 761, 767–769

munfasil, disjunct, 767

mutadakhil, overlapping, 767

muttasil, conjunct, 767

Ibn Sina

11 Melodic Modes, 681–686

12 *Dastgaha*, Persian theory, modern, 692–696

16 tetrachords, based on *strong* and *soft* genera, 676

construction of, with epimore ratios, 673–674

diatonic genus, preferred over chromatic and enharmonic genera, 673

Diatonic Genus 4 and 7, 13-limit ratios, 674–677, 679–681

Rast or Mustaqim, 727–731

modal origins

Arabian, on Al-Farabi's and Ibn Sina's *uds*, 747–754

Turkish, on Safi Al-Din *First Ud*, 732–733, 744–748

Safi Al-Din

84 Melodic Modes

First and Second Ud Tunings, 724–725

35 modes, conjunct tetrachord/
pentachord combinations of,
722–726

49 modes, conjunct tetrachord
combinations of, 722–726

of seven-by-twelve matrix system, 728–729

6 *Awazat* (secondary or derived),
730–733

12 *Shudud* (primary or principle),
726–727, 730–733

Turkish theory, modern, on *First Ud*

5 of 6 'Variant' *Maqamat*, 744–745

10 of 13 Basic *Maqamat*, 740–744

Archimedes, 265, 610Archytas, 86–89, 92, 280, 283, 288–289, 297, 311, 318Area moment of inertia. *See* Bars, rods, and tubesArel, H. Sadettin, 738Aristoxenus, 280, 289, 308–321, 624–625, 642, 646–649, 651, 655Arithmetic divisions/progressions. *See also* Cents,

harmonic series; Means

definition, 253–254

Archytas, 86–87

frequency ratios

of "double-octave and a fifth," ratio $\frac{6}{1}$,
426–428

of "fifth," ratio $\frac{3}{2}$, 437

Rameau, 434–436, 441–444

of "octave," ratio $\frac{2}{1}$, 91–93, 437

of "triple-octave," ratio $\frac{8}{1}$, Rameau, 428–433
of harmonic series, 254–255

length ratios

of "double-octave and a fifth," ratio $\frac{6}{1}$

Rameau, in context of *dual-generator*,
444–446

Salinas, 402–403

Stifel, 386–390

Zarlino, 392, 394

of "fifth," ratio $\frac{3}{2}$, 325, 396, 437

Rameau, 442–444, 446–448

Safi Al-Din, 381

Stifel, 401

Zarlino, 377–378, 382–384, 385–386,
399–401

of five tetrachords, Ptolemy, 328

of "fourth," ratio $\frac{4}{3}$, 297, 325

Ptolemy's Even Diatonic, 329

of "octave," ratio $\frac{2}{1}$, 87–89, 91, 295–296, 437
ancient length ratios *vs.* frequency ratios,
87–91

Ibn Sina, 379–381

Ptolemy, 295–297, 324

Rameau, 431–432

Zarlino, 382–383

of "whole tone," ratio $\frac{9}{8}$

Al-Farabi, 354–355, 636

Cardan, 355

Galilei, Vincenzo, 355–356

Mersenne, 356

Ptolemy, 354, 636

Zarlino, *compound unities*, 390–391

Aron, Pietro, 342, 345, 347, 352–353Ayyar, C. Subrahmanya, 586**B**Bach, Johann S., vi, 350Barbera, Andre, 82, 308Barnes, John, 349–350Bars, rods, and tubes

antinode (*clamped-free bar*)

bending (BA), 175

location of, first four modes (bar, rod, and
tube), 177

displacement (DA), 175

local reduction at, tuning process, 176–177

antinode (*free-free bar*)

bending (BA), 153–154

bending moment at, 155–157

local reduction at, tuning process, 160–163

location of, first five modes (bar, rod, and
tube), 162

displacement (DA), 153

experiment, 153–154

- area moment of inertia (bar, rod, and tube), 159
- bending
- stiffness, 158
 - and bending moment, direct
 - proportionality, 159–160
 - effect on mode frequencies, 159
 - as restoring force, 148, 152
 - tuning effect on, 160
 - wave speed, 149–150, 159
 - and bending wavelength, inverse
 - proportionality, 149
 - and dispersion, 148–149
 - effect on inharmonic frequencies, 148–149
 - experiment, 149, 153–154
 - stiffness effect on, 147–148
 - tuning effect on, 160
 - wavelength, 150
 - effect on bending moment, 154
 - experiment, 149
- dispersion
- definition, 149
 - variables, indicators of, 152–153
- mode
- frequency (*clamped-free bar*), 174–175
 - ratios of second, third, and fourth modes, 174
 - frequency (*free-free bar*), 148–149, 151–152
 - and bending moment, 154
 - and height, direct proportionality, 158–160
 - ratios of second, third, and fourth modes, 148
 - theoretical *vs.* actual, rosewood test bar, 163
 - tuning effect on, 160–161
 - shape (*clamped-free bar*), of first three modes, 175
 - shape (*free-free bar*), of first three modes, 153
- node (*clamped-free bar*)
- bending moment and shear force at, 175
 - local reduction at, tuning process, 176–177
 - locations of, first four modes (bar, rod, and tube), 177
- node (*free-free bar*)
- bar mounting considerations, 163, 170–171
 - locations of, first five modes (bar, rod, and tube), 162
 - shear force at, 155–157
- plane sections (bar, rod, and tube), 159
- radius of gyration (bar, rod, and tube), 159
- restoring forces (*clamped-free bar*), 154
- bending moment, 175–176
 - shear force, 175–176
 - tuning effect on, 176–177
- restoring forces (*free-free bar*), 154
- bending moment, 154–161
 - effect on geometry of bar elements, 156
 - shear force, 154–158, 160–161
 - tuning effect on, 160
- tuning process (*clamped-free bar*), 176–177
- mass loading, bars *vs.* reeds, 178
 - tools and techniques, 177
- tuning process (*free-free bar*), 160–161
- bass marimba bar
 - frequencies, first three modes; before, during, and after tuning, 165
 - frequency changes
 - eighteen analysis/decision steps, 168–170
 - eighteen tuning steps, 165–168
 - length limitations of, 164–165
 - triple-arch design, bar profile, 161–163
 - tuning function of first three antinodes, 161–163
 - higher mode frequencies
 - effect on pitch perception of fundamental, 163–164
 - tuning possibilities of, 164
 - tools and techniques, 164
 - treble marimba bar
 - frequencies, first two modes; before and after tuning, 171
 - length limitations of, 171
 - single-arch design, bar profile, 171
 - tuning function of first two antinodes, 170–171
- Bass Canon
- construction of, 800, 805
 - dimensions, 805
 - Plate 3, 847
- Bass Marimba
- 24 bars of, frequency ratios, 827
 - dimensions of, longest and shortest bars, 826
 - mode tuning of, 161–165
 - properties of, Honduras rosewood, 172–173, 885–886
- C₂ cavity resonator, Resonator II, 221
- dimensions
 - inside, 217
 - outside, 220
 - tuning of, 219–222
 - dimensions, 826
 - Plate 8, 852
- Beat rates. *See also* Inharmonic strings
- of 12-TET, 136
 - “fifth,” 136
 - “fourth,” 137
 - “major third,” 136–137, 139–140, 146n.28
 - beating phenomenon, 135–137, 365
 - effect on musical quality, 108, 118, 135
 - limits of human perception, 136
 - Huygens, on the consonance of 7-limit ratios: $\frac{7}{5}$, $\frac{10}{7}$, and $\frac{7}{4}$, 364–365
 - Ramatatya, description of dissonance on *vina*, 572

Beat rates (*Continued*)

- of shimmering “octave,” 509, 515
 - Balinese gamelan
 - gender rambat*, 529
 - penjorog* (beat rate), 524
 - Javanese gamelan
 - gender barung*, 516, 519–522
 - saron demung*, 509–511
- of string
 - consonance *vs.* dissonance, 365
 - flexible canon strings *vs.* stiff piano strings, 140–142
 - small integer ratios *vs.* large integer ratios, 138–140
 - harmonics
 - coincident, 136ff.
 - intermediate, 137ff.

Beeckman, Isaac, 406Benade, Arthur H., 126, 235, 247Bending moment. *See* Bars, rods, and tubes, restoring forcesBenedetti, Giovanni Battista, 415–416Bernoulli, Daniel, 426Bharata, 540–550, 552–562, 564, 572, 577, 598–599Bhatkhande, V.N., 581, 587, 594–596Boehm, Theobald, 235Boethius, 284, 380Break strength

- definition, 35
- of string
 - high-carbon spring steel music wire, 884
 - plain, 35, 111, 134
 - wound, 40

Brothers of Sincerity (Ikhwan al-Safa), 620–622Brown, Robert E., 528Brun, Viggo, 458Buskirk, Cheryl M., viii**C**Canons. *See also* Bass Canon; Harmonic/Melodic

Canon; Little Canon

- Arabian *qanun*, ancient instrument, 628
 - Abul-Salt, 628, 630–632
 - Al-Farabi, 632
 - Al-Jurjani, 629–631
 - hamila* (moveable bridge), 628, 631–632
 - Ibn Sina, 670–672
 - Ibn Zaila, 628
 - from *qanun* to *ud*, evolutionary process, 632, 669–670
- construction requirements
 - bridge, 65
 - original design, 792–794
 - string, 65, 118
- function of, 77–78, 80, 792
- Greek *kanon*
 - definitions, 65

Euclid, 76, 81–82

Plato, 91–92

Ptolemy, 78

Canright, David R., v–vi, 101, 156, 877

Plate 15, 859

Cardan, Jerome, 355Cents

calculation

adding and subtracting, 269

conversion

cents to decimal ratio, 270

cents to integer ratio. *See* Euclidean algorithm

frequency ratio to cents, 268–269

Ellis, 1200th root of 2, 267

increasing or decreasing frequency by cents, 270–271

interval comparison, 270

definition, 268

exponent

definition, 257–258

exponential function, 257

as logarithmic function, three aspects, 258

first law of, 259

four laws of, 277n.11

Stifel’s description, 276n.6

harmonic series

as arithmetic progression, 254–255

human process of adding intervals in sequences, 260–265

no human perception of interval patterns, 254

as geometric progression, 254–255

human perception of interval patterns, 254–255

mathematical process of multiplying frequencies in sequences, 256–262, 263–265

intervals of, verbal terms *vs.* mathematical terms, 64–65, 256

logarithm

antilogarithm., 259, 262ff.

common

base 1.00057779, 109, 267–268

base 2, 262–264, 274

base 10, 258–262

definition, 258

four laws of, 278n.13

guitar frets, 271–273

and human hearing, adding process, 262

logarithmic function, 258

as exponential function, three aspects, 258

musical slide rule, 273–275

Napier, inventor of, Greek *logos* + *arithmos*, 257natural, base *e*, 109, 258

- “octave” equivalence
 definition, 257
 as human trait, 257
 ratio, simplification of, 257
- Chalmers, John H., viii, 308
- Cheve System
 cipher and cipher-dot notation, 508, 535n.2
 letter-dot notation, 599
- Chiao Yen-shou, 502
- Ch'ien Lo-chih, 502
- Ch'in (qin)
 construction of, 488–489
 as microtonal instrument, 496
- Chinese scales. *See* Scales, Chinese
- Ching Fang, 502
- Chrysalis
 construction of, 788–789
 dimensions, 790
 Plate 1, 845
 Plate 13, 857
 score, excerpt from *Song of Myself*, 800–802
 stringing and tuning of, for *Song of Myself*,
 794–797
- Chrysalis Foundation, 839, 841, 843
 New Music Studio, 842–843
 Workshop, Plate 16, 860
- Chu Hsi, 492
- Chu Tsai-yü, 354, 502–504, 542, 569, 749
- Chuquet, Nicolas, 339–341
- Cleonides, 309–310, 649, 655
- Cohen, H.F., 139
- Comma of Didymus. *See* Greek ratios, comma
- Comma of Pythagoras. *See* Arabian ratios; Greek ratios, comma
- Compression and rarefaction
 alternating regions
 of plane wave, in solid, liquid, gas, 186–187
 of sound wave, 183–184
 of bar-to-cavity resonator coupling, 214, 222
 of bar-to-tube resonator coupling, 196
 definition, 130–131, 183, 200
 dependence on wave speed, 131
 in infinite tube, 200–203
 in resonator
 cavity, 213, 219
 tube, 190–196, 203–208
 of soundboard, inefficient regions, 133
 of string, leading and trailing surfaces, 130–131,
 183–184
- Conductivity. *See* Flutes, admittance
- Cremer, L., 123, 131
- Cristofori, Bartolomeo, 118, 130
- Crocker, Richard L., 308
- Crusoe, Robinson, 788
- D**
- D'Alembert, Jean le Rond, 423
- Dattila, 545, 548–549, 558
- de Fontanelle, Bernard Le Bovier, 438
- de Villiers, Christophe, 406
- D'Erlanger, Baron Rodolphe, 625, 666, 675, 755,
 758–760
- Descartes, Rene, 404
- Diamond Marimba, 13-Limit
 49 bars + 5 bars of, frequency ratios, 824–825
 dimensions of, longest and shortest bars, 825
 mode tuning of, 171–172
 properties of, Honduras rosewood, 172–173,
 885–886
 construction of, 825–826
 dimensions, 825
 expansion of
 Meyer's 7-limit Tonality Diamond, 824
 Partch's 11-limit Diamond Marimba, 824
 Plate 7, 851
 resonator
 airtight seal, making, 211
 dimensions of lowest, G₃ at 196.0 cps, 198
- Didymus, 280, 319, 329–332, 664
- Dieterici, Friedrich, 620
- Dispersion
 definition, 104, 149
 variables, indicators of, 152–153
- Downbearing force
 of string on canon bridge, 792
 string's angle of deflection, 794
- E**
- el-Hefni, Mahmud, 612
- Ellis, Alexander J., 265, 267
- End corrections. *See* Flutes, corrections;
 Resonators, cavity; Resonators, tube
- Equal temperaments
 12-TET, 361–362
 $\frac{1}{12}$ ditonic comma
 calculation of, 352–353
 cumulative reduction of twelve consecutive
 “fifths,” 353–354
 cent comparisons
 to 3-limit, 5-limit, 7-limit, and $\frac{1}{4}$ -Comma
 Meantone scales, 376–377
 Ramis *vs.* Stevin, 375–376
 discovery of
 Chu Tsai-yü, 502–504
 Stevin, 358–362
 ditonic comma (comma of Pythagoras),
 definition, 335, 350
 “fifths”
 circle of 12, *That* produces, 352–353
 spiral of 12 flat “fifths,” *That* closely
 approximates, Liu An, 500–501

Equal temperaments (*Continued*)

- frequencies of eight octaves, 879
- frequency ratios, powers of the 12th root of 2, 271
- fret equations for, 272–273
- on fretted lutes and viols, 340, 348
- “major third” of, two integer ratio approximations, 140
- “semitone” of
 - approximate value, rational, ratio $18/17$
 - Al-Farabi, 354–355
 - Cardan, 355
 - lute fretting instructions
 - Galilei, 355–356
 - Mersenne, 356–359
 - lutenists’ preference over exact value, 357
 - origin of, Ptolemy’s arithmetic division of “whole tone,” ratio $9/8$ 354
- exact value, irrational, 353–354
 - 12th root of 2, 353–354
 - Chu Tsai-yü, 503
 - Stevin, 360–361
- vs.* eleven integer ratio approximations, to the nearest cent, 459
- vs.* frequencies of the harmonic series, 63–64
- 22-TET, Indian theory, ancient
 - Bharata
 - 22-*sruti* scale, 540–543, 544–547
 - logarithmic *pramana sruti*, 541–542
 - no basis in fact, 542–543, 572–573
 - 24-TET, Arabian theory, modern, 755–758
 - formulaic imposition of, 749, 755–758
 - on fretless *ud*, as moot issue, 760–761
 - Marcus
 - ambiguity of “quarter tones” in musical practice, 760–761
 - description of Mashaqah’s twenty-four “quarter” tones, 759
 - used to justify “incorporation” of “quarter tones,” D’Erlanger and Marcus, 755–758, 760–671
- 31-TET
 - Huygens, logarithmic calculations, 362–364
 - spiral of 31 “fifths,” *That* closely approximates, 362–363
- 53-TET
 - spiral of 53 “fifths,” *That* closely approximates, Ching Fang, 502
 - Turkish theory, modern, close approximations of 4-, 5-, 8-, 9-, and 12-comma intervals, 736–737
- 60-TET, spiral of 60 “fifths,” *That* closely approximates, Chiao Yen-shou, 502
- 360-TET, spiral of 360 “fifths,” *That* closely approximates, Ch’ien Lo-chih, 502

- Eratosthenes, 280, 318, 329–332
- Euclid, 76, 81–82, 86, 92–93, 280, 283, 287–289, 297–298, 302–308, 311, 318, 337–339, 341, 610–611, 619, 624–625, 759
- Euclidean algorithm
 - 12-TET, eleven integer ratio approximations, 459
 - Brun, 458
 - and continued fractions, 484n.364
 - for converting irrational decimal ratio to rational integer ratio approximation, 458–459
- Exponent. *See* Cents

F

- Farhat, Hormoz, 677, 686–696, 744, 759, 771–772
- Farmer, Henry George, 366, 379, 401, 610, 619–621, 623–628, 635, 668, 709, 716, 727
- Flageolet tones
 - of *ch’in*, 488–492
 - experiments
 - Roberts, 421
 - Sauveur, 424–426
 - of trumpet marine, 408–409
- Fletcher, Harvey, 99–100, 113, 135
- Flexible strings
 - antinode
 - definition, 53, 55
 - formation of, as regions of maximum motion, 47–48, 51–54
 - locations of, first six modes, 58
 - equations, for stopped (bridged) string
 - frequency, 77
 - length, 77
 - equilibrium position, dynamic, 48–49
 - frequency. *See also* Flexible strings, mode
 - definition, 55, 57
 - dimensional analysis, 55
 - as function of wavelength, 80
 - harmonic series
 - definition
 - mathematical*: length ratios of subdividing string, 63–64, 427–428
 - verbal*: frequency ratios of subdividing string, 63–64, 427–428
 - discovery of
 - infinite series, first thirty-two harmonics, Sauveur, 423–424
 - limited series, first six harmonics, Mersenne, 404–408
 - first sixteen harmonics, two series, 64
 - intervals of, verbal *vs.* mathematical, 64–65, 256
 - harmonics
 - definition, 63–64
 - first six on bridged canon string, 66
 - first sixteen of subdividing string, 64

- frequency ratio, 73
 - ideal characteristics, 44
 - impedance, mechanical wave, 121
 - loop
 - count (mode number), 59, 105
 - definition, 59
 - multiple loop length, 70
 - ratio, 73
 - pattern, 59
 - single loop length, 59–60
 - ratio, 73
 - mode
 - definition, 98
 - frequency
 - definition, 59, 62
 - and mode wavelength, inverse proportionality, 61
 - shape, first six modes, 58
 - node
 - definition, 53, 55
 - formation of, as points of minimum motion, 48–49, 51–54
 - function of, 53, 116n.5
 - locations of, first six modes, 58
 - two different kinds, 55–56
 - period
 - definition, 54, 56
 - dimensional analysis, 54
 - pulse, transverse traveling
 - crest and trough, 44–46
 - collision, 46–49
 - reflection, 45–46
 - definition, 44
 - and Pythagoras, 44
 - ratios. *See also* Ancient length ratios
 - frequency
 - definition, 62, 72
 - as function of length ratio, 80, 93
 - of harmonics, 73
 - and length ratio, inverse proportionality, 63, 80
 - of non-harmonic tone, 73
 - interval
 - definition, 67
 - of harmonics, right sides of bridged canon strings, 68–69
 - of non-harmonic tone, 74
 - length
 - complementary, 69, 78
 - definition, 59, 69–70
 - and frequency ratio, inverse proportionality, 63, 80
 - human comprehension of, 79, 80, 93
 - of non-harmonic tone, 73
 - simple harmonic motion (SHM)
 - definition, 50
 - of particle motion, in cord/string, 50–51
 - superposition
 - definition, 48
 - interference
 - constructive, 47–48
 - destructive, 48–49
 - standing wave, production of, 48, 51–54
 - wave
 - standing
 - definition, 2, 48, 51–52
 - of first six modes, 58
 - motion of, 52–53
 - and sound production, 53–54
 - transverse traveling
 - in cord, 50–51
 - definition, 50, 183
 - in string, 2–3, 44–53, 58
 - superposition of, 51–54
 - wave speed, transverse traveling, 60–61
 - wave train (waveform)
 - definition, 50–51
 - frequency of, 55, 57
 - period of, 54, 56
 - as transverse traveling wave, 50–51
 - wavelength
 - definition, 54
 - equations, 60, 71
 - of first six modes, 58
 - as function of frequency, 80
- ## Flutes
- admittance
 - complex/non-complex, 233
 - definition, 233
 - terminating, 234
 - and cutoff frequency, 247
 - of embouchure hole and flute bore, 235
 - of tone hole, flute bore, and tube-piece, 236
 - conductivity
 - definition, 233
 - of duct, fictitious and actual, 234–236
 - of embouchure hole and flute bore, 234
 - of tone hole diameter, limitations, 233–234
 - corrections, 232
 - at embouchure hole, 228, 231–232
 - empirical data
 - concert flute, 235
 - simple flute, 240
 - at key pad, 228
 - Nederveen and Benade, 237
 - at open tube end (end correction), 228, 231–233
 - at tone hole, 228, 231–232, 236, 244–245
 - Nederveen, 238
 - cutoff frequency, Benade, 247
 - embouchure hole, typical dimensions
 - concert flute, 235

Flutes (*Continued*)

- simple flute, 240
 - frequency
 - when known, predicting flute dimension and tone hole location, 229–241
 - when unknown, determining from flute dimension and tone hole location, 242–246
 - impedance, acoustic, non-complex, 233
 - intonation, 227
 - at embouchure hole
 - airstream, 227, 240
 - blowing technique, 236
 - lip coverage, 227, 233, 240
 - flute tube effect on, 235–236
 - making, simple flute, 236, 246–248
 - critical variables
 - embouchure hole correction, 240
 - flute wall thickness, effect on timbre, 247
 - musical interval, Nederveen, 237–238
 - tone hole diameter, 239, 240–242
 - speed of sound in, 229
 - subharmonic series, 246
 - standing wave, 231, 234, 245, 248n.10
 - substitution tube, Nederveen, 229–230
 - two different kinds, 230–232
 - tone hole
 - dimension and location
 - concert flute, 239
 - simple flutes, 241
 - from embouchure hole, strategy for length calculation, 231–232
 - tube length
 - closed-closed (substitution tube), acoustic and effective, 230–232
 - effective
 - at embouchure hole, approximate, 234–235 Nederveen, 233
 - at tone hole, exact, 236 Nederveen, 237
 - open, measured *vs.* effective, 229–231
 - tuning process, simple flute, 236
 - variables, twenty-seven symbols, 228–229
- Force. *See also* Restoring force; Weight; Weight density
- definition, 5–6, 8
 - English Engineering System, 9, 12, 16
 - pounds-force (1 lbf), 7–9
 - inconsistent system, 9–10
 - English Gravitational System, 9
 - pounds-force (1 lbf), 10ff.
 - consistent system, 9–11, 14–15
 - experiment, 7–8, 10, 15
 - of gravity, 6–8, 10–12
 - of muscular effort, 5, 8, 16
 - Newton, 2, 5
 - of spring, 11

- of tension, 18–19

- unit, 10, 14–15

Forster

- Cris, v–viii, 839–844

- Plate 13, 857

- Plate 15, 859

- Heidi, viii, 839–844

- Plate 14, 858

- Plate 15, 859

Forster instruments. *See* Bass Canon; Bass

- Marimba; Chrysalis; Diamond Marimba,

- 13-Limit; Glassdance; Harmonic/Melodic

- Canon; Just Keys; Little Canon; Simple Flutes

Franklin, Benjamin, 828Frequency

- of bar

- clamped-free, 174–175

- mass loaded, 178

- free-free, 148, 151–152, 158

- definition, 54–55

- dimensional analysis, 55

- of longitudinal mode: in plain string, uniform

- bar, and fluid in tube, 187

- of resonator

- cavity, 215–217

- tube

- closed, 210

- closed-closed, 230

- open, 209

- of spring-mass system, 185, 219, 223n.9

- of string

- flexible, 58–62, 102–103

- stiff, 99, 105–107, 112

- and wavelength

- as function of, 80–93

- inverse proportionality, 61–62

Frequency ratios. *See* Flexible strings, ratiosFreud, Sigmund, 557**G**Gaffurio, Franchino, 367Galilei

- Galileo, 283, 338, 356

- Vincenzo, 315, 355–356, 358

Gamelan. *See also* Scales, Indonesian

- instruments

- Bali

- bar percussion instruments, built in pairs:

- pengumbang* (low) and *pengisep* (high),

- 524, 532–533

- penjorog* (beat rate) of, 524

- shimmering “octaves” of, tuned

- sharp, 524

- sharp and flat, 529–530

- gangs* (*saron*), 533

- gender*
dasa, 523
jegogan, 524
rambat, 529–530, 532
tjalung (jublag), 530–534
wayang, 522
suling gambuh, 524–528, 530, 532
trompong, 523–524, 527–528, 532
- Java
 bar percussion instruments, beat rates
 four different kinds, 522
 of *tumbuk* tuning, 519–522
gender
barung, 513–517, 519–521
panerus, 519
saron
demung, 509–511, 517–520
ritjik (saron barung), 510–511
 shimmering “octaves,” tuned
 sharp, 509–511
 sharp and flat, 515–516, 519–521
- orchestras, 508
- Bali
 Court of Tabanan, 526
 Kuta Village, 523
 Pliatan Village, 524, 530–534
 Puri Agung Gianyar, 529–530
 Tampak Gangsal, 523, 527
- Java
 Kangjeng Kyahi Sirat Madu, 520
 Kyahi Kanjutmesem, 509–510, 515–516,
 519–522
- Geometric divisions/progressions. *See also* Cents,
 harmonic series
 definition, 254
 Archytas, 86–87
 of harmonic series, 254–255
 length ratios
 of 12-TET
 Chu Tsai-yü’s solution, 503–504, 542
 Stevin’s solution, 358–362, 542
 of “fifth,” ratio $\frac{3}{2}$, Chuquet, 339–341
 of “fourth,” ratio $\frac{4}{3}$
 Aristoxenus, 309–318
 Al-Farabi’s interpretations of, especially
Jins 8, 646–655
 arithmetic-geometric asymptote,
 313–315, 318
 modern interpretations of, 309–313,
 466n.75, 655
 Ptolemy’s interpretation of, 316–317
 of “major third,” ratio $\frac{5}{4}$; meantone ratio,
 342–343
 of “minor third,” ratio $\frac{6}{5}$, Chuquet, 339–341
 of “octave,” ratio $\frac{2}{1}$, 297–299
 Chuquet, 399–341
 Euclid’s mean proportional, 297–298
 Zarlino, 338–339
 of “whole tone,” ratio $\frac{9}{8}$
 Ptolemy’s rational approximation, 320
 Zarlino, 338–339
- Ghosh, M., 549, 556–557, 559
- Glassdance
 48 crystal glasses, frequency ratios, 830–831
 Ptolemy’s Soft Diatonic, 832
 construction of, 828–830
 dimensions, 832
 Plates 9, 10, and 14, 853–854, 858
 tuning process, of brandy snifter glasses,
 830–831
- Govinda, 565, 574–576, 578–584
- Greek ratios
 classification
epimere, 285, 287
epimore, 284, 287
 Euclid, 287
 Ptolemy, 288, 321–323, 325–326, 328–329,
 354
 Pythagoreans, 288, 323
 Zarlino, 397
equal, 284
 Ptolemy, 324
multiple, 284, 286
 Euclid, 287
 Mersenne, 417
 Ptolemy, 288, 321, 323
 Pythagoreans, 288
multiple-epimere, 285–286
 Ptolemy, 288, 319
 Pythagoreans, 288, 319
multiple-epimore, 284–285
- comma
 of Didymus (syntonic comma), 315, 350, 363,
 400, 547, 697
 Aron, 344–345
 definition, 344
 Ramamatya, 572
 Safi Al-Din, 372–374
 of Pythagoras (ditonic comma), 319–320, 487,
 542
 12-TET, 352–354
 Al-Farabi, 655
tunbur of Khurasan and original *tunbur*,
 697–706
 Al-Jurjani, 722–723
 Al-Kindi, 334, 611, 615–616, 620
 Aristoxenus, 314, 318
 definition, 314–315, 334–335
 Ramamatya, 571–572
 Safi Al-Din, 372–374
 Turkish theory, modern, 736–737
 Werckmeister, 350–351

Greek ratios (*Continued*)

- Euclid, 76, 81–82, 287
interval
apotome, 314–315, 319
‘*Apotome Scale*’, 336, 418–419
Ptolemy, 314–315, 319
Ramamatya, 571
apyknon, 326–327, 659–660
Al-Farabi, 660–663
Ptolemy, 328–329, 334
diesis, modern, 464n.48
Aristoxenus, 314–315, 318
Philolaus, 299–300
diplasion, 81, 86, 286–287
diapason, 81, 407, 416
disdiapason, 407, 410
ditonon, 384
ditone, 314, 416, 633–634
epi prefix, 81, 384
epitetartos (*epitetartic*), 286–288, 657
epitritos (*epitritic*), 81–82, 86, 286–287
diatessaron, 81, 384
epogdoos (*epiogdoos*), 81–82, 86, 287
tonon, 82, 384
hemiolios, 81–82, 86, 287
diapente, 81, 382, 384, 412–413
limma, 300, 314–315, 319
Aristoxenus, 315–316
‘*Limma Scale*’, 336
Ptolemy, 314–315, 319–320
Ramamatya, 571
pyknon, 326–327, 659–660
Al-Farabi, 660–663
Didymus, 329–330
Eratosthenes, 229–330
Ptolemy, 326–329
schisma, 372–373
“schismatic fifth,” 372–375
- Nicomachus
classification, translated as Latin ratios, 486
dimoirou (two-thirds), 84, 96n.30
hemiseias (half), 84, 96n.30
sound (frequency) and string length, inverse
proportionality, 84–85
“vibration ratio,” 85
“weight ratio,” 82–84, 86
- Plato, 91–92
- Ptolemy. *See also* Arabian ratios
classification, 323
emmelic/melodic (*epimores* smaller than $\frac{4}{3}$), 288–289, 323–326
homophonic (*multiples*), 295, 323–324
symphonic (first two *epimores*, $\frac{3}{2}$ and $\frac{4}{3}$), 322–326
“quarter-tone,” origin and function of ratios $\frac{32}{31}$ and $\frac{31}{30}$, 290–292

Greek scales. *See* Scales, Greek

Greek tetrachords

- Aristoxenus, six different kinds, 309–310
fractional and integer parts of tones, 309–310
interpretation of
arithmetic-geometric asymptote, 313–315, 318
modern, in cents, 310
Ptolemy, as length ratios, 313, 316–317, 320–321
- conjunct, Terpander’s heptachord, 289, 291–293
definition, 289
disjunct, Pythagoras’ octachord, 289, 293
genera of; diatonic, chromatic, and enharmonic, 289–290
Archytas, 289
Aristoxenus, 289, 309–310
Ptolemy, 326–332
- Greater Perfect System (GPS)
composed of four different kinds, 289–292
diatonic genus, in context of
Euclid, 303–304
Philolaus, 301
Ptolemy, 333
Euclid, description of 15-tone “double-octave,” 289, 302–303
harmoniai (*modes*) of, seven different kinds, 290–292
standard “octave” of
Dorian Mode, Greek, 301
Lydian Mode, Western, 301
three scales, based on three genera, 292
- Lesser Perfect System (LPS), composed of three different kinds, 289–291
- Philolaus
Diatonic, ancient and original division, 299–301, 462n.17, 464n.47
on Al-Kindi’s *ud*, 615–617
in Euclid, 302–307
plagiarized by Plato, 91–92
- Ptolemy
Catalog of Scales, 318–319, 330–332
Even Diatonic, origin of Arabian “neutral second” and “neutral third” tetrachords, 329, 679–681
pyknon and *apyknon*, three conceptual principles for the division of tetrachords, 326–330
Soft Diatonic, unique musical quality of a minor scale, 334, 832
Tense Diatonic, origin of Western major scale, in Zarlino, 333, 832
theory of graduated consonance, mathematical/musical basis for the division of tetrachords, 321–326

Guitar frets. *See* Cents, logarithm
 Gullette, Will, viii, 845–856, 858–860
 Gupta, Abhinava, 550

H

Harmonic divisions/progressions. *See also* Means
 definition, Archytas, 86–87

frequency ratios

of “fifth,” ratio $\frac{3}{2}$, 437

Rameau, 435

of “octave,” ratio $\frac{2}{1}$, 91–93, 437

harmonic series, context of, 63–64, 80, 427–428

length ratios

of “double-octave and a fifth,” ratio $\frac{6}{1}$,
 426–427

Rameau, in context of *dual-generator*,
 444–446

Salinas, 403

Stifel, 378, 386–390

Zarlino, 378, 392–395

of “fifth,” ratio $\frac{3}{2}$, 397, 437

Rameau, 440–443, 446–448

Safi Al-Din, 381

Stifel, 401

Zarlino, 377–378, 382–383, 386, 399–401

of “octave,” ratio $\frac{2}{1}$, 89–91, 437

ancient length ratios *vs.* frequency ratios,
 87–91

Ibn Sina, 379–381

Rameau, 431–432

Zarlino, 382–383

of “triple-octave,” ratio $\frac{8}{1}$, Rameau, 430,
 432–433

Zarlino, *sonorous quantities*, 390–391

Harmonic/Melodic Canon. *See also* Canons

construction of, 790–792

bridge, original design, 792–794

dimensions, 794

Plate 2, 846

score, excerpt from *Song of Myself*, 800, 803–804

stringing and tuning of, for *Song of Myself*,
 794–795, 798–799

Harmonic series. *See* Cents; Flexible strings; Just
 intonation

Harmonics. *See* Flexible strings; Inharmonic
 strings, tone quality; Just intonation

Harpsichords. *See also* Pianos, *vs.* harpsichords

Huygens, keyboard for 31-TET, 363–364

impedance ratio

air-to-soundboard, 129

soundboard-to-string, 126

inharmonicity

analysis of, 110–111

coefficients of, plain strings, 111, 126

soundboard

critical frequency, 132–133

dimensions, typical

surface area, 129

thickness, 125

impedance

plate *vs.* soundboard, 124, 126

radiation, of air at soundboard, 129

sound radiation, 133

string, D_4 (d')

dimensions and tension, typical, 125

Hubbard, 144n.10

impedance, 125–126

Harrison, Lou, v

Heath, Thomas L., 308

Helmholtz, Hermann, 212

resonator. *See* Resonators, cavity

Hitti, Philip K., 611

Hood, Mantle, 512, 517–519, 521, 524

Hsu Li-sun, 486, 492

Huang Ti, 485

Hubbard, Frank, 125

Huygens, Christiaan, 289, 362–365

I

Ibn Al-Munajjim, 619–624, 644–645

Ibn Khallikan, 619

Ibn Misjah, 619

Ibn Sina, 93, 326, 366, 378–380, 384, 435, 610, 614,
 625, 628, 636, 666–689, 692–696, 703, 715–716,
 722, 727, 744, 749–755, 758–759

Ibn Suraj, 619–620

Ibn Zaila, Al-Husain, 628

Ikhwan al-Safa (Brothers of Sincerity), 620–622

Impedance

acoustic

complex, 197

non-complex, 197

of duct, 233

of room, 199

of tube, 198

definition, 120, 127, 129, 197, 233

mechanical

complex, 120–121, 124

radiation impedance, 129

of soundboard, *vs.* infinite plate, 122–124

non-complex

of air at soundboard (radiation impedance),
 129

of plate, 123

of string, 121

specific acoustic

complex, 127

non-complex, 127–128

of air (characteristic impedance), 128–129

Indian musical terms. *See also* Scales, Indian

ancient

Bharata, 540

grama (tone-system, tuning, or parent scale), 547*jati* (melodic mode), three different types(1) *samsarga* (combined), 549–550(2) *suddha* (pure), 548–550, 552–554(3) *vikṛta* (modified), 548–550, 552, 555–5564 hierarchical functions of tones: *graha*,*amsa*, *apanyasa*, *nyasa*, 548, 552,

560, 576, 602n.30, 691

laksana (properties) of, 547–548, 552,

554–555, 558–559, 602n.30

possible influence on, 556–557

dastgah, 691*magam*, 738*patet*, 512*rasa* (qualities) of, 547, 557–560*raga* (charm) of a note, 560–561

songs, of the theater

dhruva, 558–560*gana*, spurious text, 559–560

characteristics of, 560

gitaka, 558*sruti* (interval)of 22-*sruti* scale, ratio analysis of *Sa-grama* and *Ma-grama*, 544–547, 550–551

definition, 540

of harp-*vina* tuning experiment, 541, 543–544of *jatis*, 554–555*pramana* (typical), 541–543, 547of *Sa-grama* and *Ma-grama*, 540–541, 544–545*svaras* (notes)7 *svaras* of *Sa-grama* and *Ma-grama*, 540–541of 22-*sruti* scale, ratio analysis of *Sa-grama* and *Ma-grama*, 544–547

definition, 540

suddha (pure), 566of *svarasadharana* (overlapping note), ratio analysis of An and Ka, 549–551*vina* (harp-*vina*), 541–544, 547, 550–551, 553, 564*gramaragas*

definition, 562

and *laksana*

of Kudimiyamalai inscription, 562–564

of Matanga, 562–564

of Narada, 562–564

of Sarngadeva, 562–564

Matanga, 556

4 *suddharagas* (pure *ragas*), 562–563*raga* (elements of melodic sound), first formal definition, 561–562

Narada, 559

7 *gramaragas*, first complete compilation, 559*gramaraga*

first formal definition, 560

or *gana* (song), 559–561, 563–564*guna* (qualities) of, 560

Sarngadeva, 548

gramaraga, 562–564*suddha jatis* and *laksana*, definition, 548*svaras**suddha*, 566*vikṛta*, 566

North India

Bhatkhande, 587

10 most popular *Thats*, 581, 596*Bilaval That*, modern *suddha* scale, 592–593, 600

Narayana and Ahobala, 587

12-tone *vina* tuning, 587–592*samvadtva* (consonant intervals), 590–592*suddha svaras* (pure notes), 587*vikṛta svaras* (modified notes), 590*raga*, 558, 564, 576, 585

Roy, 595

32 *Thats*, of eight-by-four matrix system, 595–596*chromatic Thats*, not used, 593–594*sitar*

construction of, 597

melodic, drone, and *cikari* strings, 597–599

moveable frets, 597–598

definition, Sachs, 597

letter-dot notation, based on Cheve System, 598–599

Shankar's mastery of String II, Junius, 597

tuning of, example, 598

That

definition

Bhatkhande, 594

Kaufmann, 594

vs. modern *mela* of South India, 594

South India

Govinda, 565

*mela raga*14 most popular *mela ragas*, 58172 abstract *raga*-categories, 56572 musical heptatonic *scales*, first formal implementation of, 565*krama* (straight) or *vakra* (zigzag)

sequences, 565

- two-directional *krama* (straight)
 - sampurna* (complete) principle, 579–582
 - alphanumeric prefixes, 580
 - Kanakangi-Ratnangi* system, 580–581
 - critique of
 - Iyer, 582–584
 - K.V. Ramachandran, 584–585
 - N.S. Ramachandran, 580
 - Mela*
 - Chakravakam*, 580–581
 - Kharaharapriya*, 574, 581
 - Mayamalavagaula*, 574, 580–581
 - Natabhairavi*, 576, 581
 - vs.* Venkatamakhi's *melakartas*, 580
 - raga*, 558, 564, 576, 585
 - Ramamatya, 565
 - 12-tone practical scale, *suddha mela vina* tuning, 569–573
 - Antara Ga* and *Kakali Ni*, elimination of, 572
 - 14-tone theoretical scale, 566–567
 - 7 *suddha svaras*, 566–568
 - 7 *vikṛta svaras*, 566–568
 - denominative raga*, definition, 574
 - example of, *Mela Sri-raga* and *Sri-raga*, 574–576
 - janaka* (parent) *raga*, abstract *raga*-category, 574
 - janya* (born, or derived) *raga*
 - definition, 574
 - laksana* properties, absence of, 574, 576
 - musical *raga*, 565
 - Raga Mechabauli*, 574–575
 - Raga Suddhabairavi*, 574–575
 - Sri-raga*, 575–576
 - modern *Sadjagrama*, Rao, 576
 - svara*
 - anya* (foreign note), 575–576
 - vakra* (note out of order), 575
 - varjya* (omitted note), 575
 - mela* (unifier) *ragas*
 - abstract *raga*-category, 565
 - definition, 574
 - Mela Malavagaula*, 574–575
 - Mela Sri-raga*, 574–576
 - vs.* Venkatamakhi's *melakartas*, 578
 - Venkatamakhi, 565
 - 12-tone scale; new and modern *svara* names; ratios by Ramamatya, 576–577
 - 72 *Melakartas*, of twelve-by-six matrix system, 565, 577–579, 581
 - 19 *melakartas*, most popular in
 - Venkatamakhi's time, 577–579
 - 36 *vivadi* (dissonant) *melas*, or *chromatic melas*, not used, 579, 582, 593–594, 607n.119
 - Venkatamakhi's avoidance of, Iyer, 582–583
 - denominative raga*, definition, 574
 - example of, *Mela Gaula* and *Raga Gaula*, 574
 - janya* (born, or derived) *raga*
 - definition, 574
 - laksana* properties, absence of, 576
 - musical *raga*, 564–565
 - Raga Gaula*, 574–575
 - melakartas*
 - 14 most popular *mela ragas*, 581
 - abstract *raga*-categories, 565
 - definition, 577–579
 - Mela*
 - Bhairavi*, 576, 579, 581
 - Gaula*, 579, 581
 - Malavagaula*, 575, 579, 581
 - Sri-raga*, 575, 579, 581
- Indian scales. *See* Scales, Indian
- Indian tetrachords. *See also* Scales, Indian
 - North Indian theory
 - Bhatkhande, 587
 - 10 most popular *Thats*, 581, 596
 - Roy, 595
 - 32 *Thats*, of eight-by-four matrix system, 595–596
 - chromatic Thats*, not used, 593–594
 - South Indian theory
 - Govinda, 565
 - 72 musical heptatonic *scales*, first formal implementation of, 565
 - Kanakangi-Ratnangi* system, 580–581
 - Venkatamakhi, 565
 - 72 *Melakartas*, of twelve-by-six matrix system, 565, 576–579, 581
 - 36 *vivadi melas*, *chromatic melas*, not used, 579, 582, 593–594, 607n.119
- Indonesian scales. *See* Gamelan; Scales, Indonesian
- Ingard, K. Uno, 149
- Inharmonic strings
 - coefficient of inharmonicity
 - diameter effect on, 110, 118, 134–135, 808–809
 - frequency effect on, 110, 809
 - length effect on, 110, 809
 - steel stringing constants, logarithmic
 - plain string, 110
 - wound string, 115
 - of string
 - harpsichord, average values, 111
 - piano, average values, 110–111
 - piano *vs.* harpsichord, 126
 - plain, 110–111
 - wound, 114–115
 - tension effect on, 110–111

Inharmonic strings (*Continued*)

dispersion
 definition, 104
 and wave speed, effective, 104–107

inharmonicity
 analysis of, stringing and restringing, 110–111, 118
 cent calculation, strings and modes, 108–112

length, effective
 and effective wavelength, 99, 101–102
 stiffness parameter effect on, 101–102
vs. measured length, 101–102

mode
 cents equation, 110
 definition, 98
 frequency equations, 99
 shape, first four modes, 101–102

node, two different kinds, 101–102

piano tuning, 118, 135–137
 stretched octaves
 in bass, tuned flat, 112–114
 in treble, tuned sharp, 111–112

restoring force
 experiment, 104
 tension and stiffness, composite force, 104–105

stiffness effect
 experiment, 104
 on mode
 cents, 110
 frequencies, 98
 shape, 101–102
 on wave speed, effective, 99, 104–106
 on wavelength, effective, 99, 101–102

stiffness parameter
 diameter effect on, 100
 frequency effect on, 100
 length effect on, 100, 105
 steel stringing constant
 plain string, 100
 wound string, 114

of string
 plain, 99–100, 111
 wound, 113
 tension effect on, 99–100, 111

tone quality. *See also* Beat rates
 beat rate, 108, 135–136, 138, 140
 of harmonics
 coincident, 136ff.
 flexible canon strings, 141–142
 stiff piano strings, 140–141
 intermediate, 137ff.

of keyboard instrument
 Benade, 126
 Fletcher, 135

mode number effect on, 140–142

perception of, dissonance effect, 108, 118, 135, 138–141
 thick string *vs.* thin string, 104, 118, 135, 141–142

wave speed, effective, 106
 and dispersion, 104–105
 experiment, 104
 mode number effect on, 105–106
 stiffness parameter effect on, 106

Inharmonicity. *See* Inharmonic strings

Interference

definition, 48
 in infinite tube, constructive and destructive, 200–203
 in string
 constructive, 47–48, 51–52
 destructive, 48–49, 51–52

Interval ratios. *See* Flexible strings, ratios

Isothermal bulk modulus, liquids, 188

Iyer, T.L. Venkatarama, 582

J

Jairazbhoy, N.A., 541, 594, 596

Junius, Manfred M., 597

Just intonation

commensurable numbers of, 365–366
 Galileo, 283
 definitions, two different kinds, 365

Mersenne

harmonic-diatonic system, 404–405, 408, 436
 harmonic series
 as infinite series, not defined, 404–406, 420
 as limited series
 discovery of the first six harmonics, 404–408
 influenced by
 personal convictions, religious and moral, 407–408
 Ptolemy's Tense Diatonic, 407
 Zarlino's *Senario*, 407

harmonics

first 14 trumpet harmonics *vs.* length ratios of “old” trumpet marine, 410–412
 “leaps” of natural trumpet *vs.* harmonics of monochord, four identical ratios, 406
 “leaps” of trumpet marine *vs.* harmonics of monochord, five identical ratios, 406
 length ratios of “new” trumpet marine *vs.* “old” trumpet marine, 410–415
 with prime numbers greater than 5, not acknowledged, 408, 412, 415
 problems comprehending cause of; no knowledge of traveling waves, 404, 422–423
 solved by D'Alembert, 423
 three different terms for, 404

- theory of consonance
 based on Benedetti's observations,
 mechanical motions of strings, 415–416
 contemplated ratios *vs.* endorsed ratios,
 416–419
 7-limit ratios
 detailed analysis and conflicted
 rejection of, 417–420
 qualified acceptance of ratios $\frac{8}{7}$ and
 $\frac{7}{6}$, 420
 origin of, Ptolemy's theory of graduated
 consonance, 321–326
- Meyer
 7-limit Tonality Diamond, original design,
 448–452, 824
 8 tonalities of, 449–451
 13-tone scale of, 449
 minor tonality as inversion of major
 tonality, 452
 moveable boundaries of, 451–452
 origins of
 in Rameau's dual-generator, 444–446,
 448–449, 452
 in triangular tables of
 Al-Jurjani, 401–402, 448
 Salinas, 402–403, 448, 452
 “table of spans,” original description, 451
 on 11-limit ratios, 448
 plagiarized by Partch, 452–453
- Partch
 11-limit Diamond Marimba
 36 bars of, frequency ratios, 452–455
 minor tonality as inversion of major
 tonality, 454
Otonality and *Utonality*, theory of, 453–454
 43-tone scale, 454–457
 7 ratio pairs, to fill chromatic gaps of
 11-limit Tonality Diamond, 454–455
 29 ratios, 67% of 11-limit Tonality
 Diamond, 454–455
 tuning lattice of, Wilson, 455–457
 Meyer's 7-limit Tonality Diamond, three
 changes to, 452–453, 824
- Monophony, theory of, 453
 Zarlino's philosophy of *Unita* (Unity), 429,
 484n.359
- Rameau's dual-generator, 444–446, 454
- Rameau
Demonstration du principe... (1750)
 minor tonality
 attempted to rationalized, 446–447
 failed to rationalize, 447
 as inversion of major tonality, Shirlaw,
 448, 452
 ‘relative’ minor, origin of, 447–448
 subharmonic series, Rameau's recantation
 of, 446
Generation harmonique (1737)
 7-limit ratios, rejection of, 440
 generator (dual-generator), function of,
 444–446
 harmonic progression of fractional string
 lengths, modern length ratios, 440–443
 and Zarlino's ancient length ratios, same
 definition of major tonality, 441
 harmonic series
 consonances produced by nature,
 440–441
 major tonality, directly related to
 fundamental, 441
 minor tonality, not directly related to
 fundamental, 441–446
 subharmonic series, fallacy of, 441–446
Nouveau systeme... (1726)
 harmonic generation, theory of, 436
 harmonic series
 hierarchy of consonances, 438
 “major third,” directly related to
 fundamental, 438
 “minor third,” not directly related to
 fundamental, 439–440
 of single strings, 436–438
 Mersenne and Sauveur, acknowledged,
 436
Traite de l'harmonie (1722)
 7-limit ratios, rejection of, 434
 arithmetic progression of vibration
 numbers, frequency ratios, 428–434
vs. Zarlino's ancient length ratios,
 diametrically opposed descriptions
 of major and minor tonalities,
 434–436
 harmonic generation, theory of, 428–429,
 434
 harmonic progression
 of fractional string lengths (length
 ratios)
 knowledge of, 433–434
 not included, 431
 of Stifel's string length integers (length
 ratios), 428–430, 432–434
 least common multiple (LCM), of length
 numbers, 432–433
 Zarlino's
 philosophy of *Unita* (Unity), 429–431
Senario, 434, 440
- Ramis
 5-limit ratios, four different kinds, 366–367
 12-tone scale, 367–373
 with a default *schismatic fifth*, 372–375

Just intonation (*Continued*)

vs. Safi Al-Din's scale, with six *schisma* variants, 373–375

vs. Stevin's 12-TET, 375–377

Roberts

harmonic series

as infinite series

of discrete flageolet tones, 421

of natural trumpet tones, 421

as series of simultaneously sounding

harmonics, not defined, 421–422

harmonics, with prime numbers greater *That*

5, considered defective, 421

Salinas

7-limit ratios, rejection of, 403

harmonic division of ratio $\frac{6}{1}$, 403

triangular table, 402–403

arithmetic division of ratio $\frac{6}{1}$, 402–403

origin of, Al-Jurjani, 401–402

Zarlino's *Senario*, 401–403

Sauveur

'*acoustique*', science of sound, 422

discoveries on vibrating strings

'*noeuds*' (nodes) and '*ventres*' (antinodes), 422

significance to other vibrating systems, 426

simultaneous subdivisions/mode

frequencies, 422–423

'*sons harmoniques*', consistent with

intervals of *harmonic* divisions and

harmonic series, 422, 427–428

standing wave of fifth harmonic, 424–426

harmonic series, as an infinite series

discovery of, 422–424

historic significance of, 426–427

no knowledge of traveling waves, 423

first thirty-two harmonics of strings and

wind instruments, 423–424

scales

3-, 5-, and 7-limit scales *vs.* $\frac{1}{4}$ -Comma

Meantone and 12-TET scales, 376–377

prime number limit of, 282. *See also* Limit of

3; Limit of 5; Limit of 7

Stifel

division of "double-octave and a fifth,"

ratio $\frac{6}{1}$, into four arithmetic and four

harmonic means, 386–390

on the importance of both divisions, 401

plagiarized by Zarlino, 378, 389

Wallis

experiments, sympathetic resonance, 420–421

nodes

discovery of, 420–421

Sauveur's acknowledgment, 426

infinite number of, not defined, 421–422

Zarlino

major and minor tonalities, origins of, 377–378, 391–397, 399–401

arithmetic and harmonic divisions of "fifth," ratio $\frac{3}{2}$, 399–400

Stifel's arithmetic and harmonic divisions of ratio $\frac{6}{1}$, 392–397

numero Senario (Number Series 1–6), 377–378, 381–384, 390–397

rationalized consonance of "major sixth" and "minor sixth," 397–399

Western theory of consonance, modern, 400–401

philosophy of *Unita* (Unity), corporeal or spiritual manifestations, 429, 484n.359

Just Keys

Plate 6, 850

score, "The Letter," from *Ellis Island/Angel Island*

description of, 811, 824

tablature score, 812–817

traditional score, 818–823

tuning of, 808–811

K

Kanon. *See* Canons

Kasyapa, 562

Kaufmann, Walter, 490, 578, 594

Kepler, Johannes, 336, 418

Khan, Vilayat, 599

Kitharas

description

Maas and Snyder, 618–619

Sachs, 617–618

tension, only variable after stringing, 323

tuning limitations

four significant structural problems, 617–619

GPS not tunable, 619

of gut strings, 618

Krishan, Gopal, 599

Kudimiyamalai inscription, 561–564

L

Lachmann, Robert, 612

Lath, Mukund, 545, 550

Latin ratios

classification

multiple-superparticular, 284–285

multiple-superpartient, 284–286

superparticular (epimore), 284, 385

Al-Farabi, 661

Ibn Sina, 678–679

Mersenne, 417–418

Zarlino, 397

superpartient (epimere), 285, 385

Al-Farabi, 661

Zarlino, 398

- interval
- sesqui* prefix, definition, 384
 - sesquialtera*, 384–385
 - Mersenne, 417
 - Zarlino, 385–386, 392
 - sesquioctava*, 384–385
 - sesquiquarta*, 384–385
 - Mersenne, 417
 - Zarlino, 385–386, 399
 - sesquiquinta*, 385
 - Mersenne, 417
 - Zarlino, 385–386, 398–399
 - sesquitertia*, 384–385
 - Mersenne, 417
 - Zarlino, 398
- Length ratios. *See* Flexible strings, ratios
- Lieberman, Fredric, 486, 492, 496
- Limit of 3
- Al-Kindi, of original 12-tone “double-octave” *ud* tuning, 615
 - Chinese 12-tone scale, spiral of eleven ascending “fifths,” 487–488
 - compared to 12-TET, 377
 - definition, 282
 - Philolaus, of original Diatonic Tetrachord, 299–301
 - Pythagorean theory, 288, 336, 367
 - Safi Al-Din, of original 17-tone *ud* tuning, 712
 - tunbur* of Khurasan, 704
 - Western and Eastern music, two 12-tone scales, spirals of ascending and descending “fifths,” 335
- Limit of 5
- Al-Farabi, on consonance of ratios $\frac{5}{4}$ and $\frac{6}{5}$, 326, 655–658
 - Bharata, of *Sadjagrama* and *Madhyamagrama*, 546
 - compared to 12-TET, 377
 - Gaffurio, rejection of, 367
 - Mersenne, of 12-tone lute scale, 358–359
 - Ptolemy
 - argumentative acceptance of, 288
 - original *emmelic*/melodic classification of, 325–326
 - of Tense Diatonic Scale, in Zarlino, 233
 - Pythagoreans, rejection of, 288–289
 - Ramis, of original 12-tone monochord scale, 372–373, 376
 - Zarlino, of *numero Senario*, 381–384, 397–399
- Limit of 7
- compared to 12-TET, 377
 - Forster, of Just Keys, 808–811
 - Huygens, consonance of ratios $\frac{7}{5}$, $\frac{10}{7}$, and $\frac{7}{4}$, 364–365
- Mersenne
- conflicted rejection of, 417–420
 - qualified acceptance of ratios $\frac{8}{7}$ and $\frac{7}{6}$, 420
 - Meyer, of original Tonality Diamond, or “table of spans,” 448–451
 - Ptolemy, of Soft Diatonic Scale, 334, 832
 - Rameau, rejection of, 440
 - Salinas, rejection of, 403
- Limit of 11
- Al-Farabi
 - of *Jins* 2 tetrachord, 653–654, 679–680
 - of *middle finger of Zalzal* fret on *uds*, 636–339, 643
 - Javanese *slendro* and *pelog* scales, close rational approximations, 510–511
 - Partch, of Diamond Marimba, 452–454
 - Ptolemy
 - of Even Diatonic Scale, 329, 679–680
 - of Tense Chromatic Scale, 331
- Limit of 13
- Forster
 - of Bass Marimba, 827
 - of Diamond Marimba, 824–825
 - of Glassdance, 831
 - Ibn Sina
 - of *Diatonic Genus* 4 and 7 tetrachords, 676, 679–680
 - influence on modern Persian *dastgahs*, 686–689, 692–696
 - of *Zalzal frets* on *ud*, 671, 674
- Limit of 17
- Al-Farabi, of *Persian middle finger* fret on *uds*, 636–639, 643, 674
- Limit of 19
- Eratosthenes, of Enharmonic and Chromatic Scales, 330–331
 - Forster
 - of Chrysalis, 796–797
 - of Harmonic/Melodic Canon, 798–799
 - Ibn Sina, of *Chromatic Genus* 11 tetrachord, 676
- Ling Lun, 485
- Litchfield, Malcolm, 308–309, 311, 317
- Little Canon
- construction of, 834–836
 - dimensions, 837–838
 - Plate 12, 856
 - tuning of, 837
- Liu An, 500–501
- Locana, 594
- Logarithm. *See* Cents
- Loops. *See* Flexible strings
- Lutes. *See also Tunbur; Ud*
- description of, Sachs, 597
 - etymology of, Farmer, 610
 - experiment with, Wallis, 421

Lutes (*Continued*)

- inheritance of, from the Arabian Renaissance, 610–612
- moveable frets of
 - lute and *ud*, 632, 669–670
 - sitar*, 586
 - tunbur* of Khurasan, 696, 701–704
- of North India
 - sarangi*, 600
 - sitar*, 597–598
- of Persia, *tunbur* of Khurasan, 696
- of South India, *vina*, 543, 586
- tuning
 - $\frac{1}{4}$ -comma meantone *vs.* approximation of 12-TET, 340, 348–349, 354
 - of “semitone,” ratio $\frac{18}{17}$
 - Al-Farabi
 - 12 equal “semitones” per “octave,” 355
 - on two different *uds*, 354–355, 639, 643
 - approximation of 12-TET
 - in concert music, Cardan, 355
 - fretting instructions
 - Galilei, 356
 - Mersenne, 356–359
 - origin of, Ptolemy, 354–355

Lyres

- description
 - Maas and Snyder, 618–619
 - Sachs, 617–618
- tension, only variable after stringing, 323
- tuning
 - designs
 - of ancient Greece, 289
 - of Arabian *simsimiyya*, modern, 787n.228
 - of Pythagoras, 293
 - of Terpander, 291–293
 - limitations
 - four structural problems, 617–619
 - GPS not tunable, 619
 - of gut strings, 618

M

Maas, Martha, 618

Major tonality. *See also* Minor tonality

- of 7-limit ratios
 - 16-tone scale, 420
 - Meyer’s tonality diamond, 449–452
- of 11-limit ratios, Partch
 - 43-tone scale, 454–457
 - Diamond Marimba, 453
 - Otonality*, 453–454
- Al-Farabi’s ‘*Diatonic Mode*’, 645
- definition, harmonic division of “fifth,” of
 - ancient length ratios $\frac{3}{2}$, 396–397
 - Zarlino, 377–378, 399–401

Rameau, harmonic progression of modern length ratios, 440–443

of dual-generator, 444–446

Marcus, Scott Lloyd, 749, 759–762, 767–771

Marimba bars. *See* Bars, rods, and tubes

Marimbas. *See* Bass Marimba; Diamond Marimba

Martopangrawit, Raden Lurah, 513, 517, 519–521

Mashaqah, Mikhail, 759

Mass, 5–6, 12, 17, 23. *See also* Mass per unit area;

Mass per unit length; Mica mass

air mass, of cavity resonator, 212–215, 221

definition, 1–2, 8

density, 1, 23

of air, 23, 892

of the earth, 6

of fluid (liquid or gas), 4, 21

of solid, 4, 16

English Engineering System, 9, 12, 16

pounds-mass, 7ff.

experiment, 7–8

inconsistent system, 9, 16

lbm-to-mica conversion factor, 14, 880

English Gravitational System, 9–10

slug, 1, 9ff.

consistent system, 9–10

experiment, 10–11

slug-to-mica conversion factor, 880

English mass unit, undefined, 1, 13, 16

experiment, 7–10, 15

and frequency, 1, 4, 16, 18–21

of glass, snifter, 830–831

inertial property, 1–2, 4, 8

metric system

gram, 1

gram-to-mica conversion factor, 880

kilogram, 1

kg-to-mica conversion factor, 24, 880

Newton, 1–2, 5–7

of spring-mass system, 183–184, 223n.9

of string

plain, 2–3, 18–19, 338, 792

wound, 806

Mass per unit area

of plate, 123

of soundboard

harpsichord, 125–126

piano, 123–124

Mass per unit length

of bar, 158

of cylinder

hollow, 38

solid, 38

of stiffness parameter

plain string, 100

wound string, 113–114

- of string
 - plain, 4, 18–19, 27–29
 - wound, 36–41
- Mass per unit volume. *See* Mass, density; Mica
 - mass, density
- Matanga, 556, 561–562, 572
- McPhee, Colin, 522–528, 530–533
- Means, 86
 - arithmetic, and minor tonality, 90–91
 - arithmetic-geometric asymptote, 313–314
 - definition; arithmetic, harmonic, and geometric, 86
 - Archytas, 87–89
 - harmonic, and major tonality, 90
 - interpretations of, ancient length ratio *vs.* frequency ratio, 87–91, 93
 - Plato, description of, 91–92
 - explanation by
 - Nicomachus, 92
 - Plutarch, 93
 - Stifel, division of “double-octave and a fifth,” ratio $\frac{6}{1}$, into four arithmetic and four harmonic means, 386–390, 401
 - Zarlino, arithmetic and harmonic divisions of “fifth,” ratio $\frac{3}{2}$, 399–400
 - incorporation of Stifel’s arithmetic and harmonic divisions of ratio $\frac{6}{1}$, 390–397
 - origins of major and minor tonalities, 377–378, 396–397, 399–401
- Meantone temperaments
 - Aron’s $\frac{1}{4}$ -Comma Meantone Temperament, 342, 347
 - $\frac{1}{4}$ syntonic comma
 - calculation of, 344
 - cumulative reduction of four consecutive “fifths,” 344–345
 - comparisons to 3-limit, 5-limit, 7-limit, and 12-TET scales, 376–377
 - harmonic analysis of, twelve usable keys, 346–347, 349–350
 - Huygens
 - cent comparisons
 - comparable intervals of 31-TET, 363–364
 - “fifth” of 31-TET, 363
 - three septimal ratios; $\frac{7}{6}$, $\frac{7}{5}$, and $\frac{7}{4}$, 364
 - key color* of, unequal temperament with two different “semitones,” 349
 - syntonic comma (comma of Didymus), definition, 343–344
 - tuning
 - lattice, 345
 - sequence, three steps, 344–346
 - vs.* Werckmeister’s No. III Well-Temperament, 352–353
 - “wolf fifth” and “wolf fourth” of, 346–347
- Zarlino, 401
 - vs.* $\frac{2}{7}$ -comma meantone temperament, 343
- based on irrational length ratios; geometric
 - division of canon strings, 336–337
 - Euclid’s method, mean proportional, 297–298
 - interpretation of
 - Chuquet, 399–341
 - Zarlino, 338–339
- on keyboard instruments, 336, 340
 - vs.* fretted lutes and viols, 348
- meantone, definition, 342
 - geometric division of “major third,” ratio $\frac{5}{4}$, on canon string, 342–343
 - origin of, Ramis’ advocacy of 5-limit “major third,” ratio $\frac{5}{4}$, 340–342, 375
- Mersenne, Marin, 338, 349, 356, 358–359, 377, 404–408, 410–420, 422–423, 428, 436, 438, 610, 642
- Meyer, Max F., 448–453, 824
- Mica mass, 14–16, 18, 23
 - acronym, 14–15
 - consistent system, 14–15
 - definition, 14–15
 - user defined units, 23
 - density
 - of bar making materials, 885
 - of bronze, modified, 37
 - of copper, modified, 40
 - dimensional analysis, 23
 - of gases, 892
 - of liquids, 890
 - of solids, 888
 - of string making materials, 882
 - dimensional analysis, 17–20, 22–23
 - experiment, 15
 - mica-to-kg conversion factor, 24, 880
 - mica-to-lbm conversion factor, 14, 880
 - new mass unit, need for, 1, 13, 25n.19
- Minor tonality. *See also* Major tonality
 - of 7-limit ratios
 - 16-tone scale, 420
 - Mersenne, conflicted acceptance/rejection of, 418–420
 - Meyer’s tonality diamond, 448–452
 - of 11-limit ratios, Partch
 - 43-tone scale, 454–457
 - Diamond Marimba, 453
 - Utonality*, 453–454
 - Al-Farabi’s ‘*Persian Mode*’, 645
 - definition, arithmetic division of “fifth,” of ancient length ratio $\frac{3}{2}$, 396–397
 - Zarlino, 378, 399–401, 452
 - as inversion of major tonality
 - Meyer, 449–452
 - Rameau, 444–446, 452
 - Shirlaw, 448, 452

Minor tonality (*Continued*)

- Rameau, arithmetic progression of ancient length ratios, 441–444
- of dual-generator, 444–448

Mode shape, 98Modes. *See* ScalesModulus of elasticity. *See* Young's modulus of elasticityMohammed, 610Monsour, Douglas, Dedication Page, 843Morse, Philip K., 149Musical slide rule. *See* Cents, logarithm**N**Napier, John, 257, 340Narada, 559–560, 564, 572Narayana, 93, 587–592Nederveen, Cornelis, J., 227–230, 233, 235–238, 244Needham, Joseph, 487Newton, Isaac, 1, 428

laws of motion

- first, 2, 5, 8, 11, 158, 186
- second, 5–7, 9, 15

Nicomachus of Gerasa, 82–85, 92–93, 284, 286, 293–294, 299, 319, 380, 610, 624–625Nijenhuis, E. Wiersma-Te, 545, 556, 560Nodes. *See* Bars, rods, and tubes; Flexible strings;

Inharmonic strings; Just intonation, Sauveur;

Just intonation, Wallis; Resonators, tube

Numero Senario. *See* Just intonation, Zarlino**O**“Octave” equivalence. *See* CentsOld Arabian School, 619–620, 636Ornstein, Ruby Sue, 524, 530–531**P**Palisca, Claude V., 356, 415Partch, Harry, v, 452–457, 824, 840Period. *See* Flexible stringsPersian musical terms. *See also* Scales, Persian

dastan (fret), 628, 630–632

dastgah Mahur, *pishdaramad* (overture), 771–773

instruments

tar and *setar* (*three strings*), 597, 687–689

tunbur of Khurasan, 696–697, 701–704

ud al-farisi, ancient lute, 619

mode

5 hierarchical functions of tones: *aqaz*, *shahed*, *ist*, [*finalis*], and *moteqayyer*, 690–694, 772

vs. laksana, 691

12 Modern *Dastgaha* (sing. *dastgah*), Farhat, 692–696

dastgah, group of modes/dominant mode, 690ff.

gushe, individual mode, 690ff.

vs. raga, 690

musical notation sign

koron, 636–640

sori, 636, 640

Persian scales. *See* Scales, PersianPersian tetrachords, modern

all 12 *Dastgaha* on Ibn Sina's *ud*, 687–689, 692–696

Farhat

of 12 *Dastgaha*, 692–693

chromaticism, not used, 687

of *Mahur*, intervallic structure, 771–772

Philolaus, 91, 280, 288, 299–301, 304, 333, 617, 624–625, 648–649, 662, 681, 686, 758–759Pianos

Cristofori, *gravicembalo col piano e forte*, 118, 130

design change ideas, 142

impedance ratio

air-to-soundboard, 129

soundboard-to-string, 125

inharmonicity

analysis of, 110–111

coefficients of, plain strings, 111

effect on

beat rates, uncontrollable, 105, 108, 135

timbre, 110–111, 118, 135

tuning, 111–112, 118

tuning possibilities, 105, 108, 118, 135, 808–809

limitations, structural and musical

soundboard, thickness, 133–134

string tension, total force, 134–135

tuning, inharmonicity, 134–135

soundboard

bending wave speed, 131–132

components

bridges, 124

liners, 124

ribs, 123–124

critical frequency, 130–133

dimensions, typical

surface area, 129

thickness, 123

impedance

data, Wogram, 122–124, 129–130, 132–133

plate *vs.* soundboard, 123–124

radiation, of air at soundboard, 129

sound radiation, 132–133

spruce

European (*Picea abies*), 886n.8

Sitka (*Picea sitchensis*), 123, 885

stringing scale

1.88 : 1, need for, 34

2:1, structural problems of, 33

- strings
- coefficients of inharmonicity, 109–111, 126
 - dimensions and tension, typical
 - bass, G_2 , 40
 - treble
 - D_4 , 121
 - G_3 and C_4 ... G_7 and C_8 , 111
 - energy transfer, to soundboard, 118–119, 125
 - impedance, D_4 , 122
 - length *vs.* diameter, changes in, 115
 - stiffness parameters, 111
 - tension equation
 - plain, 31
 - wound, 41
 - wound, need for, 27, 34
- tuning process, general, 302
- 12-TET, 136
 - of “fifth,” 136
 - of “fourth,” 137
 - of “major third,” 136–137, 139–140, 146n.28
- vs.* harpsichords
- coefficient of inharmonicity, average values, 126
 - critical frequency of soundboard, 132–134
 - impedance ratio
 - air-to-soundboard, 129–130
 - soundboard-to-string, 125–126
 - sound radiation, 133
 - soundboard thickness, string tension, and inharmonicity, 130
- Plain strings
- equations
 - diameter, 31
 - frequency, 27, 31
 - length, 31
 - mass per unit length, 28–29
 - tension, 31
 - four laws of, 31–32
 - length
 - and frequency, inverse proportionality, 32–33
 - piano stringing, structural problems, 33–34
 - piano stringing scale, treble strings, 34
 - tension
 - break strength
 - calculation, 35, 40, 134
 - definition, 35
 - considerations; structural, technical, and musical, 35
 - instrument limitations, 35
- Plane sections. *See* Bars, rods, and tubes
- Plato, 91–93, 288, 308, 610–611, 624–625, 759
- Plutarch, 82, 85, 91–93
- Poerbapangrawit, Raden Mas Kodrat, 513, 517–519, 521–522
- Poisson’s ratio
- of hardwood plywood, 222
 - of Sitka spruce, 123
- Powers, Harold S., 580–582, 586
- Praetorius, Michael, 336, 349, 610
- Pressure
- adiabatic bulk modulus, 21–22
 - in cavity resonator, 212–214, 216–219, 226n.50
 - definition, 22
 - dimensional analysis, 22
 - driving
 - of acoustic (wave) impedance, complex, 197
 - specific, 127–128
 - of bar, 196, 214, 217–218
 - definition, 127
 - of soundboard, 128
 - of tube resonator, 198–199
- English system
- of air
 - at 1 standard atmosphere, 22
 - adiabatic bulk modulus, 22
 - psi, 13, 22
 - at flute embouchure and tone hole, 240–241
 - of gas, 188
 - wave
 - in air, 130–131, 182–184
 - of bar, 214, 217
 - in flute, 230, 234, 239, 247
 - of plane wave, 186–187
 - in tube resonator, 189–195, 198–199, 200–208
 - Young’s modulus of elasticity, 13
- Ptolemy, Claudius, vi, 78, 86, 90, 93, 280, 287–289, 295, 312–314, 316–334, 338, 354–355, 358–359, 399–401, 403–404, 407–408, 413, 415, 553, 564, 610, 624–625, 628, 636–637, 642, 646–647, 651, 657–659, 661, 664–665, 674, 677, 679–681, 707, 712, 759, 792, 832
- Pulse. *See* Waves
- Pythagoras, vi, 44, 82–83, 280, 284, 288–289, 293–295
- Pythagoreans, 288, 308, 318–319, 322–324, 336, 657
- Q**
- Quadrivium, 630
- R**
- Racy, Ali Jihad, 760–761
- Radius of gyration. *See* Bars, rods, and tubes
- Rai, I Wayan, 524–525, 529–530
- Ramachandran
- K.V., 584–585
 - N.S., 580
- Ramamatya, 93, 565–580, 582, 584, 586–587, 599
- Rameau, Jean-Philippe, 90–91, 428–436, 438–449, 451–452, 454
- Ramis, Bartolomeo, 93, 340, 342, 358, 366–377, 401

Rao, T.V. Subba, 576

Rarefaction. *See* Compression and rarefaction

Ratio of specific heats

of adiabatic bulk modulus, 22

of gases, 892

of liquids, 890

Ratios. *See also* Ancient length ratios; Arabian ratios; Flexible strings, ratios; Greek ratios; Latin ratios

integer, definition, 71, 281

numbers *That* compose

commensurable *vs.* incommensurable, 365–366
Galileo, 283

composite

definition, 281

prime factorization, 281–282

irrational

cube root of 3, Archytas, 283

definition, 283

square root of real

Chuquet, 339–341

Euclid, 283, 297–298

Zarlino, 338–339

positive natural; odd, even, and prime, 281

prime

definition, 281

factorization, 281–282

limit, definition, of intervals and scales,
282. *See also* Limit of 3

rational

definition, 281

two types, 282–283

Ray, Satyajit, 600

Resonators

cavity

air mass, in neck, 212–213

actual, 215, 221

theoretical, 214

air spring

in cavity, 212–213, 215–216

at sidewalls, 217–219, 221

air spring-air mass system, two different

kinds, 212–213, 217–219

end correction

of duct opening, 214

of flange opening, 214

and frequency, inverse proportionality, 215

frequency, 215

equation, limitations of, 212

theoretical *vs.* actual, 217

Helmholtz, 212

making, size considerations, 218–221

neck, effective length of, 212–215

reason for, 212

sidewall stiffness, 221–222

tuning process

attack tone *vs.* decay tone, 222

at opening, 215–217, 222

at sidewalls, with tuning dowels, 219–221

tube

antinode (*closed tube*), 203

displacement (DA), 205, 207–208

pressure (PA), 205

antinode (*infinite tube*)

displacement (DA), 201–203

pressure (PA), 201–203

antinode (*open tube*), 203

displacement (DA), 204

pressure (PA), 204

closed, 203

airtight seal, making, 211

end correction, 207–209

and diameter, direct proportionality, 208

and frequency, inverse proportionality,
211

frequency

actual, 210

theoretical, 207

length

cut, 210

measured *vs.* effective, 207–208

theoretical, 207

closed-closed

frequency, theoretical and actual, 230

length, measured and effective, 230

impedance, acoustic wave, 197

bar-to-resonator relation, 198

of resonator, 196–197

of room, 199

of tube, 198

tube-to-room ratio, 199

node (*closed tube*), 203

displacement (DN), 205

pressure (PN), 205, 207–208

node (*infinite tube*)

displacement (DN), 201–203

pressure (PN), 201–203

node (*open tube*), 203

displacement (DN), 204

pressure (PN), 204

open, 203

end correction, 209

frequency

actual, 209

theoretical, 206

length

cut, 209

measured *vs.* effective, 229–230

theoretical, 206

- pulse reflection
 - at closed end
 - compression, 190, 192
 - rarefaction, 190–191, 193
 - at open end
 - compression, 191, 194
 - rarefaction, 191, 195
 - reason for, 196
 - standing waves, pressure/displacement
 - closed, 203, 205, 208
 - closed-closed, 230
 - infinite, 201
 - open, 203–204, 230
- Restoring force
 - of bar
 - clamped-free, 174–177
 - free-free, 147–148, 154–162
 - bending stiffness
 - of bar, 152, 154–155, 158–160
 - of plate, 123
 - of rod and tube, 158
 - of soundboard
 - harpsichord, 125
 - piano, 123, 130–132
 - of cavity resonator, 226n.50
 - definition, 2
 - elastic property, 2–4, 21
 - of fluid, 4
 - of glass, snifter, 830–831
 - of plane wave, 186–187
 - of solid, 4
 - of spring-mass system, 226n.50
 - of string. *See also* Downbearing force
 - flexible, 1–4, 18–19
 - stiff, 104–105, 107–108
- Roberts, Francis, 421–422
- Robinson, Kenneth, 503
- Rods. *See* Bars, rods, and tubes
- Roy, Hemendra Lal, 595–597
- S**
 - Sachs, Curt, 489, 556, 597, 617–618
 - Safi Al-Din, 93, 326, 366, 373–375, 379, 381, 384,
 - 401–402, 610, 622, 626–627, 673, 696, 705,
 - 707–733, 735, 737, 740–745, 747–749, 754–755,
 - 758–759, 763–768, 771
 - Salinas, Francisco, 401–403, 448, 452
 - Sarngadeva, 548, 562, 566–567, 572, 576
 - Saunders, Lawrence, viii
 - Sauveur, Joseph, 390, 422–428, 436, 438
 - Scales, 86. *See also* Arabian musical terms;
 - Arabian ratios; Arabian tetrachords;
 - Equal temperaments; Greek ratios; Greek tetrachords; Indian musical terms; Indian tetrachords; Just intonation; Meantone temperaments; Persian musical terms; Persian tetrachords; Turkish musical terms; Turkish tetrachords; Well-temperaments
 - Arabian
 - 8 tetrachords, origins in near-equal divisions of intervals; Ptolemy, Al-Farabi, and Ibn Sina, 677–681
 - 24-TET, modern scale, 755–761
 - 46 Modern *Maqamat*, 761–771, 786n.209
 - 9 *fasail*, for the construction of, 762
 - 16-tone scale, for the playing of, 770–771
 - of Al-Farabi
 - 3 standard modes, 7-tone scales, 644–646
 - 3 tetrachordal conjunct/disjunct systems; in the context of GPS, 662–666
 - 8 *Ajnas*, tetrachords, 640–641, 646–655
 - 10-fret *ud* tuning, theoretical, 632–639
 - 15 tetrachords, 2 genera, 658–663
 - 17-tone scale
 - original *tunbur*, 703–707, 735
 - tunbur* of Khurasan, 696–697, 701–704
 - 22-tone scale, *ud* tuning, 640–643
 - of Al-Kindi, original 12-tone scale, *ud* tuning, 611–617
 - of Al-Munajjim, 8 *majari*, 7-tone scales, 621–625
 - of Brothers of Sincerity, 9-tone scale, *ud* tuning, 620–622
 - of Ibn Sina
 - 11 Melodic Modes, 681–686
 - 16 tetrachords, 3 genera, 673–676
 - 17-tone scale, *ud* tuning, 666–673
 - of Safi Al-Din
 - 17-tone scale
 - ascending spiral of “fourths,” original construction, 731–733
 - First Ud Tuning*, 707–713
 - monochord/*ud* tuning, 717–719
 - tunbur/ud* tuning, 720–721
 - Second Ud Tuning*, 714–717
 - 19-tone scale, theoretical; ascending and descending spirals of “fifths,” 710–713
 - 84 Melodic Modes, 721–726, 728–729, 768
 - 6 *Awazat*, 730–731
 - 12 genera, for the construction of, 726
 - 12 *Shudud*, 727–730, 737, 740–744, 767–768
 - Chinese
 - 1-*lu* interval
 - two different kinds, 497–499
 - of two *tiao* (mode tuning) cycles, 496–499
 - 5-tone (pentatonic) scale
 - on *ch'in*
 - 12 *tiao* (mode tunings)
 - ascending cycle of six; descending cycle of six, 496–499
 - interval sequences of, 497, 500

Scales (*Continued*)

- open strings
 - post-Ming, new scale, 490–493
 - pre-Ming, old scale, 490–493
 - String III as *chiao*, dual identity of, 492
 - tuning sequence of, 492
 - stopped strings (*hui* fractions), pre-Ming, old scale, 492–496
 - cipher notation of, Cheve System, 490, 493, 496, 508
 - original, 487–488, 490–493
 - 12-TET, discovery of, Chu Tsai-yu, 502–504
 - 12-tone scale
 - formula of, Ssu-ma Ch'ien, 486
 - generation of, 486–487
 - lu*, definition, 487
 - spirals of “fifths,” *That* closely approximate equal temperaments, 500–502
- Greek
- Archytas, three means for the construction of tetrachords, 86–87
 - of Aristoxenus, 309–311
 - Catalog of Scales, seven theorists, 330–332
 - Dorian and Lydian modes, 292, 301, 333
 - of Euclid, 304
 - Greater Perfect System (GPS), 289–292, 301–304, 333
 - Lesser Perfect System (LPS), 289–291
 - of Philolaus, 300–301, 304
 - of Ptolemy, 328–329
 - of Pythagoras, 289, 293
 - seven *harmoniai* (modes), 290, 292
 - ecclesiastical names of, 290, 292
 - of Terpander, 289, 291–293
 - three genera, Archytas and Aristoxenus, 289–292
- Indian
- ancient
 - Bharata
 - Madhyamagrama* (*Ma-grama*), 540–541, 546
 - and *svarasadharana*, 549–551
 - Sadjagrama* (*Sa-grama*), 540–541, 546
 - Suddha Jatis*, 554
 - Arsabhi* and *Naisadi* vs. Ptolemy’s Tense Diatonic, 553
 - Vikrta Jatis*, 555
 - Gandhari* vs. original Chinese pentatonic scale, 556
 - gramaragas* and *laksana*
 - of Kudimiyamalai inscription, 563
 - of Matanga, 563
 - of Narada, 563
 - of Sarngadeva, 563
 - North India
 - 12-tone scale, modern *svaras* of, 592

- Bhatkhande, 10 most popular *Thats*, 581, 596
 - letter-dot notation, after cipher-dot notation of the Cheve System, 508, 535n.6, 598–599
 - Narayana and Ahobala, 12-tone *vina* tuning, 591
 - Roy, 32 *Thats*, of eight-by-four matrix system, 581, 595–596
 - sitar* tuning and fret locations, 598
- South India
- 14 most popular *mela ragas*, 581
 - Govinda
 - 72 musical heptatonic *scales*, first formal implementation of, 565
 - Kanakangi-Ratnangi* system, 580–581
 - Ramamatya
 - 12-tone practical scale, *suddha mela vina* tuning, 573
 - 14-tone theoretical scale, 567
 - Venkatamakhi
 - 12-tone scale, 577
 - 72 *Melakartas*, of twelve-by-six matrix system, 579, 581
- Indonesian. *See also* Gamelan
- Bali
- gamelan Semar Pegulingan*, 525–530
 - patutan*, definition, 523, 537n.43
 - pelog*
 - 7-tone scale (*saih pitu*), 523–530
 - 5-tone scale (*saih lima*), derived, 524–530
 - pemero* (two auxiliary tones), *penyorog* and *pemanis*, 525
 - cipher notation of, 523
 - intervals of, 523
 - patutan Selisir*, 525–530
 - six-tone*, Pliatan Village, 530–534
 - cipher notation of, 532
 - penyorog*, 525, 531–534
 - Patutan Tembung, Selisir, Baro, Lebeng*, 525–529
 - and *Sunaren*, 527–529
 - slendro*, 5-tone scale
 - cipher notation of, 522–523
 - intervals of, 523
 - saih gender wayang*; shadow puppet theater only, 522–523
 - solmization, *slendro* and *pelog*, 525
- Java
- patet*, definition, 512–513
 - pelog*, 7-tone scale
 - 5-, 7-, and 11-limit ratio analyses of, 511
 - cipher notation of, 510–511
 - definition, 510
 - intervals of, 510–512, 517–521

- Patet Lima, Nem, and Barang*, 517–521
gong tones and cadences, 517,
519–521, 537n.29
- slendro*, 5-tone scale
5-, 7-, and 11-limit ratio analyses of, 510
cipher and cipher-dot notation of, Cheve
System, 508–511, 535n.6
definition, 509
intervals of, 509–517
Patet Nem, Sanga, and Manyura,
513–514, 516
gong tones and cadences, 513–516
solmization, *slendro* and *pelog*, 525
tumbuk (common tone) between *slendro*
and *pelog*, 519–522
- Persian
12 Modern *Dastgaha*, Farhat, 690–696
9 intervals of, 686–688
basic 17-tone scale of, 770–771
dastgah, definition, 690
on Ibn Sina's *ud*, 692–696
17-tone scale
modern cent averages of two *tar* and three
setar tunings, 687–689
tunbur of Khurasan, 696–697, 701–704
- Turkish
24-tone modern scale
6 modern musical symbols of, 735–736
53-TET, theoretical model, 736–737
comma-limma equivalents of, 736–736
vs. Al-Farabi's original *tunbur* tuning,
733–736
vs. Safi Al-Din's *First Ud Tuning*, 735
basic 18-tone scale, 770–771
- Signell
6 'Variant' *Maqamat*, 744–745
5 of 6 on Safi Al-Din's *First Ud*, 744–745
13 Basic *Maqamat*, 740–743
10 of 13 on Safi Al-Din's *First Ud*,
740–743
- Western
7-tone modes, ecclesiastical names of, 290, 292
12-tone scale
five examples of, 376–377
origins of, Chinese and Arabian sources,
334
transformation from 3-limit to 5-limit scale,
418–419
of Anonymous, in *Pro clavichordiis faciendis*,
first twelve-tone 5-limit scale, 473n.175
of Aron, 342–347, 353, 364
of Halberstadt organ, Praetorius, 336
of Huygens, 362–365
of Kepler, 336
major scale, Zarlino's advocacy of Ptolemy's
Tense Diatonic, 319, 333, 832
of Mersenne's
harmonic-diatonic system, 403–405
lute, 356–359
“old” and “new” trumpet marines,
410–415
of Meyer, 449
minor scale, Ptolemy's Soft Diatonic, 334, 832
of Partch, 454–457
Pythagorean ‘*Apotome Scale*’ and ‘*Limma
Scale*’, 336
of Ramis, 366–377
of Spechtshart, first twelve-tone 3-limit scale,
334–335
spirals of twelve “fifths,” ascending and
descending, for the construction of,
335–336
of Stevin, 358–362, 375–376
of Werckmeister, 349–353
- Schisma. *See* Arabian ratios; Greek ratios, interval;
Just intonation, Ramis; *Tunbur*
- Schlesinger, Kathleen, 227
- Seeff, Norman, 857
- Senario. *See* Just intonation, Zarlino
- Shankar, Ravi, 597, 600
- Sharma, P.L., 561
- Shear force. *See* Bars, rods, and tubes, restoring
forces
- Shirlaw, Matthew, 428, 434, 448, 452
- Signell, Karl L., 736, 738–739, 741–745
- Simple Flutes, 833
Acrylic Flute
Flute 1
construction of, 246
dimensions, 241
tuning of, 239–241, 833
Flute 3, tuning of, 833
Amaranth Flute, Flute 2
dimensions, 243
tuning of, 245–246
Plate 11, 855
- Simple harmonic motion (SHM)
definition, 50, 182
of particle motion
in air, 182–184
in cord/string, 50–51
in solid, liquid, gas, 186–187
of spring-mass system, 183–186
- Sindoesawarno, Ki, 512
- Sitar. *See* Indian musical terms, North India
- Smith, Page, Dedication Page, 839
- Snyder, Jane McIntosh, 618
- Sound waves. *See* Waves, sound
- Soundboards
instrument
Bass Canon, 800, 805
ch'in, top piece, 489
Chrysalis, 788–789
Harmonic/Melodic Canon, 790–793

Soundboards (*Continued*)

- harp-*vina*, 543
- harpichord, 123, 125–126, 129, 132–133
- Just Keys, 808–809
- kithara* and *lyre*, 617–618
- Little Canon, 834–836
- piano, 123, 125–126, 129, 132–133
- qanun*, ancient, 628–631
- sitar*, 597
- trumpet marine, 408
- ud*, top plate, 619–620, 626
- vina*, 586
- violin, top plate, 626
- piano. *See also* Pianos, soundboard
 - bending wave, 122, 128
 - speed, 119, 131
 - and bending wavelength, inverse proportionality, 131–132
 - and critical frequency, 132, 134
 - and dispersion, 130–132
 - effect on acoustic radiation, 123–124, 131–133
 - experiment, 131
 - of plate, 131–132
 - dispersion
 - definition, 131
 - effect on radiation, 130, 132
 - energy transfer (coupling)
 - soundboard-to-air, 127–128
 - string-to-soundboard, 119, 121, 127–128
 - impedance, mechanical wave, 120–121, 124
 - data, Wogram, 122–124, 129–130, 132–133
 - experiment, thick soundboard
 - and thick piano string, 133–134
 - and thin canon string, 142
 - fluctuations, 122
 - of plate, 123
 - radiation, of air at soundboard, 124, 129–130
 - ratio
 - air-to-soundboard, 129–130, 134
 - soundboard-to-string, 125–126, 134–135, 142
 - reactance and resistance, 122–123
 - resonance and resonant frequency, 122–124, 132–133
 - radiation, 119
 - bending wave speed effect on, 131–132
 - critical frequency effect on, 132–133
 - data, Wogram, 124, 129–130
 - stringing effect on, 125, 130
 - thickness effect on, 130, 134
 - wood
 - European spruce (*Picea abies*), 886n.8
 - Sitka spruce (*Picea sitchensis*), 885
 - T'ung* wood (*Paulownia imperialis*), 489

Spechtshart, Hugo, 334

- Speed of sound. *See also* Wave speed, longitudinal
 - in air, 23, 126–127, 131–132
 - at 68°F, 189
 - at 86°F, 189
 - per degree rise of, 189
 - as constant, 188, 208
 - inside played flute, 229
 - in rosewood, 21
 - in spruce *vs.* steel, 886n.8
 - temperature effect on, gas, 188–189

Spring constant

- acoustical, of cavity resonator, two different kinds, 212–219, 221
- definition, 183–185
- mechanical, 183–185, 216, 218, 221–222

Spring-mass system

- acoustical, of cavity resonator
 - definition, two different kinds, 212–213, 217–219, 221
 - elastic and inertial components, 212–213, 217–219, 221
 - excess pressure of, 226n.50
 - frequency of, 215–217
 - restoring force of, 226n.50
- mechanical
 - definition, 183–185
 - elastic and inertial components, 183–185
 - frequency of, 185, 223n.9
 - two springs in series and one mass, 218–219
 - restoring force of, 226n.50

Ssu-ma Ch'ien, 93, 334, 486Stapulensis, Jacobus Faber (*Jacques Le Febvre*), 396Stevin, Simon, 354, 358–362, 375–376, 502–503, 542, 569, 749Stifel, Michael, 276n.6, 378, 386–390, 392, 396–397, 401, 403, 427, 429, 432–433, 435String Winder. *See also* Wound strings

- construction of, basic components, 805–807
- dimensions, 807
- Plates 4 and 5, 848–849

Strings. *See* Flexible strings; Inharmonic strings; Plain strings; Wound stringsSubharmonic series. *See* Flutes; Just Intonation, RameauSumarsam, 512Superposition. *See* Flexible stringsSurjodiningrat, Wasisto, 509–511, 516, 521–522**T**Temperaments. *See* Equal temperaments;

Meantone temperaments; Well-temperaments

Temperature

- conversion factors, 189, 881
- effect on
 - density *vs.* pressure, 188

- frequency, 189
 speed of sound in gas/air, 188–189, 225n.26
 per degree rise of, in air, 189
 inside played flute, 229
- Tensile strength, 32
 and break strength calculation, 35
 definition, 35
 of spring steel, 35
 of string materials, 882
 gut strings, thick *vs.* thin, 883n.1
 high-carbon spring steel music wire, 884
 temper classifications; copper, brass, bronze, and steel, 883
- Tension. *See* Plain strings
- Tenzer, Michael, 533
- Terpander, 289, 291–293
- Tetrachords. *See* Arabian tetrachords; Greek tetrachords; Indian tetrachords; Persian tetrachords; Turkish tetrachords
- Theon of Smyrna, 284, 286, 318, 380
- Tonality Diamond. *See* Diamond Marimba; Just intonation, Meyer; Just intonation, Partch
- Tone notation, ix
- Toth, Andrew F., 524
- Touma, Habib H., 760
- Trumpet marines
 description, 408
 flageolet tones of, 408–409
- Mersenne
 harmonics of, 406
 illustration of two, 411
 as monochord, 410
 length ratios of “old” and “new” trumpet marines, 410–415
- Roberts, flageolet tones of, 421–422
- Sauveur, rejection of, 422
- Trumpets
 harmonics of, first fourteen, 412
- Mersenne, 404–405, 414–416
 harmonics of, 406, 413
 “leaps” of natural trumpet *vs.* harmonics of monochord, five identical ratios, 406–407
 with prime numbers greater than 5, not acknowledged, 408, 412, 415
- Roberts, harmonics of, infinite series, 421
- Sauveur, harmonics of, first thirty-two, 424
- Tubes
 instrument. *See* Bars, rods, and tubes resonator. *See* Resonators, tube
- Tunbur. *See also* Lutes
 bridge and string holder (hitch pin), two separate components, 626–628, 696
 of Khurasan, Al-Farabi
 frets of, fixed and moveable, 701–704
 strings and frets of, 696
- schisma* and *comma* variants of, 696–707
 of Turkey, modern tuning and musical symbols of, 733–736
- Turkish musical terms
 6 hierarchical functions of tones: *karar*, *tiz durak*, *yeden*, *guchlu*, *asma karar*, *giris*, 738
- Turkish scales. *See* Scales, Turkish
- Turkish tetrachords, modern
 of *Rast*, 727, 746–747, 749
 modal origins on Safi Al-Din *First Ud*, 726–728, 730–733, 744–748
- Signell
 6 basic tetrachords/pentachords, 738–739
 6 ‘variant’ (non-“basic”) tetrachords/pentachords, 738–739
- Tyagaraaja, 578, 583–584, 604n.73, 606n.97
- ## U
- Ud. *See also* Lutes
- bridge
 dama, 628
 faras, 626–628
 hamila, 626–628, 631–632
 hamila of canon functions as *dastan* of *ud*; *vice versa*, 628, 632, 670–672
 musht, 626–628
- construction and tuning of, earliest, 620–622, 775n.27
- fret (Pers. *dastan*), 628, 630–632
 Al-Farabi, frets of *ud* function as bridges of canon, 632
 binsir (ring finger), 615, 617ff.
 khinsir (little finger), 615, 620–622ff.
 middle finger of Zalzal
 Al-Farabi’s *uds*, 634–640, 643–645, 650–653
 Ibn Sina’s *ud*, 667–673, 681–683
 Safi Al-Din’s *First* and *Second Ud Tunings*, 709–712, 714–716
 mujannab (*neighbor*; also *anterior* or *assistant*), 615, 620, 634ff.
 Persian (ancient) middle finger
 Ibn Sina’s *ud*, 667–672
 Safi Al-Din’s *First Ud Tuning*, 709–713
 Persian middle finger
 Al-Farabi’s *uds*, 634–639, 642–645
 Safi Al-Din’s *Second Ud Tuning*, 714–717
 sabbaba (index finger), 615, 617ff.
 wusta (middle finger), 615, 617ff.
 zaid (surplus), 709ff.
- of Persia, *ud al-farisi*, precursor of classic Arabian *al-ud*, 619, 775n.27
- Pythagorean diatonic scale, playable on all ancient *uds*, 755
- string
 Bamm, String I, 612ff.

Ud (*Continued*)

- Hadd*, String V
 Al-Farabi's *ud*, 641ff.
 Ibn Sina's *ud*, 669ff.
 Safi Al-Din *uds*, 708ff.
Mathlath, String II, 612ff.
Mathna, String III, 612ff.
mutlaq (*free; open string*), 615, 617ff.
 tuning of, 612–613, 621–622, 633–634,
 641–642, 644–647, 667–668, 681–683,
 702, 708–709
- Zir*
 1st string, only on Ikhwan al-Safa's *ud*, 621
 String IV, 633ff.
 'Zir 2', String V, only on Al-Kindi's *ud*, 612ff.

V

Vaziri, A.N., 640, 691, 772

Velocity

- definition, 4
 dimensional analysis, 4, 20, 22
 equations, 5, 7
 particle, 2–3
 of bar, 156–158
 of complex impedance
 mechanical (wave), 120–121
 specific acoustic (wave), 127
 of cord, 2–3, 50–51
 of plane wave, 186–187
 of solid and fluid, 119–120
 of sound wave, in air, 183–184
- phase
 of solid and fluid, 119–120
 of string, 131
- volume, complex acoustic (wave) impedance, 197
- Venkatamakhi, 565–566, 574–584, 586, 594–596
 Muddu, 577

Vina

- ancient
alapini vina, 543, 568–569
 Bharata, 540
vina (harp-*vina*), 543–544
 tuning experiment on, 541, 544
kinnari vina, 569, 586
zither-vina, development of, 568–569
- North India
bin, modern stick-zither, 543
 Narayana and Ahobala, 587
 12-tone *vina* tuning, 591
 tuning instructions, 587, 590
vicitra vina, 585, 599
- South India
 Ramamatya, 565
suddha mela vina, 569
 12-tone tuning, 570, 573
 tuning instructions, 569–570
vina, modern lute, 543, 586

W

Wallis, John, 420–421, 426

Wave speed

- bending
 in bar, 149–150, 159–160
 and bending wavelength, inversely
 proportionality, 149
 and dispersion, 147–149
 effect on inharmonic mode frequencies,
 148–151
 experiment, 149
 tuning effect on, 160
- in soundboard (infinite plate), 131–132
 and bending wavelength, inverse
 proportionality, 131
 and critical frequency, 131–132, 134
 and dispersion, 131–132
 driving pressure of, 128
 effect on acoustic radiation, 124, 131–133
 experiment, 131
 phase velocity of, 119
 stiffness effect on, 131–132
- longitudinal
 in fluid (liquid or gas), 3–4, 21, 188
 in gases, 892
 in liquids, 890
 and mica mass unit; to simplify all
 calculations of, 21
 in solid, 3–4, 21, 188, 888
- transverse
 in solid, 3–4
 in string
 flexible, 3–4, 60–61, 102–104
 as constant, 60–61, 103–104
 stiff, 99, 104–107
 and dispersion, 104
 as variable, 105–107
- Wave train. *See* Flexible strings
- Wavelength. *See also* Dispersion
- bending
 of bar, 149–150
 experiment, 149
 of soundboard, experiment, 131
- definition, 54, 60
 dimensional analysis, 54–55
 of flute tube, half-wavelength
 approximate, 229, 231ff.
 exact, *substitution tube*, 229–232
- and frequency
 as function of, 80
 inverse proportionality, 61–62
- of longitudinal traveling wave, in air, 183–184
- of string
 flexible, 60–62, 71
 as constant, 61–62
 of transverse standing wave, 51–53, 58

- stiff, 99, 101–102
 - of transverse standing wave, 102
 - as variable, 101–102, 116n.6
- of transverse traveling wave, wave train, 50–51, 54–57
- of tube resonator
 - closed-closed, theoretical and actual, 230
 - closed (quarter-wavelength)
 - actual, 210
 - measured *vs.* effective, 207–209
 - theoretical, 207
 - open (half-wavelength)
 - actual, 209
 - measured *vs.* effective, 229–230
 - theoretical, 206
- Waves
 - bending
 - in bar, 147–151
 - in soundboard, 119, 122, 124, 131ff.
 - definition, 44
 - pulse
 - longitudinal, in tube resonator
 - at closed end, compression and rarefaction, 192–193
 - at open end, compression and rarefaction, 194–195
 - transverse, in string
 - crest and trough, 44–45
 - collision of, 46–49
 - incident and reflected, 45–46
 - sound
 - beating phenomenon, 135–137
 - as longitudinal traveling wave, 21, 130, 182–184, 186–188
 - speed of
 - in air, 23, 128, 131, 188, 203
 - and critical frequency, 131–133
 - in solid, liquid, gas, 119, 188
 - temperature effect on, in gases, 188–189
 - standing
 - displacement, in tube resonator
 - closed, 203, 205, 207–208
 - closed-closed, 230, 248n.10
 - infinite, 200–203
 - open, 203–204
 - frequency of, 55, 58–59
 - longitudinal
 - definition, pressure/displacement, 200
 - in flute, 231, 234, 245
 - in tube resonator
 - close, 196, 205, 208
 - closed-closed, 230
 - infinite, 200–203
 - open, 204, 230
 - period of, 54–56
 - pressure, in tube resonator
 - closed, 203, 205–208
 - infinite, 200–203
 - open, 203–204, 206, 230
 - transverse
 - in bar (*clamped-free*), 175
 - in bar (*free-free*), 153
 - definition, 48, 51–53
 - in soundboard, 122
 - in string
 - discovery of, Sauveur, 424–426
 - and flageolet tones, 408–409
 - flexible, 51–55, 58–59, 182–183
 - mathematical model of, D’Alembert, 423
 - stiff, 101–102
 - traveling
 - frequency of, 55
 - longitudinal
 - in air, 182–184
 - definition, 183
 - in fluid, 119
 - in solid, liquid, or gas, 186–188
 - in tube resonator, 189–191, 196–197
 - closed, 203–207
 - infinite, 200–203
 - open, 203–206
 - period of, 54–56
 - transverse
 - definition, 50–51, 182–183
 - in solid, 119
 - in string, 2, 44–53, 58, 80
 - Weight. *See also* Weight density
 - definition, 6
 - equations, 6, 8, 16, 18
 - of mica, 14
 - of object, standard weight, 11–12
 - of rosewood test bar, 173
 - of slug, 11
 - as string tension, experiment
 - flexible, 18–19, 336–337
 - stiff, 108
 - as “weight ratio”
 - Nicomachus, description of, 82–86, 93, 294
 - Ptolemy, *rejection* of, 319
 - Weight density
 - of bar making materials, 885
 - definition, 16
 - English Engineering System, inconsistent system, 16
 - of gases, 892
 - of liquids, 890
 - of rosewood test bar, 173
 - of solids, 888
 - of string making materials, 882
 - Weight per unit volume. *See* Weight density

Well-temperaments

- based on irrational length ratios; geometric divisions of canon strings, 336–337
 - Euclid's method, mean proportional, 297–298
 - interpretation of
 - Chuquet, 339–341
 - Zarlino, 338–339
 - Werckmeister's No. III Well-Temperament, 349–350, 352
 - $\frac{1}{4}$ ditonic comma
 - calculation of, 350
 - cumulative reduction of four non-consecutive "fifths," 350–351
 - Bach's *Well-Tempered Clavier* tuning, Barnes, 349
 - ditonic comma (comma of Pythagoras), definition, 335, 349–350
 - harmonic analysis of, twenty-four usable keys, 351–353
 - "key color" of, unequal temperament with four different "semitones," 352
 - tuning
 - lattice, 350–351
 - sequence, 471n.140
 - vs.* Aron's $\frac{1}{4}$ -Comma Meantone Temperament, 352–353
- Werckmeister, Andreas, 350–353
- Western scales. *See* Scales, Western
- Whitman, Walt, v, 788, 794–799, 802–804, 840
- Widdess, Richard, 558–559, 560–562
- Wienpahl, Robert W., 397
- Wilson, Erv, 455–457, 887n.12
- Winnington-Ingram, R.P., 308
- Wogram, Klaus, 122–124, 129, 132
- Wound strings
- Bass Canon, 39–40, 800, 805
 - break strength of, 40
 - coefficient of inharmonicity, 114–115
 - experiment, 61–62
 - inharmonicity, difficulties analyzing, 113
 - length, 36
 - making with String Winder, 805–807
 - mass per unit length, 36–37
 - composite, 36–38
 - of custom string, three different materials, 39–40
 - of piano string, two different materials, 40–41
 - of cylinder
 - hollow, 38
 - solid, 38
 - of stiffness parameter, 113–114
 - mode frequency, 27–28
 - stiffness parameter, 34, 113–114
 - tension, 36, 41

- of custom string, three different materials, 39–40
 - of piano string, two different materials, 40–41
 - tuning process, piano, 113
 - wrap wire, modified density
 - of bronze, 37
 - of copper, 40
- Wright, Owen, 622–625

Y

- Young
- Robert W., 99
 - Thomas, 181n.38(B)
- Young's modulus of elasticity
- of bar making materials, 885
 - definition, 21
 - elastic property, 4
 - of hardwood plywood, 221–222
 - heat effect on, 164, 177
 - of solids, 888
 - of spruce, 123, 885
 - grain direction factor, 123
 - of steel, Thomas Young, 181n.38(B)
 - of string making materials, 882
 - vibration test for, 180n.38(A)
 - rosewood bar, 172–173
 - water effect on, 172

Z

- Zalzal, Mansur, 636, 716, 755
- Al-Farabi's '*Mode of Zalzal*', 645
- Zarlino, Gioseffo, 93, 319, 333, 338–340, 342–343, 349, 377–379, 381–382, 384–387, 389–401, 403, 407–408, 428–429, 431–432, 434–436, 440–441, 446, 452, 553, 832
- Ziryab, 614